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Precoding for Multiuser Orthogonal Space-Time Block-Coded OFDM: Mean or Covariance Feedback?

Yu Fu, Witold A. Krzymień*, and Chintha Tellambura
Department of Electrical and Computer Engineering, University of Alberta
Edmonton, Alberta, Canada
* also with TR*Labs*, Edmonton, Alberta, Canada
Email:{yufu, wak, chintha@ece.ualberta.ca}

***Abstract*— This paper presents precoding design for error-rate improvement in closed-loop multiuser orthogonal space-time block-coded (OSTBC) multiple-input multiple-output (MIMO) orthogonal frequency-division multiplexing (OFDM) downlink, where both mean feedback and covariance feedback are available. We derive adaptive linear precoding and non-linear Tomlinson-Harashima precoding (THP) over a transmit-antenna-correlated, frequency-selective fading MIMO channel with estimation errors and feedback delay. In our precoder, mean-feedback precoding or covariance-feedback precoding, is adaptively chosen at the user terminal. The maximum achievable signal-to-noise power ratio (SNR) is used as the precoding-mode selection criterion. Each user calculates the selection metric and decides whether mean feedback is necessary. We confirm the intuition that mean-feedback precoding offers BER gains over covariance-feedback precoding when mean feedback becomes sufficiently accurate. Our adaptive precoding outperforms either mean-feedback precoding or covariance-feedback precoding in multiuser OSTBC OFDM, and considerably reduces the bit error rate (BER). Non-linear adaptive precoding is shown to outperform linear adaptive precoding.**

***Index Terms*— precoding, multiuser MIMO OFDM, OSTBC, mean feedback, covariance feedback**

I. Introduction

We consider a closed-loop multiuser downlink communication link with wideband multiple-input multiple-output (MIMO) orthogonal frequency-division multiplexing (OFDM), where both mean feedback (the first-order channel statistics) and covariance feedback (the long-term /second-order statistical information) can be available. For such a multiuser MIMO OFDM downlink system, transmit diversity is attractive because multiple antennas can be located at the base station (BS) and the benefits shared by all users. Orthogonal space-time block coding (OSTBC)[1] [1], [2] is a powerful transmit-diversity scheme, which can achieve full diversity with low-complexity optimal decoding, and has been adopted in several 3G standards.

OSTBC has originally been designed for an open-loop system, where channel state information (CSI) is only known at the receiver but not at the transmitter. However, if CSI can be available at the transmitter, closed-loop approaches such as transmitter precoding can exploit the channel conditions, simplify each user's receiver, and offer significant system error-rate and/or capacity improvement. OSTBC can thus be designed through the construct of precoding to optimize the MIMO transmission [3]–[5]. On the other hand, the benefits of the original OSTBC are diminished by insufficient antenna spacing at the BS, which leads to transmit-antenna correlations. Antenna correlations significantly reduce the system capacity [6] and increase the bit error rate (BER) [7]. We thus need precoding which can offer the original OSTBC the flexibility of adapting to correlated MIMO channels [8]. In OFDM systems, precoding, which enables pre-processing of the signals at a subcarrier level, improves capacity in spatially-multiplexed OFDM, and reduces the error rate in OSTBC OFDM [9].

A typical precoding design needs either mean feedback [3]–[5] or covariance feedback [8], [9]. Mean feedback can be estimated at the BS by using the reciprocity of radio channels in time-division duplex (TDD) systems. For frequency-division duplex (FDD) systems, CSI must be estimated at the user terminal and explicitly sent back to the BS, possibly via a low-rate limited-feedback link. The channel covariance can be readily obtained without overhead for both TDD and FDD systems by averaging the uplink measurements. Hence, the feedback requirements for covariance feedback could be much less than for mean feedback. Evidently, the quality of mean feedback will be degraded due to channel estimation errors, and it is more sensitive to the channel time variations and feedback delay than covariance feedback, since the long-term covariance matrices change much slower than the channel gains or even do not change at all. In contrast, covariance feedback may become less helpful when the mean feedback can be accurate. References [10] studies precoding strategies for mean and covariance feedback to approach capacity in MIMO flat-fading channels and multiple-input single-output multiuser OFDM downlink (each user has one receive antenna); spatially uncorrelated channels are considered. Efficient use of mean and/or covariance feedback in precoding design for error-rate improvement is still an open problem.

In this paper, we develop adaptive linear precoding and non-linear Tomlinson-Harashima precoding (THP) to effectively exploit both mean and covariance feedback for error-rate improvement in a multiuser OSTBC OFDM system. We consider a general frequency-selective fading channel model with transmit

[1]OSTBC here stands for orthogonal space-time block-coded and orthogonal space-time block coding, depending on the context.

