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BER of MIMO-OFDM Systems with Carrier Frequency Offset and Channel Estimation Errors

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Abstract— **Performance analysis of Multiple-input Multiple-output (MIMO) Orthogonal Frequency-Division Multiplexing (OFDM) systems with carrier frequency offset and channel estimation errors is considered in this paper. Based on the analysis of the Inter-Carrier-Interference (ICI) and Inter-Antenna-Interference (IAI) due to the residual frequency offsets, the average Signal-to-Interference-and-Noise Ratio (SINR) is derived. The bit error rate of equal gain combining (EGC) and maximal ratio combining (MRC) with MIMO-OFDM is analyzed, and an infinite-series approximation for the bit error rate is derived. Simulation results illustrate the accuracy of the theoretical analysis.**

I. INTRODUCTION

Multiple-input multiple-output (MIMO) technology can significantly increase the wireless system capacity [1]. By transforming the frequency-selective MIMO channel to a set of flat-fading MIMO channels, MIMO-OFDM can achieve a high capacity at a low cost of equalization and demodulation. However, just as single-input single-output (SISO)-OFDM systems are highly sensitive to frequency offset, so too are the MIMO-OFDM systems.

Many frequency offset estimators have already been developed for SISO-OFDM systems [2]–[6]. A synchronization algorithm for MIMO-OFDM is proposed in [7], where identical timing offset and frequency offset are assumed to each transmit-receive antenna pair. Parameter estimation for MIMO flat-fading channels with the frequency offsets is discussed in [8], where the frequency offsets for different transmit and receive antennas are assumed to be different.

Another channel impairment is the channel estimation error, which can impact the bit error rate of an OFDM system. Robust channel estimation for OFDM is discussed in [9], and the mean-square error (MSE) of channel estimation can be significantly reduced by exploiting the channel delay profile. An iterative joint frequency offset and channel estimator is proposed in [11], where a reliable estimation is performed based on the maximum likelihood principle by using pilots. Optimal training signal design for a frequency-selective block fading channel estimation in MIMO-OFDM is discussed in [10], which is based on the minimization of the MSE. The bit error rate of SISO-OFDM impaired by the frequency offset is analyzed in [12].

In this paper, we give a generalized bit-error-rate's analysis of MIMO-OFDM, taking into consideration of both the frequency offset and channel estimation errors. We exploit the fact that for an unbiased estimator, either the channel estimation error or frequency offset estimation error is a zero-mean random variable (RV). The degradation in either the receive SINR or bit error rate will be analyzed by studying the statistical characters of the residual frequency offset and channel estimation errors. As discussed in [8], the frequency offset of each transmit-receive antenna pair is assumed to be an Independent and Identically Distributed (IID) RV.

The remainder of this paper is organized as follows. MIMO-OFDM system model is given in Section II, and the SINR degradation due to the frequency offset and channel estimation errors is analyzed in Section III. The bit error rate of MIMO-OFDM with the frequency offset and channel estimation errors is derived in Section IV. Numerical results are given in Section V, followed by conclusions in Section VI.

Notation: $(\cdot)^T$ and $(\cdot)^H$ are the transpose and complex conjugate transpose of a matrix. The imaginary unit is $\jmath = \sqrt{-1}$. A circularly symmetric complex Gaussian RV with mean m and variance σ^2 is denoted by $w \sim \mathcal{CN}(m, \sigma^2)$. $\mathbf{I}_N$ is an $N \times N$ identity matrix, and $\mathbf{O}_N$ is an $N \times N$ all-zero matrix. $\mathbf{0}_N$ is an $N \times 1$ all-zero vector. $\mathbf{a}[i]$ is the i-th entry of vector $\mathbf{a}$, and $[\mathbf{B}]_{mn}$ is the mn-th entry of matrix $\mathbf{B}$. $\mathbb{E}\{x\}$ and Var$\{x\}$ are the mean and variance of x.

