On the Design of 2×2 Full-Rate Full-Diversity Space-Time Block Codes

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***Abstract*— There has been considerable research on the design of space-time block codes (STBCs) that guarantee full diversity without sacrificing the data rate. The main challenge is to maximize the coding gain by maximizing the determinant criterion. It is shown that the most of previous STBCs with full rate and full diversity order (FRFD) (e.g. threaded algebraic space-time (TAST) codes) are constructed via a unitary generator matrix. However, the unitary matrix has been represented using only a small number parameters to enable algebraic code design. In this paper, for a 2×2 STBC, we use a more general unitary matrix with a large number of parameters for STBC design. We obtain an upper bound on the coding gain and show that the maximum coding gain is attainable only with PAM signaling. Since optimum parameters for the case of QAM signaling is analytically intractable, we search using the genetic algorithm (GA) method. We also use the union bound criterion for code parameter search by GA. Our simulation results show that with both criteria, the optimum code for QAM signaling is the Golden code. The proposed code significantly outperforms other existing STBCs with the gains about 2 dB at a symbol error rate of 10^{-3} for BPSK and 4-PAM. The proposed code performs identically to the Golden code for QAM.**

I. Introduction

The design of space-time block codes (STBCs) for Multiple-input multiple-output (MIMO) wireless systems has received intense attention recently. Full rate and full diversity (FRFD) codes for a system with two transmit and two receive antennas are constructed in [1] (called the $B_{2,\phi}$) using algebraic and number-theoretic methods. Although the $B_{2,\phi}$ code had been proposed for all modulation schemes including pulse amplitude modulation (PAM) and quadrature amplitude modulation (QAM) constellations of arbitrary size, it was optimized for 4-QAM and 16-QAM only. Following the $B_{2,\phi}$ code, threaded algebraic spacetime (TAST) codes have been developed for an arbitrary number of transmit antennas [2].

It is shown that linear STBC construction can be equivalently mapped to designing an unitary encoder matrix that meets the design criteria. Yao et al. [3] considered a simple subclass of unitary matrices with a block diagonal of two real rotation matrices. They determined the angles of rotation matrices such that it leads to non-vanishing coding gain for any size of constellation. Their proposed STBC achieves full diversity and full rate for any modulation schemes (PAM and QAM) but not necessarily with better performance (coding gain) than previously proposed codes for low order of modulation. Note that the $B_{2,\phi}$ and TAST codes were only optimized for low order modulations (e.g., 4-QAM, 16-QAM and 32-QAM).

Dayal et. al [4] proposed a STBC that exhibits a significant improvement in performance compared to the $B_{2,\phi}$, Yao's code or TAST codes. Interestingly at the same time, the Golden code was proposed by Belfiore et al. [5] based on the golden number using the machinery of algebraic number theory. The Golden code is equivalent to the code in [4] with the identical performance. The Golden code not only holds non-vanishing coding gain property (i.e. the coding gain is independent of the size of constellation) for all PAM or QAM constellations but also outperforms the previously known codes. The Golden code is constructed using a unitary encoder with only two parameters so that analytical derivation of code parameters becomes tractable. On the other hand, this raises the question: is it possible to improve the performance (coding gain) if a general unitary encoder matrix with a large number of parameters is used for code design?

The main focus of this paper is to investigate the above question. By using a 4×4 general unitary matrix which has 16 parameters, we obtain an upper bound on the coding gain. We show that the upper bound is attainable with the PAM signaling only (Note that the final transmitted signals are complex.) We propose a STBC that achieves the maximum coding gain that is not achieved by Golden code. Due to the large number of design parameters, the extraction of optimum parameters for the case of QAM signaling is analytically intractable. We exploit the genetic algorithm (GA) as an exceptional search method. We also include the union bound criterion along with the coding gain criterion for the genetic search. Our simulation results show that with both criteria, the optimum code for QAM signaling is the Golden code.

The following notations are used in this paper: $|x|$ denotes the absolute value of x, bold face $\mathbf{x}$ and $\mathbf{X}$ are used for vectors and Matrices respectively. $(\cdot)^H$ denotes the conjugate transpose $((\cdot)^T)$. The trace, determinant and Frobenius norm of matrix $\mathbf{A}$ are $\mathrm{tr}(\mathbf{A})$, $\det(\mathbf{A})$ and $\|\mathbf{A}\|_F^2 = \mathrm{tr}(\mathbf{A}\mathbf{A}^H)$. A circularly symmetric complex Gaussian random variable with mean μ and variance σ^2 is denoted by $z \sim \mathcal{CN}(\mu, \sigma^2)$.