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antenna correlations and assume imperfect channel estimation. Each user's receiver sends the inaccurate estimates back to the BS via a feedback channel, which introduces delay over channel time variations. Exploiting the channel statistics, we derive the maximum achievable signal-to-noise power ratios (SNRs) for mean-feedback precoding proposed in [5] and covariance-feedback precoding proposed in [9]. Next, we propose adaptive dual-mode precoding switching between mean-feedback precoding and covariance-feedback precoding. The SNR is a useful performance indicator in OSTBC systems and is used as the precoding-selection metric. Each user's receiver calculates the metric and decides whether mean feedback is necessary or not. The decision is sent back to the BS using only one bit per subcarrier. [2] According to the decision information, the BS adaptively chooses between mean-feedback and covariance-feedback precoding. Our precoding reduces the required capacity of the feedback link since only covariance-feedback precoding is used when mean feedback cannot be sufficiently accurate, and outperforms individually either mean-feedback or covariance-feedback precoding. Non-linear adaptive precoding is shown to outperform its linear counterpart.

II. System Model

This section will introduce the system model of an N-subcarrier OFDM downlink system with M_T transmit antennas and U simultaneously active users in the presence of transmit antenna correlations. The u-th user has M_u receive antennas, and the total number of receive antennas is $M_R = \sum_{u=1}^{U} M_u$. The U users share one OFDM symbol and the u-th user is assigned a subset $\mathbb{K}_u$ containing N_u subcarriers, $0 < N_u \leq N$. At the BS, a subcarrier allocation algorithm maps the user data to the corresponding subcarriers, and this algorithm is known at both the BS and the user side. With their half-wavelength separation it is reasonable to assume the receive antennas at the mobile stations (MSs) are uncorrelated.

A. Antenna Correlations in Multiuser OSTBC OFDM

The channel between the m-th transmit antenna and the u-th user's n-th receive antenna is a wideband frequency-selective fading channel with L resolvable paths. The l-th path gain is a zero-mean complex Gaussian random variable (Rayleigh fading) with variance σ_l^2. At the time i, the set of the l-th path gains between the BS and the u-th user can be represented by an $M_u \times M_T$ matrix $\mathbf{H}_{u,l}[i]$ with entries $h_{u_{m,n},l}[i]$. We assume that the channel gains remain constant over several OFDM symbol intervals. As in [7], the $M_u \times M_T$ channel on the k-th subcarrier of the u-th user can be represented as

$$\mathbf{H}_u[k,i] = \sum_{l=0}^{L-1} \mathbf{H}_{u,l}[i] e^{-j\frac{2\pi}{N}kl} \mathbf{r}_T = \breve{\mathbf{H}}_u[k,i]\mathbf{r}_T, \quad k \in \mathbb{K}_u, \tag{1}$$

where $\mathbf{r}_T$ is an $M_T \times M_T$ matrix. Different users have the same transmit antenna correlation matrix $\mathbf{R}_T = \mathbf{r}_T^H \mathbf{r}_T$ with entries which are given by [6] $R_T(p,q) = \mathcal{J}_0(2\pi|p-q|\zeta_T)$, where $\mathcal{J}_0$ is zero-order Bessel function of the first kind; $\zeta_T = \Delta\frac{d_T}{\lambda_c}$, $\lambda_c = c/f_c$ is the wavelength at the center frequency f_c, Δ is the angle of arrival spread, and the transmit antennas are spaced by d_T.

At the receiver, the k-th received signal vector is

$$\mathbf{Y}_u[k,i] = \mathbf{H}_u[k,i]\mathbf{X}_u[k,i] + \mathbf{W}_u[k,i], \quad k \in \mathbb{K}_u, \tag{2}$$

where $\mathbf{Y}_u[k,i]$ is an M_u-dimensional vector and $\mathbf{W}_u[k,i]$ is the noise vector where the entries $W_{u_n}[k,i]$ are i.i.d. additive white Gaussian noise (AWGN) samples with zero mean and variance σ_W^2. The input data vector is $\mathbf{X}_u[k,i] = \left[X_{u_1}[k,i] \ldots X_{u_{M_T}}[k,i]\right]^T$; $X_{u_m}[k,i]$ denotes an M-ary quadrature amplitude modulation (QAM) symbol on the k-th subcarrier sent to the u-th user by the m-th transmit antenna at time i. In a multiuser OSTBC OFDM system, $\mathbf{X}_u[k,i]$ will be an OSTBC matrix instead of a data vector. The u-th user's $N_u M_u \times N_u M_T$ channel matrix is $\mathbf{H}_u[i] = \mathrm{diag}\left[\mathbf{H}_u[j,i] \ldots \mathbf{H}_u[k,i]\right]$, $j,\, k \in \mathbb{K}_u$. We consider the structure described by (2), using which precoding can be designed individually for each subcarrier.

B. Models for Mean and Covariance Feedback

This subsection introduces mathematical models for mean feedback as in [5] and covariance feedback, and gives the conditional expectation and variance of the channel matrix exploiting the channel statistics.