II. MIMO-OFDM SIGNAL MODEL

Input data bits of the MIMO-OFDM systems are mapped to complex symbols drawn from a typical signal constellation, e.g., phase-shift keying (PSK) or quadrature amplitude modulation (QAM). An OFDM symbol is generated by taking the *Inverse Discrete Fourier Transform* (IDFT) of N input subsymbols, where N is the size of IDFT. Each OFDM symbol has a useful part of duration T_s seconds and a cyclic prefix of length T_g seconds to mitigate the Inter-Symbol-Interference (ISI), where T_g is longer than the channel-response duration. For a MIMO-OFDM system with N_t transmit antennas and N_r receive antennas, a $N \times 1$ vector $\mathbf{x}_i(z)$ is used to represent the z-th block of the frequency-domain symbols sent by the i-th transmit antenna, where $i \in \{1, 2, \cdots, N_t\}$. In the following sections, when the discussion is concentrated on a single block, the temporal index z will be omitted for brevity. The time-domain vector for the i-th transmit antenna is given by $\mathbf{m}_i = \sqrt{\frac{E_s}{N_t}}\mathbf{F}\mathbf{x}_i$, where E_s is the total transmit power and $\mathbf{F}$ is the $N \times N$ IDFT matrix with entries $[\mathbf{F}]_{nk} = \frac{1}{\sqrt{N}} e^{\frac{\jmath 2\pi nk}{N}}$ for

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$0 \le n, k \le N-1$. Without loss of generality, each entry of $\mathbf{x}_i$ is assumed to be an IID RV with mean zero and variance 1; i.e., $\sigma_x^2 = \mathbb{E}\left\{|\mathbf{x}_i[n]|^2\right\} = 1$ for $1 \le i \le N_t$ and $0 \le n \le N-1$.

By using $h_{k,i}(n)$ to represent the discrete-time impulse response of the n-th tap channel between the i-th transmit and the k-th receive antennas, the channel response vector can be represented as $\mathbf{h}_{k,i} = [h_{k,i}(0), h_{k,i}(1), \cdots, h_{k,i}(L_{k,i}-1), \mathbf{0}^T_{L_{max}-L_{k,i}}]^T$, where $L_{k,i}$ is the maximum delay between the i-th transmit and the k-th receive antennas, and $L_{max} = \max\{L_{k,i} : 1 \le i \le N_t, 1 \le k \le N_r\}$. Uncorrelated taps are assumed for each (k,i); i.e., $\mathbb{E}\{h^*_{k,i}(m)h_{k,i}(n \ne m)\} = 0$. The corresponding frequency-domain channel attenuation matrix is given by $\mathbf{H}_{k,i} = \text{diag}\left\{H^{(0)}_{k,i}, H^{(1)}_{k,i}, \cdots, H^{(N-1)}_{k,i}\right\}$ with $H^{(n)}_{k,i} = \sum_{d=0}^{L_{k,i}-1} h_{k,i}(d)e^{-\jmath\frac{2\pi nd}{N}}$ representing the channel attenuation at the n-th subcarrier. Normalized channel attenuation is assumed; i.e., $\sum_{d=0}^{L_{k,i}-1} |h_{k,i}(d)|^2 = 1$ for each (k,i). The covariance of the frequency-domain channel attenuation can be derived as $C_{H^{(n)}_{k,i} H^{(l)}_{p,q}} = \sum_{d=0}^{L_{max}-1} \mathbb{E}\left\{h^*_{k,i}(d)h_{p,q}(d)\right\} e^{-\frac{\jmath 2\pi d(l-n)}{N}}$.

We use $\psi_{k,i}$ and $\varepsilon_{k,i}$ to represent the initial phase and normalized frequency offset (frequency offset normalized to a subcarrier spacing of OFDM symbols) between the i-th transmit and the k-th receive antennas. In this paper, $\varepsilon_{k,i}$ for each transmit-receive antenna pair is formulated as an IID RV with zero-mean. By considering the channel attenuations and frequency offsets, the discrete-time received vector can be represented as

$$\mathbf{y} = [\mathbf{y}_1^T, \mathbf{y}_2^T, \cdots, \mathbf{y}_{N_r}^T]^T, \tag{1}$$

where $\mathbf{y}_k = \sqrt{\frac{E_s}{N_t}} \sum_{i=1}^{N_t} \mathbf{E}_{k,i}\mathbf{F}\mathbf{H}_{k,i}\mathbf{x}_i + \mathbf{w}_k$ with $\mathbf{E}_{k,i} = \text{diag}\left\{e^{\jmath\psi_{k,i}}, e^{\jmath\left(\frac{2\pi\varepsilon_{k,i}}{N}+\psi_{k,i}\right)}, \cdots, e^{\jmath\left(\frac{2\pi\varepsilon_{k,i}(N-1)}{N}+\psi_{k,i}\right)}\right\}$, and $\mathbf{w}_k$ is a vector of additive white Gaussian noise (AWGN) where $\mathbf{w}_k[n] \sim \mathcal{CN}(0, \sigma_w^2)$. Note that the channel state information is unavailable at the transmitter, but available at the receiver. Consequently, the transmit power is equally allocated at all the transmit antennas.