II. System Model

Consider a system with M transmit and N receive antennas with signaling over a quasi-static fading channel. To transmit a symbol vector $\mathbf{s} = [s_1, \ldots, s_Q]^T$ where s_q's belong to a P-PAM or P^2-QAM constellation, i.e. $s_q = \alpha_q + j\beta_q$ and

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$\alpha_q, \beta_q \in \{-(P-1), -(P-3), \ldots, (P-1)\}$ (for PAM signals $\beta_q = 0$), a distinct $M \times T$ ST bock code (codeword) $\mathbf{X}$ is constructed and sent over the channel during T consecutive time intervals. The $N \times T$ complex received signal matrix $\mathbf{Y}$ will be

$$\mathbf{Y} = \sqrt{\frac{\rho}{ME_s}}\mathbf{HX} + \mathbf{W} \tag{1}$$

where the $N \times M$ fading channel matrix $\mathbf{H}$ and $N \times T$ additive noise matrix $\mathbf{W}$ have independent and identically distributed (i.i.d.) $\mathcal{CN}(0,1)$ elements. When E_s is the average energy of the transmit signals, ρ is the expected SNR for each received symbol. When the channel is known to the receiver the Maximum Likelihood (ML) decoding for each received block matrix is the the minimization of the Euclidean distance:

$$\hat{\mathbf{X}} = \arg\min_{\mathbf{X}} ||\mathbf{Y} - \sqrt{\frac{\rho}{M}}\mathbf{HX}||_F^2, \tag{2}$$

where the minimization is performed over all admissible codeword $\mathbf{X}$.

Although the derivation of general closed-form formula for exact pairwise error probability (PEP) is not widely reported, Tarokh et al. [6] have derived a good approximation of PEP for the asymptotically high SNR region. However, an exact PEP closed formula exists in the literature for a MIMO system with two transmit and two receive antennas [7]. Unlike the approximation of PEP, the exact formula is accurate over all range of SNR, particularly in the low SNR region. Given $\mathbf{X}_j$ is the decoded codeword instead of the transmitted codeword $\mathbf{X}_i$, and λ_i for $i = 1, 2$ is the i-th eigenvalue of the matrix $(\mathbf{X}_i - \mathbf{X}_j)(\mathbf{X}_i - \mathbf{X}_j)^H$, the exact PEP formula for equal eigenvalues case ($\lambda_1 = \lambda_2 = \lambda$) is expressed as [7]

$$P_e(\mathbf{X}_i \to \mathbf{X}_j) = (\frac{1-u}{2})^4 \sum_{k=0}^{3} \left[\binom{k+3}{k} (\frac{1+u}{2})^k \right] \tag{3}$$

where $u = \sqrt{\frac{\lambda\rho}{4M}/(1 + \frac{\lambda\rho}{4M})}$. For unequal eigenvalues case PEP is written as

$$\begin{aligned} P_e(\mathbf{X}_i \to \mathbf{X}_j) = {} & (\frac{\lambda_1}{\lambda_1 - \lambda_2})^2 \left[(\frac{1-u_1}{2})^2 (u_1 + 2) \right] \\ & + (\frac{\lambda_2}{\lambda_1 - \lambda_2})^2 \left[(\frac{1-u_2}{2})^2 (u_2 + 2) \right] \\ & - \frac{2\lambda_1^2\lambda_2}{(\lambda_1 - \lambda_2)^3}(\frac{1-u_1}{2}) + \frac{2\lambda_2^2\lambda_1}{(\lambda_1 - \lambda_2)^3}(\frac{1-u_2}{2}) \end{aligned} \tag{4}$$

where $u_1 = \sqrt{\frac{\lambda_1\rho}{4M}/(1 + \frac{\lambda_1\rho}{4M})}$ and $u_2 = \sqrt{\frac{\lambda_2\rho}{4M}/(1 + \frac{\lambda_2\rho}{4M})}$.