1) Covariance Feedback Only: The u-th user's channel on subcarrier k in (1) can be considered as a zero-mean complex Gaussian matrix with variance

$$\mathbf{C}_{\mathbf{H}_u\mathbf{H}_u}[k] = \mathrm{E}\left[\mathbf{H}_u^H[k,i]\mathbf{H}_u[k,i]\right] = \mathbf{R}_T, \tag{3}$$

where $\mathbf{R}_T = \mathbf{r}_T^H \mathbf{r}_T$; $\mathbf{r}_T$ is given by (1), $k \in \mathbb{K}_u$, and $\mathbf{R}_T = \mathbf{R}_T^H$. The proof of (3) is given in [5]. $\mathbf{R}_T$ changes slower than the channel response or even does not change at all. It is easily available at both the BS and the user terminal. When only covariance feedback is available, i.e., the conditional mean $\mathrm{E}\left[\mathbf{H}_u[k,i] \,|\, \mathbf{H}_{T_u}[k,i]\right] = \mathbf{0}$, where $\mathbf{H}_{T_u}[k,i]$ is the channel matrix at the BS.

2) Mean and Covariance Feedback: In this case, both mean and covariance feedback are available. Each user's receiver has inaccurate channel estimates $\mathbf{H}_{R_u}[k,i]$ of the current actual, but unknown, channel $\mathbf{H}_u[k,i]$, $k \in \mathbb{K}_u$; the imperfect channel estimates are sent to the BS via a feedback channel which introduces delay τ_u, and $\tau_u \neq \tau_{u'}$, $\forall\, u \neq u'$. Consequently, the BS has the inaccurate estimate $\mathbf{H}_{T_u}[k,i]$ of the actual (unknown) but outdated channel matrix $\mathbf{H}_u[k,i-\tau_u]$, in which the $\{m,n\}$th channel of the u-th user is τ_u seconds older than that in the current channel $\mathbf{H}[k,i]$. The actual channel matrices $\mathbf{H}_u[k,i]$ and $\mathbf{H}_u[k,i-\tau]$ are unknown at both the transmitter and the receiver, and $\mathbf{H}_u[k,i] \neq \mathbf{H}_u[k,i-\tau] \neq \mathbf{H}_{T_u}[k,i] \neq \mathbf{H}_{R_u}[k,i]$.

We model the frequency-selective fading channel as follows:

- The entries in a tap vector for the $\{m,n\}$th antenna pair of the u-th user $\mathbf{h}_{u_{m,n}}[i] = \left[h_{u_{m,n},0}[i], \ldots, h_{u_{m,n},L-1}[i]\right]^T$

[2]This one bit is in addition to the mean or covariance information feedback.

are time-varying according to Clarke's 2-D isotropic scattering model with maximum Doppler shift f_{D_u} [11]. The delayed version $\mathbf{h}_{u_{m,n}}[i-\tau_u]$ and $\mathbf{h}_{u_{m,n}}[i]$ are jointly Gaussian with an autocovariance matrix

$$\mathrm{E}\left[\mathbf{h}_{u_{m,n}}[i]\mathbf{h}_{u_{m,n}}^H[i-\tau_u]\right] = J_u\left[\sigma_0^2,\ldots,\sigma_{L-1}^2\right], \quad (4)$$

where $J_u = \mathcal{J}_0(2\pi\epsilon_u)$ and $\epsilon_u = f_{D_u}\tau_u$.

- The channel estimates at the receiver are maximum likelihood (ML) estimates and can be expressed as

$$\mathbf{H}_{R_u}[k,i] = \mathbf{H}_u[k,i] + \mathbf{e}_u[k], \quad (5)$$

where $\mathbf{e}_u[k]$ is the estimation error with entries $e_{m,n}[k] \sim \mathcal{CN}(0,\,\Omega_{e_u}), \forall\, i,k,u$, and $\mathbf{e}_u[k]$ is independent of all other stochastic processes.

- The transmitter channel matrix $\mathbf{H}_{T_u}[k,i]$ is an imperfect estimate of the actual but unknown $\mathbf{H}_u[k,i-\tau_u]$, and can be modeled by

$$\mathbf{H}_{T_u}[k,i] = \mathbf{H}_{R_u}[k,i-\tau] = \mathbf{H}_u[k,i-\tau_u] + \mathbf{e}_u[k]. \quad (6)$$

Since both mean and covariance feedback are available, the conditional mean and variance of the actual channel matrix given $\mathbf{H}_{T_u}[k,i]$ can be calculated

$$\begin{aligned}\mathbf{H}_{u|T_u}[k,i] &= J_u\mathbf{H}_{T_u}[k,i]\mathbf{R}_T\left(\mathbf{R}_T+\Omega_{e_u}\mathbf{I}_{M_T}\right)^{-1},\\ \mathbf{C}_{u|T_u} &= \mathbf{R}_T - J_u^2\mathbf{R}_T\left(\mathbf{R}_T+\Omega_{e_u}\mathbf{I}_{M_T}\right)^{-1}\mathbf{R}_T.\end{aligned} \quad (7)$$