III. SINR Analysis in MIMO-OFDM Systems

In this paper, space-time coding is not considered at the transmitter; instead, IID data sub-streams are mapped to the OFDM symbols and are transmitted from the transmit antennas. Each received vector $\mathbf{y}_k$ is thus a superposition of the transmit signals from all the N_t transmit antennas. In order to demodulate $\mathbf{x}_i$, signal from the transmit antennas other than the i-th transmit antenna should be eliminated (IAI cancellation).

Here we first assume that $\varepsilon_{k,j}$ and $\mathbf{H}_{k,j}$ for each $(1 \le j \le N_t, j \ne i)$ have been estimated; i.e., $\hat{\varepsilon}_{k,j} = \varepsilon_{k,j} + \Delta\varepsilon_{k,j}$ and $\hat{\mathbf{H}}_{k,j} = \mathbf{H}_{k,j} + \Delta\mathbf{H}_{k,j}$, where $\Delta\varepsilon_{k,j}$ and $\Delta\mathbf{H}_{k,j} = \text{diag}\left\{\Delta H^{(0)}_{k,j}, \Delta H^{(1)}_{k,j}, \cdots, \Delta H^{(N-1)}_{k,j}\right\}$ are estimation errors of $\varepsilon_{k,j}$ and $\mathbf{H}_{k,j}$ ($\Delta H^{(n)}_{k,j} = \hat{H}^{(n)}_{k,j} - H^{(n)}_{k,j}$ represents the estimation error of $H^{(n)}_{k,j}$). We also assume that the demodulate error for each $\mathbf{x}_{j\ne i}$ is negligible. After estimating $\varepsilon_{k,i}$, i.e., $\hat{\varepsilon}_{k,i} = \varepsilon_{k,i} + \Delta\varepsilon_{k,i}$, $\varepsilon_{k,i}$ can be compensated for and $\mathbf{x}_i$ can be demodulated as

$$\begin{aligned}\mathbf{r}_{k,i} &= \mathbf{F}^H\hat{\mathbf{E}}^H_{k,i}\left(\mathbf{y}_k - \sqrt{\frac{E_s}{N_t}}\sum_{j=1, j\ne i}^{N_t} \hat{\mathbf{E}}_{k,j}\mathbf{F}\hat{\mathbf{H}}_{k,j}\mathbf{x}_j\right) \\ &= \sqrt{\frac{E_s}{N_t}}\underbrace{\mathbf{F}^H\hat{\mathbf{E}}^H_{k,i}\mathbf{E}_{k,i}\mathbf{F}\mathbf{H}_{k,i}\mathbf{x}_i}_{\mathbf{s}_{k,i}} + \boldsymbol{\Upsilon}_{k,i} + \tilde{\mathbf{w}}_{k,i},\end{aligned} \tag{2}$$

where $\hat{\mathbf{E}}_{k,j}$ is derived from $\mathbf{E}_{k,j}$ by replacing $\varepsilon_{k,j}$ with $\hat{\varepsilon}_{k,j}$, $\boldsymbol{\Upsilon}_{k,i}$ and $\tilde{\mathbf{w}}_{k,i}$ are the residual IAI and AWGN components of $\mathbf{r}_{k,i}$, respectively.

A. SINR Analysis without Combining at the Receive Antennas

In this subsection, we analyze the SINR for the i-th transmit signal at the k-th receive antenna. Signals transmitted by the antennas other than the i-th antenna are interference, which should be eliminated before demodulating the desired signal of the i-th transmit antenna.

Based on (2), the n-th subcarrier $(0 \le n \le N-1)$ for the i-th transmit antenna can be demodulated as

$$\begin{aligned}\mathbf{r}_{k,i}[n] &= \sqrt{\frac{E_s}{N_t}}\mathbf{s}_{k,i}[n] + \boldsymbol{\Upsilon}_{k,i}[n] + \tilde{\mathbf{w}}_{k,i}[n] \\ &= \sqrt{\frac{E_s}{N_t}} m^{(n)}_{k,i} H^{(n)}_{k,i}\mathbf{x}_i[n] + \eta^{(n)}_{k,i} \\ &\quad + \underbrace{\lambda^{(n)}_{k,i} - \hat{\lambda}^{(n)}_{k,i}}_{\Delta\lambda^{(n)}_{k,i}} + \underbrace{\xi^{(n)}_{k,i} - \hat{\xi}^{(n)}_{k,i}}_{\Delta\xi^{(n)}_{k,i}} + \tilde{\mathbf{w}}_{k,i}[n],\end{aligned} \tag{3}$$

