A Technique for Multiuser and Intercarrier Interference Reduction in Multiple-Antenna Multiuser OFDM Downlink

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***Abstract*— We propose a two-stage precoder/equalizer to suppress intercarrier interference (ICI) and multiuser interference (MUI) in downlink multiuser OFDM with multiple transmit antennas. The first stage, non-linear Tomlinson-Harashima precoding (THP) at the base station (BS) transmitter, mitigates the effect of the spatial inter-stream interference caused by transmission from multiple transmit antennas to decentralized users. In the second stage, each user's receiver employs low-complexity iterative linear minimum mean-square error (MMSE) equalization to suppress the ICI due to frequency offset. Our proposed technique virtually eliminates the bit error rate (BER) degradation due to normalized frequency offsets as high as 10%.**

***Index Terms*— Multiuser OFDM, multiple antennas, intercarrier interference (ICI), multiuser interference (MUI), Tomlinson-Harashima precoding (THP), minimum mean-square error (MMSE) equalization.**

I. Introduction

CURRENT trends in wireless system design focus on the use of multiple-input multiple-output (MIMO) links to provide capacity gains, orthogonal frequency-division multiplexing (OFDM) to facilitate the utilization of these gains on frequency-selective channels, and closed-loop techniques to offer a bit error rate (BER) improvement [1]. Closed-loop multiuser MIMO OFDM systems are of interest in this letter. In multiuser OFDM, frequency offset, which may be caused by mismatch of oscillators, and/or the Doppler effect due to users' mobility, leads to intercarrier interference (ICI) and multiuser interference (MUI), and results in an increase in the system BER. This problem has been addressed in [2] and [3] for the uplink, but remains to be solved for the downlink case, characterized by the difficulty of signal detection caused by lack of coordination among independent mobile users.

This letter focuses on ICI and MUI suppression for the downlink (broadcast channel) of multiuser spatially-multiplexed OFDM. We consider a typical closed-loop system, in which the base station (BS) knows the spatial channel gain matrix $\mathbf{H}$. We show (see Section II) that the overall channel matrix can be separated into two parts: $\mathbf{H}$ and the interference matrix $\mathbf{S}$ determined by the frequency offsets. In the downlink case, each mobile station (MS) knows the frequency offset and channel response affecting its receiver only, but not those of other users. Due to the lack of coordination among receiving users, spatial layer separation is not possible at the receiver end for the usual case of transmission to several users simultaneously. On the other hand, since the users are decentralized, it may be difficult to know at the BS the frequency offsets of all users. In the time-division duplex (TDD) mode the channel matrix $\mathbf{H}$ can be estimated at the transmitter but the frequency offset information has to be fed back to the transmitter on a separate channel. In frequency-division duplex (FDD) systems, both frequency offset and channel gains have to be fed back. Not only is the feedback channel capacity limited, but also imperfect feedback results in frequency offset and/or channel gains mismatch, i.e., the values of frequency offsets/channel gains available at the BS are different from the actual values at the time of transmission, which increases the BER. Hence, the application of only transmitter precoding for ICI and MUI mitigation, which requires full channel state information (CSI) including both $\mathbf{H}$ and $\mathbf{S}$ at the BS, is problematic.

A novel two-stage technique for ICI and MUI suppression is thus proposed, in which the first stage applies non-linear Tomlinson-Harashima precoding (THP) [4]–[7] at the BS transmitter to mitigate the MUI and the second stage employs an iterative minimum mean-square error (MMSE) equalizer at each user's receiver to suppress the ICI due to the frequency offset. The spatial channel gain matrix $\mathbf{H}$ is available at the BS in a typical closed-loop system. Since the precoder only needs $\mathbf{H}$ at the BS, the feedback load is reduced. The MMSE equalizer at each user's receiver has low complexity due to the unitary property of the ICI matrix demonstrated in [8]. Our scheme significantly reduces the BER increase due to frequency offsets in closed-loop multiuser spatially-multiplexed OFDM downlink. When the feedback link is perfect, the proposed technique almost completely cancels the ICI and MUI, and experiences the same BER as in the case when only full-CSI (channel gains and frequency offset) precoding is used at the BS. Hence, sending frequency offset information to the BS does not offer additional BER improvement. When the feedback is inaccurate, our technique outperforms the case of full-CSI feedback since we avoid the possible frequency-offset mismatch.