The proof of (7) is given in [5]. Since the receiver has the information $\mathbf{H}_{R_u}[k,i-\tau_u] = \mathbf{H}_{T_u}[k,i]$ (6), the mean and variance in (7) can be calculated at both the BS and each user's receiver according to $\mathbf{H}_{T_u}[k,i]$ and $\mathbf{H}_{R_u}[k,i]$. Similarly, at each user's receiver, we can also obtain

$$\begin{aligned}\mathbf{H}_{u|R_u}[k,i] &= \mathbf{H}_{R_u}[k,i]\mathbf{R}_T\left(\mathbf{R}_T+\Omega_{e_u}\mathbf{I}_{M_T}\right)^{-1},\\ \mathbf{C}_{u|R_u} &= \Omega_{e_u}\mathbf{R}_T\left(\mathbf{R}_T+\Omega_{e_u}\mathbf{I}_{M_T}\right)^{-1}.\end{aligned} \quad (8)$$

The conditional means $\mathbf{H}_{u|T_u}[k,i]$ and $\mathbf{H}_{u|R_u}[k,i]$ can be described as equivalent channels exploiting the channel statistics and uncertainty structure to mitigate the impact of imperfect CSI at the BS and the receivers [12]. The covariances $\mathbf{C}_{u|T_u}$ and $\mathbf{C}_{u|R_u}$ indicate the CSI at the transmitter (CSIT) and CSI at the receiver (CSIR) uncertainty given by the equivalent channels, respectively. The CSIR uncertainty is determined by the channel correlation matrix and the estimation error. If CSIR is perfect, i.e., $\Omega_{e_u}=0, \forall\, u$, we have $\mathbf{H}_{u|R_u}[k,i] = \mathbf{H}_{R_u}[k,i] = \mathbf{H}_u[k,i]$. At the BS, beside $\mathbf{R}_T$ and estimation errors, the uncertainty also depends on the autocovariance factor J_u, which is the function of the normalized maximum Doppler shift. As the maximum Doppler shift increases, which may be caused by rapidly growing mobility of the user, the CSIT uncertainty may become significant.

III. Precoding with Mean and Covariance Feedback

In this section, we introduce mean-feedback precoding in multiuser OSTBC OFDM [5], including linear precoding and non-linear THP. We also generalize covariance-feedback precoding [9] from single to multiple-user case.

A. Mean-Feedback Precoding

In this case, [5] presents a minimum-BER precoder which takes into account estimation errors and the channel time variation. For simplicity, we omit the time index i. The general form of the linear precoding matrix $\mathbf{E}_{\mathrm{MF}_u}[k]$ can be given by

$$\mathbf{E}_{\mathrm{MF}_u}[k] = \widetilde{\mathbf{V}}_u[k]\mathbf{\Lambda}_{\mathrm{MF}_u}[k]\widetilde{\mathbf{V}}_u^H[k], \quad (9)$$

where $\mathbf{\Lambda}_{\mathrm{MF}_u}[k]$ is a positive semi-definite diagonal matrix representing the power distribution with the main diagonal entries λ_{u_m}. $\widetilde{\mathbf{V}}_u[k]$ is an $M_T \times M_T$ unitary matrix yielded by singular value decomposition (SVD) of the u-th user's equivalent channel matrix $\mathbf{H}_{u|T_u}[k,i]$ (7), which is $\mathbf{H}_{u|T_u}[k,i] = \widetilde{\mathbf{U}}_u[k]\widetilde{\mathbf{\Gamma}}_u[k]\widetilde{\mathbf{V}}_u^H[k]$. $\widetilde{\mathbf{\Gamma}}_u[k]$ is the diagonal singular value matrix with the real, non-negative entries $\widetilde{\gamma}_{u_m}$. According to the maximum SNR criterion, the error-rate minimization precoding matrix is [5]

$$\mathbf{E}_{\mathrm{MF}_u}[k]_{\mathrm{opt}} = \arg\max_{\mathbf{\Lambda}_{\mathrm{MF}_u}[k]} \mathrm{tr}\left[\widetilde{\mathbf{V}}_u^H[k]\left(\widetilde{\mathbf{V}}_u[k]\mathbf{\Gamma}_u^2[k]\widetilde{\mathbf{V}}_u^H[k] + \mathbf{C}_{u|T_u}\right)\widetilde{\mathbf{V}}_u[k]\mathbf{\Lambda}_{\mathrm{MF}_u}^2[k]\right], \quad (10)$$

subject to $\mathrm{tr}\left(\mathbf{\Lambda}_{\mathrm{MF}_u}^2[k]\right) = M_T$. The solution of the optimization problem (10) can be obtained numerically. The effective channel $\mathbf{H}_{u|R_u}[k,i]\mathbf{E}_{\mathrm{MF}_u}[k]_{\mathrm{opt}}$ is used at the ML receiver, where $\mathbf{H}_{u|R_u}[k,i]$ is the receiver equivalent channel matrix given in (8).