III. Existing Code Construction Criteria

A comprehensive design objective is to minimize the error rate at a given SNR and data rate. Since the minimization of the error rate is not always easy, one can use the related measures such as *coding gain* or *union bound*.

Typical STBC designs have focused the PEP between $\mathbf{X}_i$ and $\mathbf{X}_j$ as a measure of error performance. They minimize the maximum PEP among all pairs of codewords. This approach leads to the rank criterion and determinant criterion, which were proposed in [6] and [8].

In the determinant criterion, the minimum of $\prod_{j=1}^{r} \lambda_j$, taken over all possible codeword pairs $\mathbf{X}_i$ and $\mathbf{X}_j$, is the coding gain and must be maximized where r is the rank of the matrix $(\mathbf{X}_i - \mathbf{X}_j)$. If $M = T$, the coding gain δ is defined as

$$\delta = \min_{\mathbf{X}_i, \mathbf{X}_j} |\det(\mathbf{X}_i - \mathbf{X}_j)|. \tag{5}$$

On the other hand, it has been established that the union bound reflects the actual error performance closer than the coding gain [9]. The union bound (UB) is an upper bound on error probability where the average is taken over all possible transmitted codewords. When all codewords are equally likely to be transmitted, the UB may be defined as [10]

$$P_U = \frac{1}{L} \sum_{\mathbf{X}_i} \sum_{\mathbf{X}_j \neq \mathbf{X}_i} r(\mathbf{X}_i, \mathbf{X}_j) P_e(\mathbf{X}_i \to \mathbf{X}_j). \tag{6}$$

Here L is the number of possible codwords and $r(\mathbf{X}_i, \mathbf{X}_j)$ is the error rate from $\mathbf{X}_i$ to $\mathbf{X}_j$ in terms of bit error rate (BER) or symbol error rate (SER). Therefore it is advisable to minimize the union bound rather than maximum PEP or maximize coding gain. In a 2×2 MIMO systems, $P_e(\mathbf{X}_i \to \mathbf{X}_j)$ can be replaced by either exact PEP in (3) and (4) or by an approximation of PEP in [6]. However, it is shown that the latter union bound is a looser bound for both BER and SER [7]. Although minimizing the union bound makes more sense than maximizing the coding gain, the computation complexity of calculating the coding gain is much less than computing the union bound. In this paper, we consider both criteria for completeness of our investigation.

IV. FRFD Space-Time codes

Most of the previous 2×2 FRFD codes are linear, i.e. for each transmission, based on the independent input data bits, a quadruple of symbols are carved from a PAM or QAM constellation and form the vector $\mathbf{s}_i = [s_{i,1}, s_{i,2}, s_{i,3}, s_{i,4}]^T$. This vector is transformed (rotated) by the unitary encoder matrix $\mathcal{M}$ to produce

$$\mathbf{x}_i = [x_{i,1}, x_{i,2}, x_{i,3}, x_{i,4}]^T = \mathcal{M}\mathbf{s}_i \tag{7}$$

and the rate 2 (symbol per channel use) transmit matrix $\mathbf{X}_i$ is generated as

$$\mathbf{X}_i = \begin{bmatrix} x_{i,1} & x_{i,3} \\ x_{i,4} & x_{i,2} \end{bmatrix} \triangleq \mathrm{mat}(\mathbf{x}_i) \tag{8}$$

where the function $\mathrm{mat}(\mathbf{b})$ shapes the 4×1 vector $\mathbf{b}$ into a 2×2 matrix as shown in (8). The encoding matrix $\mathcal{M}$ that completely specifies the STBC, is restricted to unitary matrices i.e. $\mathcal{M}^H\mathcal{M} = \mathbf{I}$ subject to the transmit power constraint. Note that one may relax the unitary condition on the encoder matrix to $\mathrm{tr}\{\mathcal{M}^H\mathcal{M}\} = 4$ which also meets the power constraint and may obtain better codes [11]. The tradeoff is that in the unitary encoder case, the total transmit power in each signaling interval is fixed, while in the second scenario only the total transmit power in both signaling intervals is fixed not

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in each interval. Therefore, the code constructed by the second scenario may have better performance but will have pick to average power ratio problems in practical implementations. We focus on the case that the encoder matrix in unitary and so the STBC design problem is reduced to find the optimum encoder matrix $\mathcal{M}$ such that it meets the code construction criteria. For the coding gain criterion, by substituting (8) into (5), we have:

$$\delta_{ij} = |\det(\mathbf{X}_i - \mathbf{X}_j)| = |\mathcal{M}\mathbf{u}| \tag{9}$$

where $\mathbf{u} \triangleq \mathbf{s}_i - \mathbf{s}_j$. Note that the real and imaginary parts of elements of $\mathbf{u}$ are always even integers and also all elements of $\mathbf{u}$ can not be zero simultaneously ($\mathbf{u} \neq \mathbf{0}$).