This letter is organized as follows. In Section II we describe a multiple-antenna multiuser spatially-multiplexed OFDM system model with frequency offsets. Section III proposes a two-stage precoder/equalizer for ICI and MUI suppression. Simulation results are given in Section IV. Section V concludes the letter.

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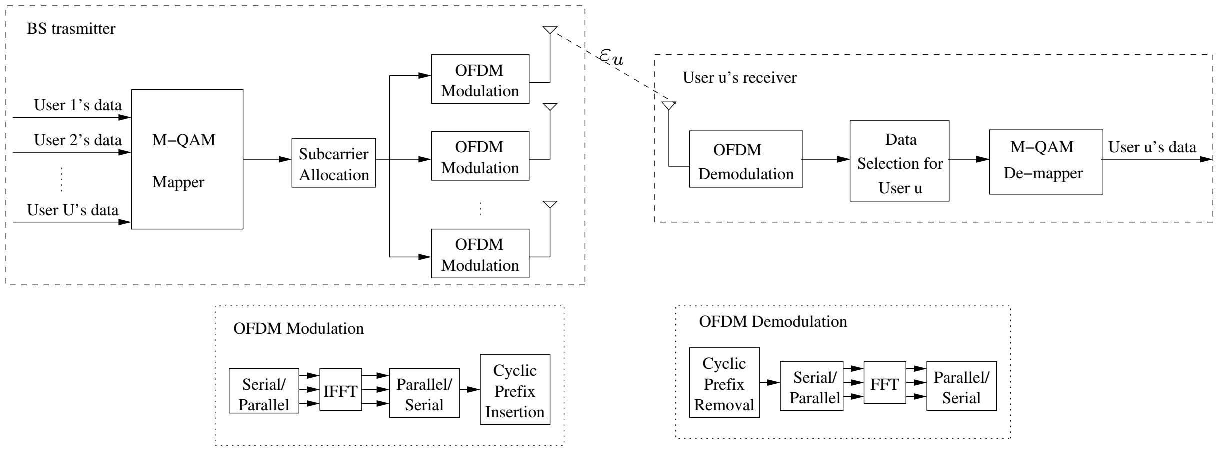

Fig. 1. Block diagram of a multiuser MIMO OFDM downlink.

II. System Model

We consider a multiuser OFDM system employing N subcarriers with M_T transmit antennas and U simultaneously active users. Each user has a single receive antenna and the u-th user is assigned a subset $\mathbb{K}_u$ containing $N_u \le N$ subcarriers. The same subcarriers can be used for transmission to different users (spatial multiplexing). The transmitter and the u-th user's receiver are shown in Fig. 1. At the BS, a subcarrier allocation algorithm maps the user data to the corresponding subcarriers, and this algorithm is known at both the BS and the user side. The cyclic prefix is assumed longer than the expected maximum excess delay, eliminating intersymbol interference (ISI).

We consider a wideband frequency-selective fading channel with L resolvable paths, which we assume constant during at least one OFDM symbol interval T_s. The discrete-time domain received signal can be represented as

$$y_{v,u}(k) = e^{\jmath\frac{2\pi}{N}\varepsilon_u k}\sum_{l=0}^{L-1} h_{v,u}(l)x_v(k-l) + w_{v,u}(k), \quad (1)$$

where $k = 0, \ldots, N-1$, $v = 1, \ldots, M_T$ and $u = 1, \ldots, U$; $\varepsilon_u = \Delta f_u T_s$ is the normalized frequency offset; the Δf_u is the u-th user's frequency offset. Since different users experience different fading channels and may move at different speeds, we assume $\varepsilon_u \ne \varepsilon_{u'}$, $\forall\ u \ne u'$. $w_{v,u}(k)$ is an additive white Gaussian noise (AWGN) sample. The complex channel gain $h_{v,u}(l), l = 0, \ldots, L-1$, refers to the l-th path between the u-th user and v-th transmit antenna. Each path gain is a zero-mean complex Gaussian random variable (Rayleigh fading) with variance σ_l^2. $x_v(m)$ is the m-th entry of the time-domain signal vector $\mathbf{x}_v = \mathcal{F}_N\mathbf{X}_v$, where $\mathcal{F}_N$ is the $N \times N$ IDFT matrix with entries $\mathcal{F}(m,n) = \frac{1}{N}e^{\jmath\frac{2\pi}{N}mn}$. $\mathbf{X}_v = \left[X_v[0] \ldots X_v[N-1]\right]^T$ is the length-N input data vector where $X_v[k]$ is the k-th M-ary quadrature amplitude modulation (QAM) symbol sent by the v-th transmit antenna. The received signal for the k-th subcarrier of the u-th user can be expressed as