where $m^{(n)}_{k,i} = \frac{\sin[\pi(\Delta\varepsilon_{k,i})]}{N\sin\left[\frac{\pi(\Delta\varepsilon_{k,i})}{N}\right]}$. $\eta^{(n)}_{k,i}$ is decomposed as $\eta^{(n)}_{k,i} = H^{(n)}_{k,i}\alpha^{(n)}_{k,i} + \beta^{(n)}_{k,i}$, which is the ICI contributed by the subcarriers other than the n-th subcarrier of transmit antenna i. $\alpha^{(n)}_{k,i}$ and $\beta^{(n)}_{k,i}$ are zero-mean RVs with variances

$$\text{Var}\left\{\alpha^{(n)}_{k,i}\right\} \cong \frac{\pi^2\sigma^2_{res}E_s}{N_t}\sum_{l\ne n}\frac{\left|C_{H^{(n)}_{k,i}H^{(l)}_{k,i}}\right|^2}{N^2\sin^2\left[\frac{\pi(l-n)}{N}\right]} \tag{4}$$

and

$$\text{Var}\left\{\beta^{(n)}_{k,i}\right\} \cong \frac{\pi^2\sigma^2_{res}E_s}{3N_t} - \text{Var}\left\{\alpha^{(n)}_{k,i}\right\}. \tag{5}$$

$\Delta\lambda^{(n)}_{k,i} = \lambda^{(n)}_{k,i} - \hat{\lambda}^{(n)}_{k,i}$ is the interference contributed by the n-th subcarrier of the interfering transmit antennas, i.e., Co-Subcarrier Inter-Antenna-Interference (CSIAI), and $\Delta\xi^{(n)}_{k,i} = \xi^{(n)}_{k,i} - \hat{\xi}^{(n)}_{k,i}$ is the ICI contributed by subcarriers other than the n-th subcarrier of the interfering transmit antennas, i.e., Inter-Carrier-Inter-Antenna-Interference (ICIAI). It is easy to prove that $\Delta\lambda^{(n)}_{k,i}$ and $\Delta\xi^{(n)}_{k,i}$ are zero-mean RVs, and their variances are given by

$$\begin{aligned}&\mathbb{E}\left\{\left|\Delta\lambda_{k,i}^{(n)}\right|^2\right\}\\&\quad\cong\frac{(N_t-1)\pi^4E_s}{9N_t}\left(2\sigma_\epsilon^2\sigma_{res}^2+\sigma_{res}^4+\frac{\mathbb{E}\left\{\Delta\varepsilon_{k,j}^4\right\}}{4}\right)\\&\qquad+\frac{(N_t-1)E_s\cdot\sigma_{\Delta H}^2}{N_t}\\&\qquad\cdot\left[1+\frac{\pi^4\left(\mathbb{E}\left\{\varepsilon_{k,j}^4\right\}+8\sigma_\epsilon^2\sigma_{res}^2+2\sigma_\epsilon^4+2\sigma_{res}^4\right)}{18}\right]\\&\qquad-\frac{2\pi^2\left(\sigma_\epsilon^2+\sigma_{res}^2\right)(N_t-1)E_s\cdot\sigma_{\Delta H}^2}{3N_t}\end{aligned}\tag{6}$$

and

$$\begin{aligned}&\mathbb{E}\left\{\left|\Delta\xi_{k,i}^{(n)}\right|^2\right\}\\&\quad\cong\frac{\pi^2\sigma_{res}^2(N_t-1)E_s}{3N_t}\\&\qquad-\frac{(N_t-1)E_s}{3N_t}\left[\pi^4\left(2\sigma_\epsilon^2\sigma_{res}^2+\sigma_{res}^4+\frac{\mathbb{E}\left\{\Delta\varepsilon_{k,j}^4\right\}}{4}\right)\right]\\&\qquad+\frac{2\pi^2(N_t-1)\left(\sigma_\epsilon^2+\sigma_{res}^2\right)E_s\cdot\sigma_{\Delta H}^2}{3N_t}.\end{aligned}\tag{7}$$

