

POWER CONTROL FOR WIRELESS CELLULAR SYSTEMS VIA D.C. PROGRAMMING

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ABSTRACT

Power control at the base station is typically used in wireless cellular networks in order to optimize the transmission subject to quality of service (QoS) constraints. It has been shown that the power control problem in the wireless cellular network framework can be efficiently solved using the so-called *geometric programming*. However, in order to enable the application of geometric programming the signal to interference ratio (SIR) has been considered instead of signal to interference plus noise ratio (SINR). Such problem reformulation is imprecise and might be loose because it does not take into account the noise component, especially for low signal to noise ratio (SNR) operation. In this paper, we show that the power control problem for wireless cellular systems can be efficiently solved via the so-called *difference of two convex functions* (D.C.) programming. Numerical simulation example demonstrates significant performance advantages of the proposed approach.

Keywords: Wireless communications, cellular networks, quality of service (QoS), difference of two convex functions, D.C. programming

1. INTRODUCTION

The efficient management of radio resource is essential for wireless networks, which are characterized by scarce radio spectrum, an unreliable propagation channel, and user mobility. One important issue of the radio resource management is power control. Power control and resource allocation techniques for cellular communication systems have been a recent focus of intensive studies [1]- [7]. It has been proposed to use the user signal to interference plus noise ratio (SINR) to adjust the transmitted power [3]. In this way, power control is used to control interference, and therefore, to control also individual users' quality of services (QoS). Various objectives have been considered for developing power control algorithms. Particularly, one can maximize the minimum

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SINR, minimize total transmitted power, or minimize outage probability in a cellular network [3]- [7]. Although various iterative methods have been developed to solve the power control problem in cellular wireless systems, these methods are not general to allow a diverse set of QoS constraints and objective functions.

A general framework for the power control based on *geometric programming* was developed in [5]- [7]. However, in order to enable the application of geometric programming, SINR has to be approximated by the signal to interference ratio (SIR) and sum of $\log(\text{SIR})$ is considered as the optimization objective function. Unfortunately, such approximation might be imprecise and loose, especially when the operation signal-to-noise ratio (SNR) is low. Moreover, note that the individual user throughput, that is $\log(1 + \text{SINR})$, is a monotonic function of the user SINR, and maximization of the aggregate system throughput, that is a sum of $\log(1 + \text{SINR})$ for all users, is the actual target for network optimization.

In this paper, we directly use the the aggregate system throughput as objective function in solving the power control problem for cellular systems. We show that the corresponding optimization problems belong to the class of so-called *difference of two convex functions* (D.C.) programming problems which can be efficiently solved using modern optimization methods [8]. The D.C. framework can accommodate a variety of realistic QoS and fairness constraints. Moreover, the fairness parameters can also be jointly optimized with QoS criteria using this framework. Note that the D.C. programming has been recently introduced in communication community to solve the spectral management problem in digital subscriber line (DSL) [9].

2. SYSTEM MODEL

We consider a downlink channel in a cellular network with K users (links¹) and a single base station.² Extension to multiple base stations is straightforward.

¹Each link represents a unidirectional path from the transmitter to the receiver.

²We consider only downlink transmission in this paper since the uplink case can be treated similarly.

The effects of three signal strength attenuation factors: path loss, shadowing, and multipath fading are considered in the following propagation model. Let P_k be the transmitted power level of the k th user. Then, the propagation model for the k th user can be written as [7]

$$\tilde{P}_k = P_k F_k \left(\frac{d_0}{d_k} \right)^{\beta_k} \quad (1)$$

where \tilde{P}_k is the received power, d_k is the propagation path length, d_0 is a reference distance for the antenna far-field, F_k is multipath fading gain, and β_k is the path loss exponent for the k th receiver. Note that in the aforementioned model we ignore the effect of shadowing for brevity. Using (1), the SINR for the k th receiver can be defined as

$$\text{SINR}_k = \frac{P_k F_k d_k^{-\beta_k} \alpha_k}{\sum_{j \neq k}^K P_j F_j D_s^{-1} d_j^{-\beta_j} \alpha_j + \sigma_k^2} \quad (2)$$

where the factors α_j , $j = 1, \dots, K$ are introduced to accommodate normalization constant d_0 and other factors, such as the effect of beamforming in multiantenna systems, and σ_k^2 is the noise power of the k th user. In (2), the decrease of the interfering users' power is modeled by the inverse of D_s that can be viewed, for example, as a spreading gain for CDMA systems with matched-filter receivers.