$$\begin{aligned} Y_u[k] = &\underbrace{\sum_{v=1}^{M_T} S_u[0]H_{v,u}[k]X_v[k]}_{\text{desired signal}} + \underbrace{\sum_{v=1}^{M_T}\sum_{\substack{n\ne k,\\ n=n_1}}^{n_{Q_u}} S_u[n-k]H_{v,u}[n]X_v[n]}_{\text{ICI}} \\ &+ \underbrace{\sum_{v=1}^{M_T}\sum_{\substack{n\ne k,\\ n=j_1}}^{j_J} S_u[n-k]H_{v,u}[n]X_v[n]}_{\text{MUI}} + W_u[k], \end{aligned} \quad (2)$$

where n_q is the index of the subcarriers assigned only to the u-th user, belonging to the subset $\mathbb{Q}_u = \{n_q,\, q = 1, \ldots, Q_u,\ Q_u \le N_u\} \subset \mathbb{K}_u$; $j_p \in \overline{\mathbb{Q}}_u$ is the index of subcarriers assigned to other users; $\overline{\mathbb{Q}}_u$ is the complementary set of $\mathbb{Q}_u$. $H_{v,u}[k] = \sum_{l=0}^{L-1} h_{v,u}[l]e^{-\jmath\frac{2\pi}{N}lk}$; $W_u[k]$ is an AWGN sample with zero mean and variance $\sigma_{W_u}^2$. Similarly to notation used in [8], [9], $S_u[m]$ is an interference coefficient of the u-th user, given by

$$S_u[m] = \sum_{n=0}^{N-1} e^{\jmath\frac{2\pi}{N}n(\varepsilon_u+m)} = \frac{\sin\pi(\varepsilon_u+m)}{N\sin\frac{\pi}{N}(\varepsilon_u+m)}e^{\jmath\pi(1-\frac{1}{N})(\varepsilon_u+m)}. \quad (3)$$

For a multiuser OFDM system with multiple transmit antennas and the transmitted vector $\mathbf{X} = \left[\mathbf{X}_1^T \ldots \mathbf{X}_{M_T}^T\right]^T$ the received signal vector of the u-th user is

$$\mathbf{Y}_u = \mathbf{S}_u\mathbf{H}_u\mathbf{X} + \mathbf{W}_u, \quad (4)$$

where $\mathbf{S}_u$ is an $N \times N$ interference matrix with the $\{m,n\}$th entry $S_u[n-m], n, m = 0, \ldots, N-1$; the $\{m,n\}$th entry is an MUI coefficient if $n \in \overline{\mathbb{Q}}_u$, otherwise it is an ICI coefficient. $\mathbf{S}_u$ is given by (7) in [8], [9], which is repeated here for convenience

$$\mathbf{S}_u = \begin{bmatrix} S_u[0] & S_u[1] & \ldots & S_u[N-1] \\ S_u[-1] & S_u[0] & \ldots & S_u[N-2] \\ \vdots & & \ddots & \vdots \\ S_u[-(N-1)] & S_u[-(N-2)] & \ldots & S_u[0] \end{bmatrix}. \quad (5)$$