B. Covariance-Feedback Precoding

When mean feedback is not available, the BS knows only the channel correlation matrix. Here, we generalize covariance-feedback precoding from single [9] to multiuser downlink cases.

The transmit-antenna correlation matrix can be written as

$$\mathbf{R}_T = \mathbf{r}_T^H\mathbf{r}_T = \mathbf{V}_T\mathbf{\Gamma}_T^2\mathbf{V}_T^H, \quad (11)$$

where $\mathbf{V}_T$ is an $M_T \times M_T$ unitary matrix and $\mathbf{\Gamma}_T$ is the diagonal singular-value matrix of $\mathbf{r}_T$ with real, non-negative entries γ_{T_m}, $m = 1,\ldots,M_T$, in descending order. From [9] , the optimal covariance-feedback precoding matrix minimizing the pairwise error probability is

$$\mathbf{E}_{\mathrm{CF}_u}[k]_{\mathrm{opt}} = \arg\max_{\tilde{\mathbf{Z}}_u[k]} \log\det\left(\mathbf{\Gamma}_T\tilde{\mathbf{Z}}_u[k]\mathbf{\Gamma}_T + \mathbf{I}_{M_T}\right) \quad (12)$$

subject to $\mathrm{tr}\left(\tilde{\mathbf{Z}}_u[k]\right) = \xi M_T$, where $\xi = \frac{\mu_{\min}}{4\sigma_W^2}$, $\mu_{\min}$ is the minimum distance over all pairs of the codewords at the transmitter, $\tilde{\mathbf{Z}}_u[k] = \xi\mathbf{V}_T^H\mathbf{E}_{\mathrm{CF}_u}[k]\mathbf{E}_{\mathrm{CF}_u}^H[k]\mathbf{V}_T$. The optimal $\tilde{\mathbf{Z}}_u[k]$ results from a waterfilling solution and is only determined by $\mathbf{r}_T$. We thus have $\tilde{\mathbf{Z}}_u[k]_{\mathrm{opt}} = \tilde{\mathbf{Z}}_{\mathrm{opt}}$. The optimal precoding matrix is

$$\mathbf{E}_{\mathrm{CF}_{\mathrm{opt}}} = \frac{1}{\sqrt{\xi}}\mathbf{V}_T\mathbf{\Lambda}_{\mathrm{CF}}\mathbf{V}_T^H, \quad (13)$$

where $\mathbf{\Lambda}_{\mathrm{CF}} = \sqrt{\tilde{\mathbf{Z}}_{\mathrm{opt}}}$ is an $M_T \times M_T$ diagonal matrix with the m-th main entry λ_{CF_m}. The solution is the identical for all subcarriers.

The precoding design, using either mean-feedback (10) or covariance-feedback (13) approach, aims to minimize the system error rate, which depends on SNR in OSTBC systems. Mean feedback needs more feedback capacity and is more sensitive to the channel time variations and feedback delay. On the other hand, covariance-feedback precoding is a one-size-fits-all solution which does not represent the instantaneous and varying channel conditions. Covariance feedback thus may become less helpful when mean feedback is accurate. Naturally, how to efficiently use mean and covariance feedback and optimize the SNR determines the error probability.

C. Non-Linear Tomlinson-Harashima Precoding

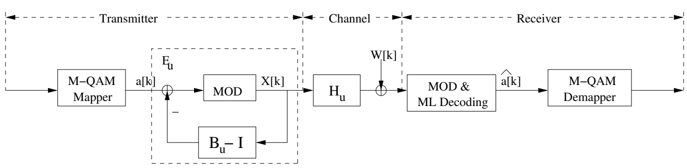

Fig. 1. Tomlinson-Harashima precoding in multiuser OSTBC OFDM downlink.

The structure of non-linear THP for mean-feedback precoding [5] and covariance-feedback precoding [9] is shown in Fig. 1. Regardless of the modulo device, the the u-th user's feedback structure is equivalent to $\mathbf{B}_u^{-1}[k]$, which can be optimally designed as, $\mathbf{B}_u[k]_{\text{opt}} = \mathbf{E}_u^{-1}[k]_{\text{opt}}$, where $\mathbf{E}_u[k]_{\text{opt}}$ can be the precoding mode in (10) or (13). The effective channel is $\mathbf{H}_u[k]\mathbf{B}_u^{-1}[k]_{\text{opt}}$ and ML decoding is used at the receiver. After ML decoding and discarding the modulo congruence, the unique estimates of the data symbols can be obtained. If the input sequence $a[k]$ is a sequence of i.i.d. samples, the output of the modulo device is also a sequence of i.i.d. random variables, and the real and imaginary parts are independent, i.e., we can assume $\mathrm{E}\left[\mathbf{X}_u[k]\mathbf{X}_u^H[k]\right] = E_s\mathbf{I}_{M_T}, \forall\, k$. The details of operations in THP can be found in [5], [9], [13].