Proposition 1: The maximum possible coding gain for all PAM and QAM constellations is $\delta_{max} = 2$, where a unitary encoder matrix is used to construct the transmit matrix.

Proof: As a special case in (9), assume $\mathbf{u} = [u_1, 0, 0, 0]^T$ and the encoder matrix $\mathcal{M}$ has the form of $\mathcal{M} = [a_{ij}e^{j\alpha_{ij}}]$, $i, j = 1, \ldots, 4$ where $0 \leqslant a_{ij} \leqslant 1$ and $0 \leqslant \alpha_{ij} < 2\pi$. Thus,

$$\begin{aligned}\delta_{ij} &= \left|u_1^2 a_{11}a_{21}e^{j(\alpha_{11}+\alpha_{21})} - u_1^2 a_{31}a_{41}e^{j(\alpha_{31}+\alpha_{41})}\right| \\ &= |u_1|^2 \left|a_{11}a_{21} - a_{31}a_{41}e^{j\alpha}\right|\end{aligned}$$

where $\alpha = \alpha_{31} + \alpha_{41} - \alpha_{11} - \alpha_{21}$. Since $\min u_1 = 2$,

$$\delta = \min_{u_1} \delta_{ij} = \left|a_{11}a_{21} - a_{31}a_{41}e^{j\alpha}\right|$$

Since $a_{ij} \geqslant 0$, δ is maximized when $\alpha = \pi$ or equivalently

$$\delta_{max} = \max_{a_{11}^2+a_{21}^2+a_{31}^2+a_{41}^2=1} |a_{11}a_{21} + a_{31}a_{41}| \tag{10}$$

where the condition is because the norm of each column of unitary matrix $\mathcal{M}$ is one. By partial differentiation, it is easy to verify that $\delta_{max} = 2$ and this maximum is obtained when

$$a_{11} = a_{21}\,,\ a_{31} = a_{41}\,,\ a_{11}^2 + a_{31}^2 = \frac{1}{2} \tag{11}$$

■

The unitary encoder matrix $\mathcal{M}$ has 16 independent parameters that should be optimized based on the coding gain or union bound criteria. We applied the well-known genetic algorithm to find the optimum solutions. Our simulation results show that the optimum code obtained with all 16 parameters of $\mathcal{M}$ is the same as the optimum code obtained by using the block diagonal unitary structure defined as

$$\begin{aligned}\mathcal{M} = &\operatorname{diag}(e^{j\phi_{11}}, e^{j\phi_{12}}, e^{j\phi_{13}}, e^{j\phi_{14}})\times \\ &\operatorname{diag}(RF(\theta_1), RF(\theta_2))\times \\ &\operatorname{diag}(e^{j\phi_{21}}, e^{j\phi_{22}}, e^{j\phi_{23}}, e^{j\phi_{24}})\end{aligned} \tag{12}$$

where

$$RF(\theta_i) = \begin{bmatrix}\cos(\theta_i) & -\sin(\theta_i)\\ \sin(\theta_i) & \cos(\theta_i)\end{bmatrix}$$

and θ_i's and ϕ_{ij}'s are the rotation and phase parameters, respectively. Note that the unitary structure in (12) potentially can satisfy the conditions in (11) to reach the maximum coding gain.

Proposition 2: Considering the encoder matrix $\mathcal{M}$ defined in (12), the parameters $\phi_{11}, \phi_{12}, \phi_{14}, \phi_{21}$ and ϕ_{23} can set to be zero without any effect on the coding gain in (5).