Therefore $\mathbf{S}_u$ is unitary as shown in [8], [9]. $\mathbf{H}_u = \left[\mathbf{H}_{1,u} \ \ldots \ \mathbf{H}_{M_T,u}\right]$ is the channel gain matrix of the u-th user, where $\mathbf{H}_{v,u}$ is a diagonal matrix $\mathbf{H}_{v,u} =$

diag $\left[H_{v,u}[0]\ \ldots\ H_{v,u}[N-1]\right]$. $\mathbf{W}_u$ is the noise vector received by the u-th user. The received signals of all users can hence be expressed as

$$\left[\mathbf{Y}_1\ \ldots\ \mathbf{Y}_U\right]^T = \mathbf{SHX} + \mathbf{W}, \tag{6}$$

where $\mathbf{S} = \text{diag}\left[\mathbf{S}_1\ \ldots\ \mathbf{S}_U\right]$, and $\mathbf{H}$ is the $NU \times NM_T$ channel gain matrix, including channel gains of all user channels:

$$\mathbf{H} = \left[\mathbf{H}_1 \quad \mathbf{H}_2 \quad \ldots \quad \mathbf{H}_U\right]^T. \tag{7}$$

Obviously, the ICI and MUI components in (2) include frequency offsets and channel gains. We thus need to suppress the impact on ICI and MUI due to both $\mathbf{H}$ and $\mathbf{S}$.

III. MUI and ICI Reduction

¿From the observation $\mathbf{Y}_u$ in (4), each user attempts to detect the transmitted symbols $\mathbf{X}[k]$, $\forall\, k \in \mathbb{K}_u$. We propose a non-linear TH precoder at the BS to mitigate the ICI and MUI components due to the spatial channel gain matrix $\mathbf{H}$, and a linear equalizer at each user's receiver to suppress the remaining ICI and MUI due to frequency offset. With proper design, the transmitted data symbols can be directly detected from the equalized samples.

A. Non-linear Tomlinson-Harashima precoding

We propose the non-linear TH precoder [4]–[7] as the first stage to mitigate the impact of the spatial channel gain matrix $\mathbf{H}$ (7) at the BS. THP can be interpreted as moving the feedback part of decision feedback equalization (DFE) to the transmitter, eliminating error propagation and achieving better performance. Furthermore, non-linear THP outperforms linear precoding since it avoids possible unbounded increases in the average transmit power. For detection convenience, we assume that the number of transmit antennas is greater than or equal to the number of users, i.e., $M_T \geq U$.

The structure of the TH precoder in multiuser OFDM downlink is shown in Fig. 2. Because the users are distributed, the received symbols $Y_u[k]$, $\forall\ u$, cannot be processed jointly by the feedforward receiver filter. We hence move the feedforward matrix $\mathbf{D}$ to the transmitter side as in [6]. With the knowledge of channel responses of all users at the transmitter, the interference caused by $\mathbf{H}$ can be completely suppressed from the received samples. The channel response estimates are available at the BS in TDD systems, or they can be sent back to the BS via a feedback link in FDD systems. Given the spatial channel matrix $\mathbf{H}$ (7), we design the feedforward matrix $\mathbf{D}$, the scaling matrix $\mathbf{P}$ and the feedback matrix $\mathbf{B}$ using the zero-forcing (ZF) criterion. A QR factorization [10] of the channel matrix yields $\mathbf{H} = \mathbf{TD}^H$ where $\mathbf{T}$ is an $NU \times NU$ lower triangular matrix with the $\{m,n\}$th entry $T(m,n)$; $\mathbf{D}$ is an $NM_T \times NU$ matrix and $\mathbf{D}^H\mathbf{D} = \mathbf{I}_{NU}$. The complexity of QR decomposition increases as the number of users, subcarriers and/or transmit antennas grow. A complexity-reduced QR decomposition can be used for a large-size channel matrix [11]. The feedforward filter $\mathbf{D}$ is applied at the BS side. The $NU \times NU$ scaling matrix is $\mathbf{P} = \text{diag}\left[1/T(1,1)\ \ldots\ 1/T(NU,NU)\right]$, and the feedback matrix $\mathbf{B} = \mathbf{PT}$. The scaling matrix keeps the average transmit power constant. If there is no modulo device, the output vector of the feedback filter is $\mathbf{X} = \mathbf{B}^{-1}\mathbf{A}$, and $\mathbf{DX}$ is transmitted instead of $\mathbf{A}$, where $\mathbf{A} = \left[\mathbf{A}_1^T\ \ldots\ \mathbf{A}_{M_T}^T\right]^T$ is the input data vector. The entries in $\mathbf{A}_v = \left[a_v[0]\ \ldots\ a_v[N-1]\right]^T$ are from the original M-ary QAM constellation.