Although SINR is often used as a QoS parameter, it is the network throughput which is of concern. Indeed, it is well known that the capacity of Gaussian channel with Gaussian interference is a function of SINR. Then the throughput for the k th user is given by

$$R_k = \frac{1}{T} \log(1 + K^{\text{ber}} \text{SINR}_k) \quad (3)$$

where $K^{\text{ber}} = \frac{-1.5}{\log(5\text{BER})}$, BER is the bit error rate, and T is the symbol duration which we set to be equal to 1, i.e., $T = 1$, for the sake of brevity. Let us denote $G_j = F_j D_s^{-1} d_j^{-\beta_j} \alpha_j > 0$, $j = 1, \dots, K$ for path attenuation of interfering user j . One popular power control problem is based on maximizing the data rate for some particular user under QoS constraints for other users. Mathematically, this problem can then be formulated as follows.

Problem 1:

$$\text{maximize } R_k = \log \left(1 + K^{\text{ber}} \frac{P_k G_k}{\sum_{j \neq k}^K P_j G_j + \sigma_k^2} \right) \quad (4)$$

$$\text{subject to } R_j \geq \gamma_j^{\text{LB}}, \quad \forall j \neq k \quad (5)$$

$$\sum_{j \in \mathcal{I}_{1,l}} P_j G_j < c_l \quad (6)$$

$$0 \leq P_j \leq P_j^{\text{UB}}, \quad \forall j \quad (7)$$

where γ_j^{LB} is the lower bound on the the required data rate of j th user, $\mathcal{I}_{1,l}$ is the index set including interfering users

for which the assigned QoS values are relatively low, c_l is a positive constant (QoS value), and P_j^{UB} is the upper bound on the transmitted power P_j .

Note that the constraints (5) are used to guarantee that the QoS requirements on the minimum data rates are satisfied for the existing users, while the constraint (6) limits the interference from the corresponding group of users $\mathcal{I}_{1,l}$. Finally, the constraint (7) guarantees that the powers P_j , $j = 1, \dots, K$ are positive and do not exceed the peak limit powers P_j^{UB} , $j = 1, \dots, K$.

The optimization problem (4)-(7) is clearly a nonlinear nonconvex optimization problem which is extremely hard to solve. The goal of the following discussion is to show that the optimization problem (4)-(7) can be rewritten in the form of the so-called *D.C. programming* problem [8].

3. D.C. PROGRAMMING: AN OVERVIEW

Definition 1: A real-valued function $f(\mathbf{x})$ defined on a convex set $\mathcal{C} \subseteq \mathbb{R}^n$ is called D.C. (difference of two convex functions) on \mathcal{C} if, for all $\mathbf{x} \in \mathcal{C}$, $f(\mathbf{x})$ can be expressed as

$$f(\mathbf{x}) = f_1(\mathbf{x}) - f_2(\mathbf{x}) \quad (8)$$

where $f_1(\mathbf{x})$ and $f_2(\mathbf{x})$ are convex functions on \mathcal{C} . The representation (8) is called a D.C. decomposition of $f(\mathbf{x})$. The class of D.C. functions is closed under many operations frequently encountered in optimization. Let f, f_i , ($i = 1, \dots, m$) be D.C. functions. Then the following functions are also D.C. [8]:

- (i) $\sum_{i=1}^m \lambda_i f_i(x)$, for any real number λ_i ,
- (ii) $\max_{i=1, \dots, m} f_i(x)$ and $\min_{i=1, \dots, m} f_i(x)$,
- (iii) $|f(x)|$, $f^+(x) := \max\{0, f(x)\}$, $f^-(x) := \min\{0, f(x)\}$,
- (iv) the product $\prod_{i=1}^m f_i(x)$.

Definition 2: A global optimization problem is called a D.C. programming problem if it has the form

$$\text{minimize } f(\mathbf{x}) \quad (9)$$

$$\text{subject to } g_j(\mathbf{x}) \leq 0, \quad \forall j \quad (10)$$

$$\mathbf{x} \in \mathcal{C} \quad (11)$$

where \mathcal{C} is a closed convex subset of \mathbb{R}^n and all functions $f(\mathbf{x})$ and $g_j(\mathbf{x})$ are D.C. functions.

An important property of D.C. programming is that any problem of the form (9)-(11) can be reduced to a canonical problem of minimizing a linear function over the intersection of a convex set with the complement of an open convex set, where the complement of an open convex set is usually described by a so-called reverse convex constraint which has the form $g(x) \geq 0$ and $g(x)$ is convex.