IV. Mean or Covariance Feedback?

This section develops adaptive precoding which switches between mean-feedback precoding (10) and covariance-feedback precoding (13). We derive the maximum achievable SNR in both cases of (10) and (13). The SNRs are calculated at each user terminal and proposed as the selection metric.

A. Maximum Achievable SNR

In a precoded OSTBC system, exploiting the channel statistics, the SNR on the k-th subcarrier of the u-th user can be given by

$$\begin{aligned}\text{SNR}_u[k] &= \frac{E_s}{\sigma_W^2}\mathrm{E}\left[\mathrm{tr}\left(\mathbf{E}_u^H[k]_{\text{opt}}\mathbf{H}_u^H[k]\mathbf{H}_u[k]\mathbf{E}_u[k]_{\text{opt}}\right)\right] \\ &= \frac{E_s}{\sigma_W^2}\mathrm{tr}\left(\mathbf{E}_u^H[k]_{\text{opt}}\boldsymbol{\Upsilon}_u[k]\mathbf{E}_u[k]_{\text{opt}}\right),\end{aligned} \tag{14}$$

where $\boldsymbol{\Upsilon}_u[k] = \mathbf{H}_{u|T_u}^H[k]\mathbf{H}_{u|T_u}[k] + \mathbf{C}_{u|T_u}$. If only covariance feedback is available, the BS only knows the correlation matrix, i.e., the conditional mean of the actual channel is zero. Substituting the variance from (3), the precoding matrix (13) and (11) into (14), the SNR of a covariance-feedback precoded system is identical for all users, and becomes

$$\text{SNR}_{\text{CF}} = \frac{E_s}{\xi\sigma_W^2}\mathrm{tr}\left(\boldsymbol{\Gamma}_T^2\boldsymbol{\Lambda}_{\text{CF}}^2\right) = \frac{E_s}{\sigma_W^2}f. \tag{15}$$

Clearly, the SNR of covariance-feedback precoded OSTBC OFDM is independent on the index of subcarriers and the users'mobile speeds. Different users will have identical SNRs.

When mean feedback is available, the BS can calculate the equivalent channel matrix $\mathbf{H}_{u|T_u}[k]$ and the SNR becomes

$$\text{SNR}_{\text{MF}_u}[k] = \frac{E_s}{\sigma_W^2}\mathrm{tr}\left(\mathbf{E}_{\text{MF}_u}^H[k]_{\text{opt}}\boldsymbol{\Upsilon}_u[k]\mathbf{E}_{\text{MF}_u}[k]_{\text{opt}}\right). \tag{16}$$

The SNR for a mean-feedback precoded OSTBC system is a function of J_u. Since the precoding matrix (10) is determined by the transmitter equivalent channel matrix $\mathbf{H}_{u|T_u}[k]$, whose accuracy is dominated by J_u and Ω_{e_u}, SNR in (16) is sensitive to the channel estimation errors and time variations.

B. When to Use Mean Feedback

In this subsection, we find an indicator of the accuracy needed for mean feedback to give a higher SNR over covariance feedback, i.e., a lower BER. The accuracy of mean feedback can be gauged by its correlation with the actual channel response, which primarily depends on the autocorrelation function J_u and estimation errors Ω_{e_u}. There may exist certain values of J_u and Ω_{e_u} such that for some channel realizations, the SNR of mean-feedback precoded systems given by (16) is greater than the SNR of covariance-feedback precoded systems given by (15), i.e., mean feedback is helpful to achieve a lower BER.

To use mean feedback on the subcarrier k for user u, we need

$$\text{SNR}_{\text{MF}_u}[k] > \text{SNR}_{\text{CF}}. \tag{17}$$

We first consider a simple case, in which the receiver has perfect channel estimates, i.e., $\Omega_{e_u} = 0, \forall\, u$. The mean and covariance of the actual channel in (7) are thus simplified to

$$\mathbf{H}_{u|T_u}[k] = J_u\mathbf{H}_{T_u}[k] = J_u\mathbf{H}_u[k], \quad \mathbf{C}_{u|T_u} = (1 - J_u^2)\mathbf{R}_T. \tag{18}$$

We thus have SNRs

$$\begin{aligned}\text{SNR}_{\text{MF}_u}[k] &= \frac{E_s}{\sigma_W^2}(1 - J_u^2)\mathrm{tr}\left(\boldsymbol{\Lambda}_{\text{MF}_u}[k]\widetilde{\mathbf{V}}_u^H[k]\mathbf{R}_T\widetilde{\mathbf{V}}_u[k]\boldsymbol{\Lambda}_{\text{MF}_u}[k]\right) \\ &+ \frac{E_s}{\sigma_W^2}J_u^2\mathrm{tr}\left(\boldsymbol{\Lambda}_{\text{MF}_u}^2[k]\widetilde{\boldsymbol{\Gamma}}_u^2[k]\right) = \frac{E_s}{\sigma_W^2}\left((1 - J_u^2)c + J_u^2 b\right).\end{aligned} \tag{19}$$

The condition (17) can thus be simplified to $J_u^2(b - c) > f - c$, i.e., if J_u satisfies this inequality, mean feedback is sufficiently accurate to offer an error-rate gain. If the receiver has inaccurate channel estimates, each user calculates (15) and (16), and selects the precoding mode which has a higher SNR. The selection metric (17) is a function of J_u and Ω_{e_u}, which assesses the channel conditions and tells us whether mean feedback is helpful. The same selection metric is used for non-linear precoding.