Proof: Using (12) and (9) and factoring, we obtain:

$$\begin{aligned}\delta_{ij} = \Big| &\left(u_1\cos(\theta_1) - u_2\sin(\theta_1)\,e^{j(\phi_{22}-\phi_{21})}\right) \\ &\left(u_1\sin(\theta_1) + u_2\cos(\theta_1)\,e^{j(\phi_{22}-\phi_{21})}\right) - \\ &e^{j(\phi_{13}+\phi_{14}+\phi_{23}-\phi_{11}-\phi_{12}-\phi_{21})} \\ &\left(u_3\cos(\theta_2) - u_4\sin(\theta_2)\,e^{j(\phi_{24}-\phi_{23})}\right) \\ &\left(u_3\sin(\theta_2) + u_4\cos(\theta_2)\,e^{j(\phi_{24}-\phi_{23})}\right)\Big|\end{aligned}$$

Clearly, without loss of generality, we can set:

$$\phi_{11} = \phi_{12} = \phi_{14} = \phi_{21} = \phi_{23} = 0$$

and by just keeping the rest of angles and defining $\phi \triangleq \phi_{13}, \beta_1 \triangleq \phi_{22}$ and $\beta_2 \triangleq \phi_{24}$, we can summarize that

$$\begin{aligned}\delta_{ij} = | \quad &\left(u_1\cos(\theta_1) - u_2\sin(\theta_1)e^{j\beta_1}\right) \\ &\left(u_1\sin(\theta_1) + u_2\cos(\theta_1)e^{j\beta_1}\right) - \\ e^{j\phi} &\left(u_3\cos(\theta_2) - u_4\sin(\theta_2)e^{j\beta_2}\right) \\ &\left(u_3\sin(\theta_2) + u_4\cos(\theta_2)e^{j\beta_2}\right) |\end{aligned} \tag{13}$$

where the equivalent encoder matrix is

$$\begin{aligned}\mathcal{M} = &\operatorname{diag}(1, 1, e^{j\phi}, 1) \\ &\operatorname{diag}(RF(\theta_1), RF(\theta_2)) \\ &\operatorname{diag}(1, e^{j\beta_1}, 1, e^{j\beta_2}).\end{aligned} \tag{14}$$

■

Note that the encoder matrix (14) is a general form of all encoders presented in the literature to design 2×2 FRFD codes. For instance, if $\phi = \beta_1 = \beta_2 = 0$, (14) is equivalent to the block diagonal encoder in [3], or if $\theta_1 = \theta_2$ and $\beta_1 = \beta_2 = 0$, (14) is equivalent to the unitary structure of the Golden code presented in [5] and [4]. The design goal is thus to find the optimum set of parameters $\{\phi, \theta_1, \theta_2, \beta_1, \beta_2\}$, which yields the largest coding gain or the smallest union bound (6) depending on the constellation.

Proposition 3: The optimum design parameters in (14) for all PAM constellations are $\theta_1 = \theta_2 = \pi/4$, $\beta_1 = \beta_2 = \pi/2$ and $\phi = \pi$. In this case $\delta = \delta_{max} = 2$.

Proof: For all PAM constellations, the elements of $\mathbf{u} = \mathbf{s}_i - \mathbf{s}_j$ are even integers so that $u_i^2 = |u_i|^2$. On the other hand, Proposition 1 clearly states in (11) that $\theta_1 = \theta_2 = \pi/4$ is the necessary condition to reach the δ_{max}. In this case, from (13) we have:

$$\delta_{ij} = \frac{1}{2}\left||u_1|^2 - |u_2|^2 e^{j2\beta_1} - |u_3|^2 e^{j\phi} + |u_4|^2 e^{j(2\beta_2+\phi)}\right|$$

By taking $\beta_1 = \beta_2 = \pi/2$ and $\phi = \pi$,

$$\delta_{ij} = \frac{1}{2}\left||u_1|^2 + |u_2|^2 + |u_3|^2 + |u_4|^2\right| \geqslant 2$$

TABLE I
IMPLEMENTATION PARAMETERS FOR GA

Population size(p)	700	Stall generation limit	200
Time Limit	5 hours	Crossover fraction	0.8
Mutation type	Uniform	Crossover type	Scattered
N_g	1e5	Parameters bound	LB=0, UB=2π

where the inequality comes from the fact that $|u_i| \geqslant 2$ for nonzero u_i. Finally, $\delta = \min \delta_{ij} = 2$. ∎