After averaging out $\varepsilon_{k,i}$, $\Delta\varepsilon_{k,i}$ and $\Delta H_{k,i}^{(n)}$ for each (k,i), the average SINR of $\mathbf{r}_{k,i}[n]$ (parameterized by only $H_{k,i}^{(n)}$) is

$$\bar{\gamma}_{k,i}\left(n\middle|H_{k,i}^{(n)}\right)\cong\frac{\frac{E_s}{N_t}\cdot\sigma_m^2\cdot\left|H_{k,i}^{(n)}\right|^2}{\left|H_{k,i}^{(n)}\right|^2\cdot\text{Var}\left\{\alpha_{k,i}^{(n)}\right\}+\nu},\tag{8}$$

where $\nu=\text{Var}\left\{\beta_{k,i}^{(n)}\right\}+\mathbb{E}\left\{\left|\Delta\lambda_{k,i}^{(n)}\right|^2\right\}+\mathbb{E}\left\{\left|\Delta\xi_{k,i}^{(n)}\right|^2\right\}+\sigma_w^2$ and $\sigma_m^2=\mathbb{E}\left\{\left|m_{k,i}^{(n)}\right|^2\right\}=1-\frac{\pi^2\sigma_{res}^2}{3}+\frac{\pi^4\mathbb{E}\left\{\Delta\varepsilon_{k,j}^4\right\}}{36}$.

Note that when demodulating the signal transmitted by the i-th transmit antenna in MIMO-OFDM, the diversity reception can be exploited to improve the receive SINR. In the following, we mainly consider two receiver combining methods: equal gain combining (EGC) and maximal ratio combining (MRC).

B. SINR Analysis with EGC at Receive Antennas

In order to demodulate the signal transmitted by the i-th transmit antenna in MIMO-OFDM, the received signals at all the N_r receive antennas can be co-phased by using EGC to improve the receiving diversity, and, therefore, we have

$$\mathbf{r}_i^{\text{EGC}}[n]=\sum_{k=1}^{N_r}e^{-\jmath\theta_{k,i}^{(n)}}\mathbf{r}_{k,i}[n],\tag{9}$$

where $\theta_{k,i}^{(n)}=\arg\left\{m_{k,i}^{(n)}H_{k,i}^{(n)}\right\}$. The average SINR of $\mathbf{r}_i^{\text{EGC}}[n]$ is given by

$$\begin{aligned}&\bar{\gamma}_i^{\text{EGC}}\left(n\middle|H_{1,i}^{(n)},\cdots,H_{N_r,i}^{(n)}\right)\\&\quad\cong\frac{\frac{E_s}{N_t}\cdot\sigma_m^2\cdot\left(\sum_{k=1}^{N_r}\left|H_{k,i}^{(n)}\right|^2+\sum_{k\neq l}\left|H_{k,i}^{(n)}\right|\cdot\left|H_{l,i}^{(n)}\right|\right)}{\sum_{k=1}^{N_r}\left|H_{k,i}^{(n)}\right|^2\cdot\text{Var}\left\{\alpha_{k,i}^{(n)}\right\}+N_r\nu}.\end{aligned}\tag{10}$$

C. SINR Analysis with MRC at Receive Antennas

In a MIMO-OFDM system with N_r receive antennas, based on the channel estimation $\hat{H}_{k,i}^{(n)}=H_{k,i}^{(n)}+\Delta H_{k,i}^{(n)}$ for each (k,i,n), the received signal at all the N_r receive antennas can be combined by using MRC, so that

$$\mathbf{r}_i^{\text{MRC}}[n]=\frac{\sum_{k=1}^{N_r}\omega_{k,i}\mathbf{r}_{k,i}[n]}{\sum_{k=1}^{N_r}\left|\omega_{k,i}\right|^2},\tag{11}$$

where $\omega_{k,i}=\left(\hat{H}_{k,i}^{(n)}m_{k,i}^{(n)}\right)^*$. The average SINR of $\mathbf{r}_i^M[n]$ is

$$\begin{aligned}&\bar{\gamma}_i^{\text{MRC}}\left(n\middle|H_{1,i}^{(n)},\cdots,H_{N_r,i}^{(n)}\right)\\&\quad\cong\frac{\frac{E_s}{N_t}\cdot\sigma_m^2\cdot\left(\sum_{k=1}^{N_r}\left|H_{k,i}^{(n)}\right|^2\right)^2}{\sum_{k=1}^{N_r}\left|H_{k,i}^{(n)}\right|^4\cdot\text{Var}\left\{\alpha_{k,i}^{(n)}\right\}+\sum_{k=1}^{N_r}\left|H_{k,i}^{(n)}\right|^2\nu'+N_r\nu\cdot\sigma_{\Delta H}^2},\end{aligned}\tag{12}$$

where $\nu'=\left[\nu+\left(\frac{E_s}{N_t}+\text{Var}\left\{\alpha_{k,i}^{(n)}\right\}\right)\sigma_{\Delta H}^2\right]$.