THP employs modulo operation at both the transmitter and the receiver. The modulo $2\sqrt{M}$ reduction at the BS, which is applied separately to the real and imaginary parts of the input, is to restrict the transmitted signals to within the region $\Re\{X[k]\} \in \left(-\sqrt{M}, \sqrt{M}\right]$ and $\Im\{X[k]\} \in \left(-\sqrt{M}, \sqrt{M}\right]$. If the input sequence $a[k]$ is a sequence of i.i.d. samples with variance E_s, the output of the modulo device is also a sequence of i.i.d. random variables with variance E_x, and the real and imaginary parts are independent [7]. E_x is slightly larger than E_s, but as the constellation size M increases, the difference between E_s and E_x becomes negligible. At the receiver, the received signal-to-noise power ratio (SNR) on the subcarrier k becomes $\frac{E_s}{\sigma^2_{W'_k}}$, where $\sigma^2_{W'_k} = P[k]\sigma^2_{W'}$, i.e., each subcarrier may have a different SNR. A slicer, which applies the same modulo operation as that at the transmitter, is used at the receiver. After discarding the modulo congruence, the unique estimates of the data symbols $\hat{a}[k]$ can be generated. Further details of the operation of the TH precoder can be found in [6] and [7].

Because only $\mathbf{H}$ is needed at the BS, the feedback capacity requirement is reduced. Since the linear pre-distortion via $\mathbf{B}^{-1}$ equalizes the cascade $\mathbf{PHD}$, the TH precoder in multiuser OFDM completely suppresses the interference caused by $\mathbf{H}$.

B. Iterative ICI and MUI equalization

This subsection introduces iterative linear MMSE equalization to suppress the remaining MUI and ICI due to frequency offset at the individual receiver level. We assume that the u-th user knows its frequency offset, ε_u, and its channel gains $h_{v,u}(l)$, $\forall\, l$.

After modulo reduction at the users' receivers, as shown in Fig. 2, the received signals become:

$$\begin{aligned}\left[\mathbf{Y}_1\ \ldots\ \mathbf{Y}_U\right]^T &= \mathbf{PSHDB}^{-1}\mathbf{A} + \mathbf{PW} \\ &= \mathbf{PSTD}^H\mathbf{DT}^{-1}\mathbf{P}^{-1}\mathbf{A} + \mathbf{PW} \\ &= \mathbf{PSP}^{-1}\mathbf{A} + \mathbf{PW}.\end{aligned} \tag{8}$$

Clearly, the spatial channel gain matrix $\mathbf{H}$ becomes a diagonal matrix, i.e., the spatial channel has been converted into NU parallel, independent sub-channels. Since $\mathbf{S} = \text{diag}\left[\mathbf{S}_1\ \ldots\ \mathbf{S}_U\right]$ and $\mathbf{P}$ is a diagonal matrix, the received signal vector of the u-th user in (4) is reduced to the single transmit-antenna case with the interference due to frequency offsets only, which is

$$\mathbf{Y}_u = \mathbf{P}_u\mathbf{S}_u\mathbf{P}_u^{-1}\mathbf{A} + \mathbf{W}'_u, \tag{9}$$

where $\mathbf{P}_u$ is an $N \times N$ diagonal matrix with the main-diagonal entry $P_u[k] = T^{-1}(N(u-1)+k, N(u-1)+k)$, $\mathbf{W}'_u = \mathbf{P}_u\mathbf{W}_u$. Here we omit the index of the transmit antenna in $\mathbf{A}$ for simplicity. The MMSE linear estimator of $a[k]$ given $\mathbf{Y}_u$

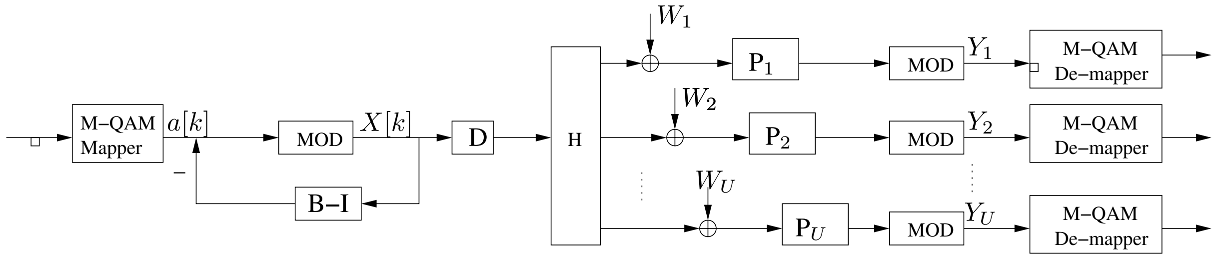

Fig. 2. Block diagram of THP in multiuser MIMO OFDM for decentralized receivers.