Proposition 4: For all QAM constellations, $\delta_{max} = 2$ is not attainable

Proof: is omitted due to limited space. It appear in the journal version. ∎

V. OPTIMUM PARAMETERS FOR QAM SIGNALING

Analytical derivation of the optimum parameters in (14) is intractable for QAM constellations. Moreover, due to the number of parameters and the range of parameters in (14), exhaustive search seems to be practically impossible. Therefore, we resort to the use of *Genetic Algorithm.*

A. Genetic Algorithm

The genetic algorithm [12] is an exceptional search technique inspired from biological processes for finding the optimum solutions to the optimization and search problems. Generation in each iteration inherits properties from the best precedent solutions. In each iteration, a population of p solutions which are interpreted as the *parents* are considered. Assume $\mathcal{Z}_t$ denotes the parent population in iteration t:

$$\mathcal{Z}_t = \{z_t^1, \cdots, z_t^p\} \tag{15}$$

where z_t^i is a possible solution in optimization function. In order to form a possible solution manipulated by the GA, the optimization parameters should be represented in binary format and concatenated to each other such that a string of 0s and 1s is formed. A fitness function (cost function) is also required to measure the quality of solutions. The Fitness function in our case would be either the coding gain in (5) or the union bound in (6). From set (15), c new solutions are generated which are interpreted as the children. The children population may be expressed as

$$\overline{\mathcal{Z}}_t = \{\overline{z}_t^1, \cdots, \overline{z}_t^c\}. \tag{16}$$

Generally, there are two ways to generate the children:
(a) Crossover: This type of children are created by swapping parts of two parents. Scattered crossover technique is the most cited crossover method. This technique first creates a random binary vector with the same size as parents. Then if the i-th bit of generated random vector is 0, corresponding gene (bit) is selected from the first parent, otherwise it is selected from the second parent. Ultimately, all selected genes are combined to form a child.
(b) Mutation: Mutation operator creates new children by randomly changing the bits (genes) of each solution in the set $\mathcal{Z}_t$. In order to mute a parent, a fraction of bits of that parent is selected and replaced by new uniformly generated random bits. The aim of mutation and crossover in the GA is to allow the algorithm to avoid local optima by preventing the population from becoming too similar to each other.

After generating a population $\overline{\mathcal{Z}}_t$, the new population of parents $\mathcal{Z}_{t+1}$ for $(t+1)$'s iteration must be chosen. Typically, p individuals (solutions) out of $p+c$ parents and children who have higher fitness values are selected to make the new parent generation $\{z_{t+1}^1, \cdots, z_{t+1}^p\}$.

If any of the following conditions happens, GA stops.
(1) Allocated time is reached (Time Limit),
(2) The specified number of generation is reached (N_g),
(3) There is no improvement in the objective function for certain number of successive iterations (stall generation). Other implementation parameters, which were used in our program, have been presented in Table I. Interested reader may refer to reference [12] for more details.

TABLE II
OPTIMUM PARAMETERS OF UNITARY ENCODER MATRIX $\mathcal{M}$ FOR BPSK, 4-PAM AND 4-QAM.

Signaling	ϕ	$[\beta_1, \beta_2]$	$[\theta_1, \theta_2]$
BPSK / Min P_{UB}	59.031	$[90.007, 60.757]$	$[44.723, 134.99]$
4-PAM / Min P_{UB}	65.152	$[90.146, 147.56]$	$[45.178, 44.914]$
4-QAM / Max δ	264.12	$[89.989, 2.920]$	$[31.705, 211.69]$
4-QAM / Min P_{UB}	286.49	$[89.984, 150.93]$	$[353.23, 96.5]$

TABLE III
COMPARISON OF THE CODING GAIN OF SOME FRFD STBCS WITH $M = 2, N = 2, T = 2$

Signaling	δ ST code [3]	δ ST code [5]	δ-Our code
BPSK	0.8944	1.7889	2.000
4-PAM	0.8944	1.7889	2.000
4-QAM	0.8944	1.7889	1.782

B. Optimum parameters for QAM from GA

Table II shows the parameter results found by the GA search based on maximizing the coding gain and minimizing the union bound for the proposed ST code in (8) with the encoder matrix in (14). For comparison, the maximum coding gains of the previous STBCs and the proposed code are presented in Table III for BPSK, 4-PAM and 4-QAM. Results show that our proposed code achieves the maximum possible coding gain δ_{max} for all PAM constellations. This gain is larger than the coding gains reported previously in the literature [3], [5]. On the other hand, for QAM signaling, our results are better than [3] but the same as the Golden code [5].