4. POWER CONTROL VIA D.C. PROGRAMMING

4.1. Problem Formulations Using D.C. Optimization

The following theorem is in order.

Theorem 1: The optimization problem (4)-(7) is a D.C. programming problem.

Proof: It is easy to show that the power constraints (6)-(7) are convex. Using the basic property of the logarithmic function, that is $\log(A/B) = \log A - \log B$, the objective function (4) can be rewritten as

$$\begin{aligned} R_k &= \log\left(1 + \frac{K^{\text{ber}} P_k G_k}{\sum_{j \neq k}^K P_j G_j + \sigma_k^2}\right) \\ &= f_1^k(P_1, \dots, P_K) - f_2^k(P_1, \dots, P_K) \end{aligned} \quad (12)$$

where

$$f_1^k(P_1, \dots, P_K) = -\log\left(\sum_{j \neq k}^K P_j G_j + \sigma_k^2\right) \quad (13)$$

$$f_2^k(P_1, \dots, P_K) = -\log\left(\sum_{j \neq k}^K P_j G_j + K^{\text{ber}} P_k G_k + \sigma_k^2\right) \quad (14)$$

are logarithmically convex functions³. The rate constraints (5) can be also expressed as D.C. constraints on P_1, \dots, P_K in a similar way. Therefore, the problem (4)-(7) is D.C. programming problem. \square

We can rewrite the optimization problem (4)-(7) in standard form as follows

$$\begin{aligned} &\text{minimize } f_2^k - f_1^k \\ &\text{subject to } f_2^j - f_1^j \leq -\gamma_j^{\text{LB}}, \quad \forall j \neq k \\ &\quad \sum_{j \in \mathcal{I}_{1,t}} P_j G_j < c_t \\ &\quad 0 \leq P_j \leq P_j^{\text{UB}}, \quad \forall j. \end{aligned} \quad (15)$$

Another power control goal can be to find $P_k \geq 0, k = 1, \dots, K$ such that the total transmitted power $\tilde{P} = \sum_{k=1}^K P_k$ would be minimized while the required QoS is guaranteed for each user. Then the corresponding optimization problem can be written as

Problem 2:

$$\begin{aligned} &\text{minimize } \tilde{P} \\ &\text{subject to } \text{The constraints (5)-(7)} \end{aligned} \quad (16)$$

Power control in wireless cellular networks often has to take into account the fairness consideration since the fairness among different users is also a major issue in a QoS policy. In other words, additionally to providing a preferential treatment to high priority connections, fairness issues must also be

³Note that a function $f(z)$ is logarithmically convex on the interval $[a, b]$ if $f(z) > 0$ and $\log f(z)$ is convex on $[a, b]$.

taken into account for low priority users. Two types of fairness can be considered: proportional fairness and maximin fairness. Both these types of fairness can be accommodated in the framework of D.C. programming. Then, the corresponding problem formulations are given as

Problem 3:

$$\begin{aligned} &\text{maximize } \sum_{k=1}^K w_k R_k \\ &\text{subject to } \text{The constraints (5)-(7)} \end{aligned} \quad (17)$$

where $w_k, k = 1, \dots, K$ are some weights, and

Problem 4:

$$\begin{aligned} &\text{maximize } \min_{k=1, \dots, K} R_k \\ &\text{subject to } \text{The constraints (6)-(7)}. \end{aligned} \quad (18)$$

We can see that the power control based on *Problem 3* maximizes weighted sum-rate of the wireless networks, while for *Problem 4* it optimizes the minimum user rate. The maximin fair power allocation is useful in the situations when the worst case is of concern to the cellular network operator. Using the properties (i) and (ii) of D.C. functions, it is easy to show that *Problem 3* and *Problem 4* are D.C. optimization problems. It worths noting that in *Problem 3*, the weights can be any real positive numbers other than integer values. It helps to characterize better the fairness issue among users.

We should also note here that for weighted optimization, the fairness parameters, i.e. $w_k, k = 1, \dots, K$ can be jointly-optimized with power levels. Using the property (iv) of D.C. functions, we can show that $w_k R_k$ is a D.C. function on variables w_k, P_1, \dots, P_K . Thus, $\sum_k w_k R_k$ is also a D.C. function on variables $w_1, \dots, w_K, P_1, \dots, P_K$. It shows that D.C. optimization framework is capable for joint optimization of fairness parameters with QoS criteria.