IV. BER Performance

The bit error rate as a function of SINR in MIMO-OFDM is derived in this section. Each subcarrier is modulated by using the M-ary square QAM with Gray bit mapping. In Rugini and Banelli [12], the bit error rate of SISO-OFDM with the frequency offset is developed. Their bit error rate analysis is now extended to the MIMO-OFDM.

As discussed in [12], the bit error rate for the i-th transmit antenna with the input constellation being M-ary square QAM (Gray bit mapping) can be represented as

$$\bar{P}_{BER}(\gamma_i)=\sum_{j=1}^{\sqrt{M}-1}a_j^M\int_{\gamma_i}\text{erfc}\left(\sqrt{b_j^M\gamma_i}\right)f(\gamma_i)\,\text{d}\gamma_i,\tag{13}$$

where a_j^M and b_j^M are specified by the signal constellation, γ_i is the average SINR of the i-th transmit antenna, $\text{erfc}(x)=\frac{2}{\sqrt{\pi}}\int_x^\infty e^{-u^2}\text{d}u$ is the error function, and $f(\gamma_i)$ represents the probability density function of γ_i. Since it appears impossible to obtain a close-form solution of (13), an infinite series approximation of $\bar{P}_{BER}$ is developed. In [12], the average is expressed an infinite series of generalized hypergeometric functions. Our solution is different and perhaps a less complicated solution.

From [13, page 939], (13) can be rewritten as an infinite series

$$\bar{P}_{BER}(\gamma_i) = \frac{2}{\sqrt{\pi}} \sum_{j=1}^{\sqrt{M}-1} a_j^M \sum_{m=1}^{\infty} \frac{(-1)^{(m+1)}(b_j^M)^{(m-\frac{1}{2})}}{(2m-1)(m-1)!} D_{i;m}, \tag{14}$$

where $D_{i;m}$ is a factor that accounts for the type of reception. Detailed derivation of $D_{i;m}$ is impossible due to the page limit and will be provided in a journal submission. We next give a recursive definition for $D_{i;m}$. We first define $\varpi = \frac{E_s}{N_t} \cdot \sigma_m^2$ and $\mu = \mathrm{Var}\left\{\alpha_{k,i}^{(n)}\right\}$, which will be used in the following subsections. Three reception methods are considered next: (1) demodulation without combining, (2) EGC, and (3) MRC.

A. BER without Receiving Combining

The bit error rate measured at the k-th receive antenna for the i-th transmit antenna can be approximated by (14) with $D_{i;m}^k$ instead of $D_{i;m}$ being used here. When $m > 2$, we have $D_{i;m}^k = \frac{\varpi\left[(2m-3)\mu+\nu\right]}{\mu^2(m-\frac{3}{2})} \cdot D_{i;m-1}^k - \frac{\varpi^2}{\mu^2} \cdot D_{i;m-2}^k$. The initial condition is given by

$$D_{i;1}^k = \int_0^{\infty} \frac{\varpi^{\frac{1}{2}} h^{\frac{1}{2}}}{(\mu h+\nu)^{\frac{1}{2}}} e^{-h} \mathrm{d}h. \tag{15}$$

B. BER with EGC

For a MIMO-OFDM system with EGC at the receiver, the average bit error rate can be approximated by (14) with $D_{i;m}^{\mathrm{EGC}}$ instead of $D_{i;m}$ being used here. Defining $\nu^E = N_r\nu$, $\sigma_H^2 = \frac{(N_r!)^2}{8\left[(N_r-\frac{1}{2})\cdots\frac{1}{2}\right]^2}$, $\tilde{\nu}^E = \nu^E - \frac{\mu N_r(N_r-1)\pi}{4}$ and $\tilde{\mu} = 2\sigma_H^2 \cdot \mu$, when $m > 2$, $D_{i;m}^{\mathrm{EGC}} = \frac{2\sigma_H^2\varpi\left[(2m+N_r-4)\tilde{\mu}(N_r-1)!+\tilde{\nu}^E\right]}{\tilde{\mu}^2(m-\frac{3}{2})(N_r-1)!} \cdot D_{i;m-1}^{\mathrm{EGC}} - \frac{(2\sigma_H^2\varpi)^2(m+N_r-\frac{5}{2})}{\tilde{\mu}^2(m-\frac{3}{2})} \cdot D_{i;m-2}^{\mathrm{EGC}}$. The initial condition is given by