VI. SIMULATION RESULTS

In this section, we provide simulation results for a multiple antenna system with 2 transmit antennas, 2 receive antennas and 2 signaling intervals, using the proposed FRFD STBC in 12 and previously proposed FRFD codes in [4], [13], [3] and [5].

Fig (1) shows the performance of the proposed code, the Golden code [5], [4], and Yao's code [3] for BPSK and 4-PAM. As expected, the proposed code outperforms significantly other

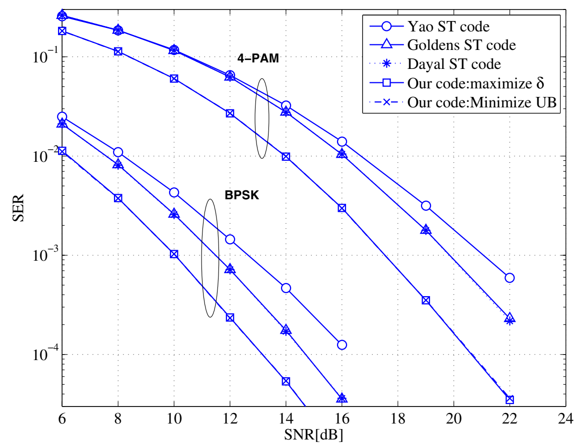

Fig. 1. The symbol error rate of different FRFD ST codes for BPSK and 4-PAM signalings.

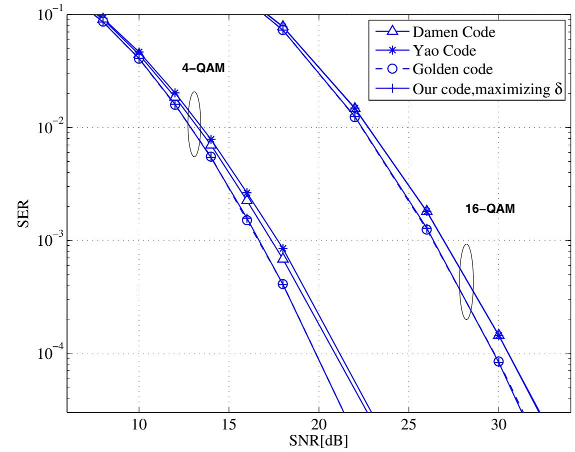

Fig. 2. The symbol error rate of different ST Codes for 4-QAM and 16-QAM signalings.

existing ST codes with the gains of almost 1.5 dB and 2 dB at the SER of 10^{-3} for BPSK and 4-PAM, respectively. Note that the performance of Golden code [5] and Dayal's code [4] are the same.

Fig (2) compares the performance of our proposed code 12 with the TAST code in [13], the Golden code [5] and Yao's code [3] for 4-QAM and 16-QAM signalings. The proposed code performs the same as the Golden code [5] and outperforms the codes presented in [13] and [3].

VII. Conclusion

In this paper, we proposed a new FRFD STBC for a system with two transmit and two receive antennas. This code can be extended to a larger number of transmit antennas by stacking it horizontally or vertically [4]. This code structure is more general than existing codes in that all the previous STBC designs are found to be special cases of it. We analytically determined the optimal parameters in the STBC structure (14) that maximize the coding gain for all PAM constellations. The maximum possible coding gain with any unitary encoder matrix $\mathcal{M}$ is $\delta_{max} = 2$ which is attainable only in PAM. Due to the number of parameters in (14), we introduced the genetic algorithm as a powerful optimization method to find the best parameters for QAM since analytical optimization is intractable. Based on theoretical values of coding gain and simulation results, the proposed STBC offers considerable performance improvement in comparison to the other known PAM codes. (Note that the final transmitted symbols are complex though.) In the case of QAM, the proposed code shows the same performance as the Golden code. Although we considered two distinct criteria for optimization, i.e. minimizing the union bound and maximizing the coding gain, simulation results show that the codes designed from both criteria perform almost the same. Therefore, due to its lower computational complexity, the coding-gain based optimization is preferable for obtaining optimal codes.

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