is [12, page 382]

$$\hat{a}[k] = \mathrm{E}\left(a[k]\right) + \mathbf{Cov}\left(a[k], \mathbf{Y}_u\right) \mathbf{Cov}\left(\mathbf{Y}_u, \mathbf{Y}_u\right)^{-1} \left(\mathbf{Y}_u - \mathrm{E}(\mathbf{Y}_u)\right). \tag{10}$$

If we assume that $\mathrm{E}\left[\mathbf{W}_u\right] = \mathbf{0}$, $\mathrm{E}\left[\mathbf{W}'_u \mathbf{W}'^H_u\right] = \sigma_W^2 \mathbf{P}_u \mathbf{P}_u^H$, $\mathrm{E}\left[\mathbf{A}\mathbf{W}_u^H\right] = \mathbf{0}$ and independence among $a[k]$ with $\mathrm{E}\left[a[k]\right] = \bar{a}[k]$, $\bar{a}[k] \in \bar{\mathbf{A}} = \mathrm{E}\left[\mathbf{A}\right]$, we can have

$$\begin{aligned} \mathrm{E}\left[\mathbf{Y}_u\right] &= \mathbf{P}_u \mathbf{S}_u \mathbf{P}_u^{-1} \bar{\mathbf{A}} \\ \mathbf{Cov}\left(a[k], \mathbf{Y}_u\right) &= C_a[k] (P_u^*[k])^{-1} \mathbf{S}_u^H[k] \mathbf{P}_u^H \\ \mathbf{Cov}\left(\mathbf{Y}_u, \mathbf{Y}_u\right) &= \sigma_W^2 \mathbf{P}_u \mathbf{P}_u^H + \mathbf{P}_u \mathbf{S}_u \mathbf{P}_u^{-1} \mathbf{C}_a \mathbf{P}_u^{-H} \mathbf{S}_u^H \mathbf{P}_u^H, \end{aligned} \tag{11}$$

where $\mathbf{C}_a = \mathrm{diag}\left[C_a[0] \ \ldots \ C_a[N-1]\right] = E_s \mathbf{I}_N$ with entries $C_a[k] = E[|a[k]|^2] = E_s$, and $\mathbf{S}_u[k] = \left[S_u[k] \ S_u[k-1] \ \ldots \ S_u[k-(N-1)]\right]^T$ is the k-th column in $\mathbf{S}_u$. The linear iterative MMSE estimate of $a[k]$ is

$$\hat{a}[k] = \bar{a}[k] + \mathbf{\Delta}_u[k] \left(\mathbf{Y}_u - \mathbf{P}_u \mathbf{S}_u \mathbf{P}_u^{-1} \bar{\mathbf{A}}\right), \tag{12}$$

where $\mathbf{\Delta}_u[k] = E_s (P_u^*[k])^{-1} \mathbf{S}_u^H[k] \mathbf{P}_u^H \left(\mathbf{Cov}\left(\mathbf{Y}_u, \mathbf{Y}_u\right)\right)^{-1}$.

With the first-stage precoding at the BS, the design for second-stage equalization at each user's receiver is simplified to process a single-input OFDM system. The complexity of the receiver's MMSE equalization is hence primarily determined by the operation of the inversion of $\mathbf{Cov}\left(\mathbf{Y}_u, \mathbf{Y}_u\right)$ in (12). Since $\mathbf{S}_u$ is unitary [8], [9], we have

$$\begin{aligned} \mathbf{Cov}\left(\mathbf{Y}_u, \mathbf{Y}_u\right) &= \sigma_W^2 \mathbf{P}_u \mathbf{P}_u^H + \mathbf{P}_u \mathbf{S}_u \mathbf{P}_u^{-1} \mathbf{C}_a \mathbf{P}_u^{-H} \mathbf{S}_u^H \mathbf{P}_u^H \\ &= \mathbf{P}_u \mathbf{S}_u \left[\sigma_W^2 \mathbf{I}_N + E_s \mathbf{P}_u^{-1} \mathbf{P}_u^{-H}\right] \mathbf{S}_u^H \mathbf{P}_u^H \\ &= \mathbf{P}_u \mathbf{S}_u \mathbf{\Psi}_u \mathbf{S}_u^H \mathbf{P}_u^H. \end{aligned} \tag{13}$$