4.2. Power Control for Queuing Delay Optimization

Delay is a crucial part of QoS for a wireless cellular network. In general, there are three main components of the overall delay: propagation delay, transmission delay and queuing delay. Queuing delay is common and especially important for scenarios where the short term data rate may exceed the data rate supported by the wireless link, and thus data needs to be buffered. Hence, queuing delay is sometimes the main source of the overall delay. Assuming that for each user k , packets with variable length arrive at the base station according to a Poisson distribution with rate λ_k , the system can be modeled as an M/M/1 queue. The average queuing delay, QD_k can then be expressed as

$$QD_k = \frac{1}{R_k - \lambda_k}. \quad (19)$$

If an existing QoS agreement specifies a maximum average delay QD_k^{UB} , then such type of constraint can be transformed into the constraint on the rate R_k . However, when the system's total delay is of concern, it is much more difficult to handle, for example, the total delay $\sum_{i=1,\dots,K} QD_k$ minimization. Fortunately, the total queuing delay can be written as a fraction of two D.C. functions. Specifically, let us write

$$\sum_{i=1,\dots,K} QD_k = \frac{f(P_1, \dots, P_k)}{g(P_1, \dots, P_k)} \quad (20)$$

where f and g are D.C. functions. Then, the power control problem to minimize the total queuing delay can be formulated as

Problem 5:

$$\begin{aligned} & \text{minimize} && f/g \\ & \text{subject to} && \text{The constraints (5)-(7).} \end{aligned} \quad (21)$$

which can be equivalently rewritten as a D.C. problem

$$\begin{aligned} & \text{minimize} && t \\ & \text{subject to} && f - tg \leq 0, t \geq 0, \\ & && \text{The constraints (5)-(7).} \end{aligned} \quad (22)$$

$$\text{subject to} \quad f - tg \leq 0, t \geq 0, \quad (23)$$

It is easy to see that the constraint (23) is a D.C. constraint. We should stress here that minmax fairness optimization on queuing delay, that is

$$\text{minimize} \max_{k=1,\dots,K} QD_k$$

can also be done similarly. To the best of our knowledge, no method have been developed for solving such problems before.

4.3. Power Control: A Probabilistically Constrained Approach

So far, we have considered the deterministic optimization approach to the power control problem, when the controller needs to acquire the instantaneous channel state information (CSI) through feedback channel from the receiver. The latter requirement on the availability of the instantaneous CSI at the transmitter can significantly consume the bandwidth of the system, especially in the case of a time-varying channel. In communications over fading environment, one important QoS parameter for long-term users' requirements is the outage probability. The power control scheme with outage probability constraints arises in cellular networks when the power does not need to be updated whenever the channel varies from one state to another. Taking into account users' outage probability constraints and the statistical variation of the channel, we can optimally allocate power to users and meet the QoS constraints.

Using the model (1), the signal power at the k th receiver is given by $P_k G_k F_k$ and the total interference from other users is given by $\sum_{j \neq k} P_j G_j F_j$, where G_j , $j = 1, \dots, K$ represent the path gains not including fading, and F_j , $j = 1, \dots, K$ model the *Rayleigh fading* and are assumed to be independent exponentially distributed random variables with unit mean [5]. Thus, the SINR of the k th receiver becomes

$$\text{SINR}_k = \frac{P_k G_k F_k}{\sum_{j \neq k} P_j G_j F_j + \sigma_k^2}. \quad (24)$$

An outage is declared for k th user when the data rate R_k falls below a given threshold R_k^{th} . The outage probability associated with the k th user is then given by

$$\begin{aligned} O_k &= Pr(R_k \leq R_k^{\text{th}}) \\ &= Pr\left(P_k G_k F_k \leq \varphi_k^{\text{th}} \left[\sum_{j \neq k} P_j G_j F_j + \sigma_k^2 \right]\right) \end{aligned} \quad (25)$$

where $\varphi_k^{\text{th}} = (K^{\text{ber}})^{-1} (\exp\{R_k^{\text{th}}\} - 1)$. The outage probability can be expressed as [5]

$$O_k = \exp\left\{-\frac{\varphi_k^{\text{th}} \sigma_k^2}{P_k G_k}\right\} \prod_{j \neq k} \frac{P_k G_k}{P_k G_k + \varphi_k^{\text{th}} P_j G_j}. \quad (26)$$

Then the power control problem can be written as the following nonlinear optimization problem of maximizing total network throughput.