The precoding mode is thus adaptively selected at each user terminal. Each user can calculate (7) and (17), compare the SNRs for both cases, and decide whether mean feedback is necessary. The decision can be sent back to the transmitter via a one-bit information per subcarrier. The BS chooses mean-feedback precoding (10) or covariance-feedback precoding (13) on different subcarriers, and allows distinct users to use different precoding modes when sharing one OFDM symbol. When the channel time variation is high or the channel conditions are poor, covariance-feedback precoding may outperform mean-feedback precoding. In this case, mean feedback does not need to be sent back, which significantly reduces the feedback requirements. If mean feedback can be accurate, mean-feedback precoding may perform better. The user can thus transfer the conditional mean back to the BS to achieve a lower BER.

V. Simulation Results

This section presents simulation results to show how our proposed precoders improve the system performance in a 64-subcarrier multiuser OSTBC OFDM system with transmit-antenna correlations. The vehicular B channel specified by ITU-R M. 1225 [14] is used where the channel taps represent zero-mean complex Gaussian random processes with variances of $-4.9\,$dB, $-2.4\,$dB, $-15.2\,$dB, $-12.4\,$dB, $-27.6\,$dB, and $-18.4\,$dB relative to the total power gain of the frequency-selective channel. ML decoding is used at each user's receiver. The BS and the user terminals know the correlation matrix $\mathbf{R}_T$ with the parameter $\zeta_T = \Delta\frac{d_T}{\lambda}$; the angle of arrival spread is assumed $12°$, i.e., $\Delta \approx 0.2$.

We consider the interval $\mathbb{I} = [0.9, 1]$ for the autocorrelation function J_u, and its values are assumed to be uniformly distributed in this interval. The maximum possible number of distinct Doppler shift values is U. In the interval $\mathbb{I} = [0.9, 1]$, the Doppler shifts $\epsilon_u = f_{D_u}\tau_u$ normalized with respect to feedback delays τ_u are in the range $[0, 0.1]$. For a wideband OFDM system with the carrier frequency $5\,$GHz, if the feedback delay is $100\,\mu s$, the users' mobile speeds are in the range from zero, walking speed to $216\,$km/h (train speed).

A. Maximum Achievable SNR

In Fig. 2, we compare the maximum achievable SNR of mean-feedback precoding and covariance-feedback precoding in Alamouti-coded OFDM systems. To reveal the relationship between the SNR and J_u, we consider the single-user case with 2 receive antennas and perfect channel estimation. We use mean-to-covariance-SNR ratio (MCSR) to show the gain that mean feedback can obtain gains over covariance feedback. Clearly, as J increases, i.e., the time variation decreases, the MCSR monotonously grows. On the other hand, as the correlation parameter ζ_T increases, i.e., the transmit-antenna correlations weaken, the MCSR also increases. When ζ_T is great than 0.3 and J is greater than 0.8, the value of MCSR is larger than 1, i.e., mean-feedback precoding achieves higher SNR. This confirms the intuition that when mean feedback can be sufficiently accurate, it offers a lower error rate than covariance feedback. Furthermore, THP performs better than linear precoding.

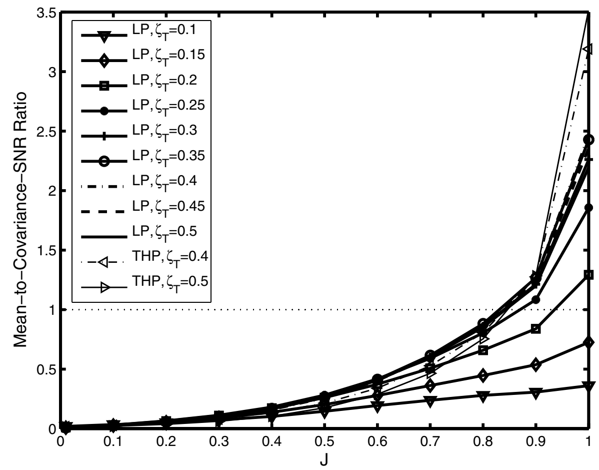

Fig. 2. The mean-to-covariance SNR ratio with linear precoding (LP) and THP as a function of the time-varying autocorrelation for 2×2 4-QAM Alamouti-coded 64-subcarrier single-user OFDM systems over transmit-antenna-correlated channels with different values of ζ_T, $\Omega_e = 0$.