Since $\mathbf{P}_u$ is a diagonal matrix constant for all $k = 0, \ldots, N-1$ over at least one OFDM symbol interval, $\mathbf{\Psi}_u$ is a diagonal matrix as well. Therefore, we can easily obtain $\left(\mathbf{Cov}\left(\mathbf{Y}_u, \mathbf{Y}_u\right)\right)^{-1} = \mathbf{P}_u^{-H} \mathbf{S}_u \mathbf{\Psi}_u^{-1} \mathbf{S}_u^H \mathbf{P}_u^{-1}$. The inversion in (12) only requires simple operations, while a typical MMSE estimator needs at least $\mathcal{O}(N^2)$ operations [13]. Complexity of calculations in (12) is significantly reduced.

To initialize the iterative algorithm, we set $\bar{a}[k] = 0$ and $C_a[k] = 1$. We calculate $\hat{a}[0]$ via (12) and immediately update the initial values $\bar{a}[0]_{\text{new}} = \hat{a}[0]$. We next calculate $\hat{a}[1]$ and then immediately update $\bar{a}[1]_{\text{new}} = \hat{a}[1]$. This calculation continues until $\hat{a}[N-1]$ has been computed, and then repeats again starting from $\hat{a}[0]$. $\bar{a}[k]_{\text{new}}$ will be used for next calculation instead of the initial $\bar{a}[k]$. The algorithm terminates when the estimate of $a[k]$ converges or a specified number of iterations elapses. After the algorithm terminates, the u-th user selects the $\hat{a}[k]$, $k \in \mathbb{K}_u$, which are the data symbols transmitted on the k-th subcarrier assigned to the u-th user, and discards the other $\hat{a}[k]$.

IV. Simulation Results

Simulation results show how the proposed two-stage precoder/equalizer suppresses ICI and MUI in multiuser multiple-antenna OFDM. A multiuser 4-QAM MIMO OFDM system with 64 subcarriers and 4 or 8 transmit antennas over a 6-tap Rayleigh fading channel is considered. The channel model is vehicular B as defined in ITU R-M1225 [14]. Each user exactly estimates the frequency offset and channel response affecting its receiver, and the channel gains are sent back to the BS. We consider 2 intervals for the normalized frequency offsets, $\mathbb{I} = [-0.1, 0.1]$, and $\mathbb{II} = [-0.3, -0.1) \cup (0.1, 0.3]$, and the frequency offset values are assumed to be uniformly distributed in these intervals. The maximum possible number of distinct frequency offset values is U.

We provide two benchmark cases for reference. In the first case (Case 1), the BS employs non-linear THP with perfect or imperfect full knowledge of CSI, i.e., both $\mathbf{S}$ and $\mathbf{H}$ are available at the BS transmitter. Full-CSI THP pre-equalizes both $\mathbf{S}$ and $\mathbf{H}$, and no individual equalization at each user's receiver is applied. Since the feedback channel bandwidth is usually much smaller than the downlink traffic channel capacity, we assume the noise variance of the feedback link to be $\sigma_F^2 = \sigma_W^2/100$. In the second case (Case 2), THP or linear precoding is used at the BS to pre-equalize $\mathbf{H}$; time-domain compensation (phase-rotation in (1)) for frequency offset is used at users' terminals.