$$D_{i;1}^{\mathrm{EGC}} = \frac{(2\sigma_H^2\varpi)^{\frac{1}{2}}}{(N_r-1)!} \int_0^{\infty} \frac{h^{(N_r-\frac{1}{2})}}{(\tilde{\mu}h+\tilde{\nu}^E)^{\frac{1}{2}}} e^{-h} \mathrm{d}h. \tag{16}$$

C. BER with MRC

For a MIMO-OFDM system with channel knowledge at the receiver, the receiving diversity can be optimized by using MRC, and the average bit error rate can be approximated by (14) with $D_{i;m}^{\mathrm{MRC}}$ instead of $D_{i;m}$ being used here. Defining $\nu' = \left[\nu + \left(\frac{E_s}{N_t}+\mu\right)\sigma_{\Delta H}^2\right]$ and $\nu^M = \nu' + \nu \cdot \sigma_{\Delta H}^2$, $D_{i;m}^{\mathrm{MRC}}$ with $m > 2$ being given by $D_{i;m}^{\mathrm{MRC}} = \frac{\varpi\left[(2m+N_r-4)\mu(N_r-1)!+\tilde{\nu}^M\right]}{\mu^2(m-\frac{3}{2})(N_r-1)!} \cdot D_{i;m-1}^{\mathrm{MRC}} - \frac{\varpi^2(m+N_r-\frac{5}{2})e^{-(N_r-1)}}{\mu^2(m-\frac{3}{2})} \cdot D_{i;m-2}^{\mathrm{MRC}}$. The initial condition is given by

$$D_{i;1}^{\mathrm{MRC}} = \frac{e^{-(N_r-1)}\varpi^{\frac{1}{2}}}{(N_r-1)!} \int_0^{\infty} \frac{h^{(N_r-\frac{1}{2})}}{(\mu h+\tilde{\nu}^M)^{\frac{1}{2}}} e^{-h} \mathrm{d}h. \tag{17}$$

V. Numerical Results

In this section, the quasi-static MIMO-OFDM wireless channels are assumed, i.e., the channel impulse response is fixed over one OFDM symbol period but changes across the symbols.

In Fig. 1 to Fig. 4, we compare the bit error rates of QPSK and 16QAM with different combining methods in MIMO-OFDM. Note that IAI cannot be totally eliminated due to the non-zero frequency offset and channel estimation errors, which makes a bit-error-rate floor appearing at a high SNR. IAI can be reduced considerably by exploiting the receiving diversity using either EGC or MRC.

Without the receiving combining, the bit error rate at each receive antenna will be much worse than that in SISO-OFDM, simply because of the average SINR degradation due to the IAI. For example, when $N_t = N_r = 2$ and $\sigma_{\Delta H}^2 = 10^{-4}$, the bit error rate with QPSK is about 5.5×10^{-3} when $\sigma_{res}^2 = 10^{-4}$, and this error is much higher than that in SISO-OFDM (the bit error rate of SISO-OFDM is 1.8×10^{-3}), as shown in Fig. 1. For a given number of receive antennas, MRC can achieve a lower bit error rate than that with EGC, provided that an accurate channel estimation is assumed at the receiver. For example, in Fig. 2, when $\sigma_{\Delta H}^2 = 10^{-4}$ with $N_t = N_r = 2$ and 16QAM, the performance improvement of EGC (MRC) over that without combining is about 5.5 dB (6 dB), and that performance improvement will increase to 7.5 dB (8.5 dB) if σ_{res}^2 is increased to 10^{-3}. By increasing the number of receive antennas to 4, this performance improvement will be about 8.2 dB (9 dB) for EGC (MRC) with $\sigma_{\Delta H}^2 = 10^{-4}$, or 11 dB (13.9 dB) for EGC (MRC) with $\sigma_{\Delta H}^2 = 10^{-3}$, as shown in Fig. 4. If accurate frequency offset and channel estimation is performed at the receiver, the proposed theoretical analysis can also be accurate.