B. Adaptive Precoding

In this subsection, we show how our new adaptive precoding improves the BER for multiuser OSTBC OFDM with transmit antenna correlations and imperfect channel estimation.

Fig. 3 shows the BER of the proposed adaptive linear precoder for 4-QAM Alamouti-coded 2-user OFDM when the correlation parameter ζ_T is 0.5. Perfect and imperfect channel estimation are considered. The variance of the estimation errors is $\Omega_e = 1/16$, $\forall\, u$. The BERs of mean-feedback linear precoding (MFB-LP) and covariance-feedback linear precoding (CFB-LP) are shown for reference. Adaptive precoding performs mode switching to find the better SNR. We therefore have $1.5\,$dB gain over covariance-feedback precoding when channel estimation is prefect and $2\,$dB gain when estimation is imperfect and eliminate the error floors in mean-feedback precoding at the BER of 10^{-4}.

In Fig. 4, we show the BERs of the proposed adaptive THP in 16-QAM $1/2$-rate OSTBC 4-user 4-transmit-antenna OFDM with perfect ($\Omega_e = 0$) and imperfect ($\Omega_e = 1/16$) channel estimation, when ζ_T is 0.4. The adaptive THP outperforms adaptive linear precoding, and outperforms MFB-THP and CFB-THP for both perfect and imperfect estimation cases.

VI. Conclusion

We have considered precoding design in multiuser OSTBC OFDM downlink over a general transmit-antenna-correlated, frequency-selective fading MIMO channel, when both mean and covariance feedback can be available. We have developed dual-mode precoding, in which mean-feedback precoding or

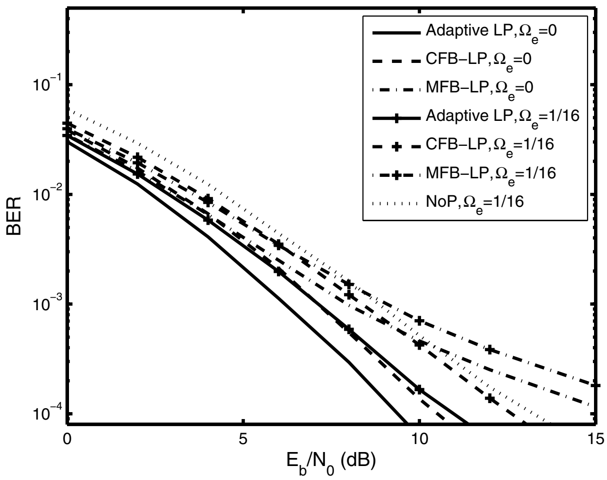

Fig. 3. BER as a function of SNR for mean-feedback linear precoding (MFB-LP), covariance-feedback linear precoding (CFB-LP), adaptive linear precoding and no precoding (NoP) in 64-subcarrier 4-QAM Alamouti-coded 2-user OFDM systems with perfect and imperfect estimation. $\zeta_T = 0.5$, $M_T = 2$, $M_u = 2$, $J_u \in \mathbb{I}$.

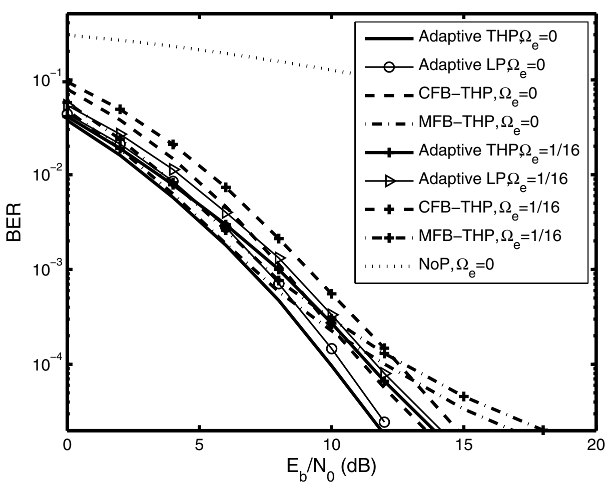

Fig. 4. BER as a function of SNR for mean-feedback THP (MFB-THP), covariance-feedback THP (CFB-THP), adaptive non-linear precoding, adaptive linear precoding and no precoding (NoP) in 64-subcarrier 16-QAM $1/2$-rate OSTBC 4-user OFDM systems with perfect and imperfect estimation. $\zeta_T = 0.4$, $M_T = 4$, $M_u = 2$, $J_u \in \mathbb{I}$.

covariance-feedback precoding is adaptively chosen at the receiver according to the channel conditions. Each user calculates the mode-switching metric and decides whether mean feedback is necessary. Our adaptive precoding considerably reduces the error rate, and outperforms individually either mean-feedback precoding or covariance-feedback precoding in multiuser OSTBC OFDM.

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