Problem 6:

$$\text{maximize} \quad \sum_k w_k R_k \quad (27)$$

$$\text{subject to} \quad R_k \geq \gamma_k^{\text{LB}}, \quad \forall k \quad (28)$$

$$O_k = \exp\left\{-\frac{\varphi_k^{\text{th}} \sigma_k^2}{P_k G_k}\right\} \cdot \prod_{j \neq k} \frac{P_k G_k}{P_k G_k + \varphi_k^{\text{th}} P_j G_j} \leq O_k^{\text{UB}}, \quad \forall k \quad (29)$$

$$0 \leq P_k \leq P_k^{\text{UB}}, \quad \forall k \quad (30)$$

where the second constraint is used to limit the outage probability for QoS requirement of each user.

We have previously shown that the objective function (27) in *Problem 6* is a D.C. function, and the constraints (28) and (30) are D.C. constraints. Therefore, in order to prove that *Problem 6* is a D.C. programming problem, we need to show that constraint (29) can be written as D.C. constraint. Taking the logarithm of the outage probability expression (29), we can write

$$\log(O_k) = \tilde{f}_1(P_1, \dots, P_K) - \tilde{f}_2(P_1, \dots, P_K) \quad (31)$$

where the functions

$$\tilde{f}_1(P_1, \dots, P_K) = - \sum_{j \neq k} \log(P_k G_k + \varphi_k^{\text{th}} P_j G_j) \quad (32)$$

$$\tilde{f}_2(P_1, \dots, P_K) = \frac{\varphi_k^{\text{th}} \sigma_k^2}{P_k G_k} - \log(P_k G_k) \quad (33)$$

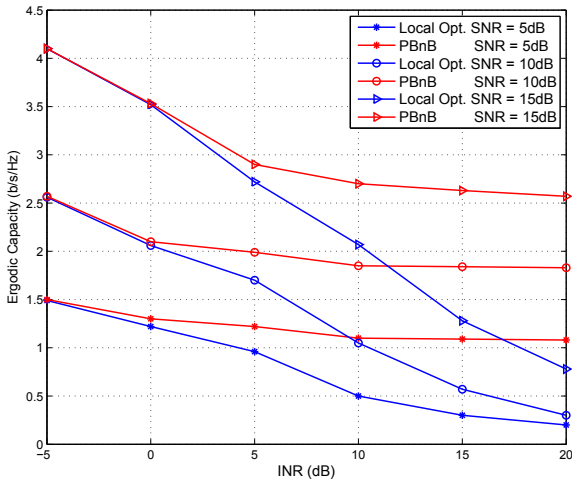


Fig. 1. Ergodic sum rate capacity for $K = 2$.

are logarithmically convex functions of P_1, \dots, P_K . Thus, we can formulate *Problem 6* as a D.C. programming problem.

5. SIMULATION RESULTS

We consider a simple system comprised of a single cell and two users and no inter-cell interferences. The power control problem which we consider in our simulations aims at maximizing the system's sum-rate, i.e., (*Problem 3* with weights equal to 1's), under the constraints on the upper power consumptions on each user. The prismatic branch-and-bound (PBnB) algorithm [8] is used to solve the corresponding D.C. programming problem. For simplicity reason, we rewrite the formula for the rate of the user k as follows

$$R_k = \log\left(1 + \frac{P_k \tilde{G}_k F_k}{\sum_{j \neq k} P_j G_j F_j + 1}\right) \quad (34)$$

where \tilde{G}_k, G_j are normalized SNR and interference-to-noise ratio (INR), respectively. We assume that $\tilde{G}_1 = \tilde{G}_2$ and $G_1 = G_2$. The maximum power is constrained for all users, i.e., $P^{\text{UB}} = 1W$. We perform 500 runs to obtain the ergodic sum rate capacity. Fig. 1 demonstrates the sum rate of the system versus INR. For comparison, the local optimization method is also performed.

We can see that the PBnB algorithm significantly outperforms the local optimization search based algorithm in all cases, especially when the INRs are large. In the full paper and during the presentation, we show more simulation results on the proposed D.C. applications.

6. CONCLUSION

In this paper, we have developed various QoS provisioning problems for wireless cellular networks based on the resource allocation perspective. The individual user data rate or aggregate system throughput are used as performance metrics. The optimization of the queuing delay is also considered. It is shown that the developed problems can be posed as D.C. programming problems. Numerical simulation example demonstrates significant performance advantages of the proposed approach.

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