VI. Conclusions

The bit error rate impaired by the Frequency offset and channel estimation errors in MIMO-OFDM is discussed. IAI due to the residual Frequency offsets will degrade the bit error rate of MIMO-OFDM, which is analyzed in this paper. The bit error rates with different receiving combiners, i.e., EGC and MRC, for MIMO-OFDM are also analyzed. In a quasi-static MIMO-OFDM wireless channel, an infinite-series approximation of the bit error rate that is derived in this paper is accurate, as shown by the simulation results.

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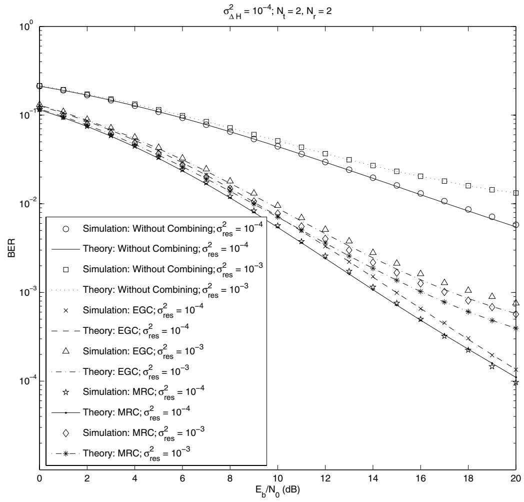

Fig. 1. BER performance with QPSK modulation when $(N_t = 2, N_r = 2)$.

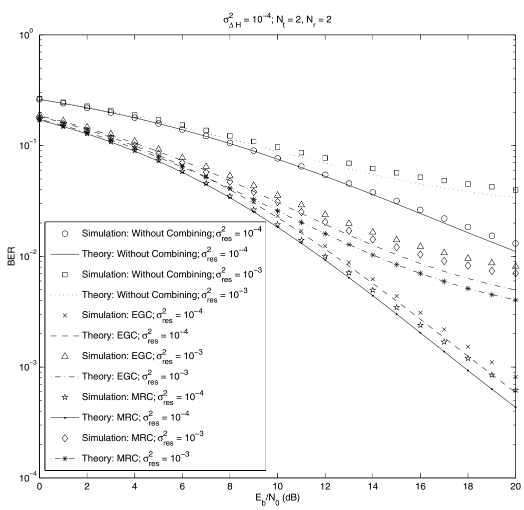

Fig. 2. BER performance with 16QAM modulation when $(N_t = 2, N_r = 2)$.

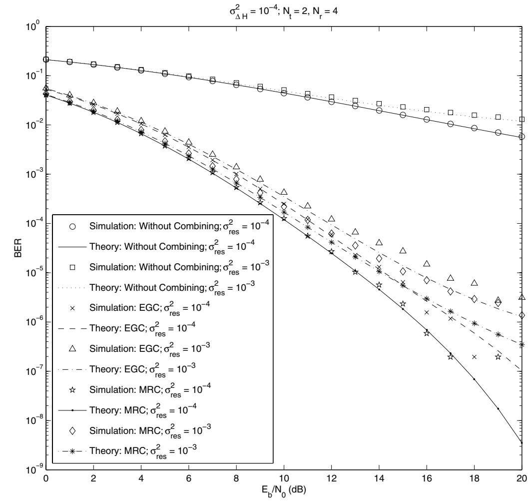

Fig. 3. BER performance with QPSK modulation when $(N_t = 2, N_r = 4)$.

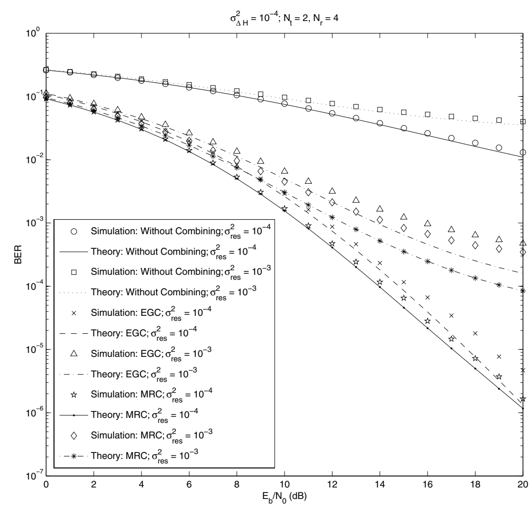

Fig. 4. BER performance with 16QAM modulation when $(N_t = 2, N_r = 4)$.

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