Fig. 3 gives the BERs of our technique and Case 1 (zero-forcing full-CSI THP only) with perfect or imperfect feedback; 4 transmit antennas and 4 users are considered. The performance of 4-user MIMO OFDM without any precoder/equalizer and with zero-frequency offset are also shown for reference. With perfect feedback, our technique exhibits practically the same BER as Case 1 when full-CSI THP only is used, and almost completely cancels the ICI and MUI, even for normalized frequency offsets as high as in the interval $\mathbb{II}$. Hence, Case 1, in which all users' frequency offsets are sent back to the BS, does not offer any BER improvement over our scheme. Therefore, frequency offsets do not need to be fed back, which reduces feedback bandwidth requirements. When the feedback link is noisy, our scheme has a noise-corrupted $\mathbf{H}$ at the BS and accurate frequency offset at each

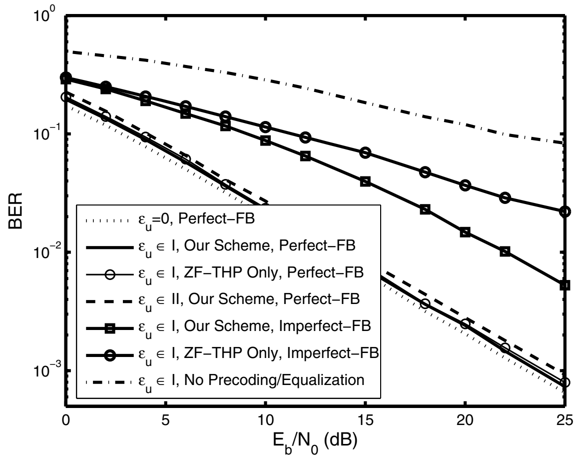

Fig. 3. BER with the proposed precoder/equalizer and full-CSIT THP as a function of the SNR for different values of the normalized frequency offset for 64-subcarrier 4-QAM 4-user MIMO OFDM with perfect and imperfect feedback; $M_T = 4, U = 4$.

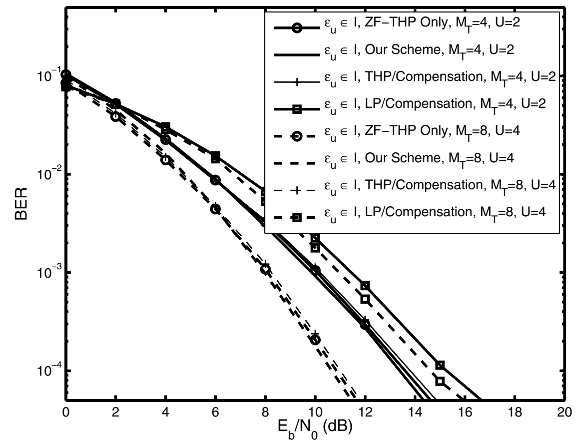

Fig. 4. BER with the proposed precoder/equalizer, full-CSIT THP, THP/frequency-offset-compensation and LP/frequency-offset-compensation as a function of the SNR for different values of the normalized frequency offset for 64-subcarrier multiuser 4-QAM MIMO OFDM with perfect feedback; $M_T = 4, U = 2$ and $M_T = 8, U = 4$.

user's receiver, while both $\mathbf{S}$ and $\mathbf{H}$ are inaccurate at the BS in Case 1. Hence, our scheme outperforms Case 1 since we avoid the frequency-offset mismatch at the BS. It is also easier to implement, because for Case 1 the BS needs to know the frequency offset of each user, which may be difficult to achieve.

Fig. 4 shows the BERs of our technique and for both reference cases in multiuser MIMO OFDM with 4 transmit antennas, 2 users, and 8 transmit antennas, 4 users; $\varepsilon_u \in \mathbb{I}$. Zero-forcing linear precoding (LP) and ZF-THP are considered. For simplicity, perfect feedback is assumed. THP with full perfect CSI (Case 1) and THP/frequency-offset-compensation (Case 2) do not offer BER gain over our scheme. Furthermore, in Case 2, THP/FO-compensation outperforms LP/FO-compensation, i.e., lower BER can be expected if non-linear precoding is used at the BS.

V. Conclusion

We have proposed two-stage transmitter/receiver processing to reduce ICI and MUI in downlink multiuser OFDM with multiple transmit antennas. The first stage employs a TH precoder at the BS to mitigate MUI in a spatial MIMO channel. The second stage applies a low-complexity linear equalizer to suppress ICI and MUI due to frequency offset at each user's receiver. Our proposed precoder/equalizer significantly reduces the BER increase due to frequency offset.

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