Receive Antenna Selection Based on Union-Bound Minimization Using Convex Optimization

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***Abstract*—Despite their high spectral efficiencies, multiple-input multiple-output (MIMO) systems suffer from high cost and complexity due to multiple radio frequency chains at both link ends. A possible solution is to select a subset of the available antennas at transmitter and/or receiver based on maximal capacity or minimal error rates. In this letter, we propose a receive antenna selection algorithm based on the minimization of the union bound on the vector error rate. By relaxing the antenna selection variables from discrete to continuous, we arrive at a convex optimization problem. Efficient numerical methods such as interior-point algorithms can be applied to solve this optimization problem with polynomial complexity**

***Index Terms*—Antenna selection, convex optimization, interior-point algorithm, log barrier method, union bound.**

I. Introduction

Multiple-input multiple-output (MIMO) systems, employing multiple antennas at both the transmitter and the receiver, achieve remarkably high spectral efficiencies in rich-scattering multipath environments. However, MIMO systems suffer from high cost and complexity due to multiple radio frequency chains at both link ends. A potential solution is to select a subset of the available antennas at transmitter and/or receiver while keeping the advantages of using all antennas [1].

MIMO antenna selection techniques have thus been extensively studied, and there are several antenna selection criteria. For full-diversity space-time codes, a subset of available antennas can be selected to maximize the channel norm [2]. For spatial-multiplexing systems, antennas can be selected to minimize the error rates [3]. Heath and Love [4] propose several approaches to pick any number of transmit antennas (termed multimode selection). A cross-layer approach to transmit antenna selection is given in [5]. A useful tutorial paper on antenna selection can be found in [6].

Exhaustive search based on maximum output SNR is proposed in [3] and [7] when the system uses linear receivers. Since exhaustive search is computationally expensive for large MIMO systems, several sub-optimal algorithms with lower complexity are derived at the expense of performance. A selection algorithm based on accurate approximation of the conditional error probability of quasi-static MIMO systems is derived in [8]. The authors in [1] and [9] develop upper and lower bounds for channel capacity with antenna selection. Near-optimal approaches for capacity maximization are given in [10] and [11].

In [12], the authors formulate the receive antenna selection problem as a combinatorial optimization problem and relax it to a convex optimization problem. They employ an interior point algorithm, i.e., the barrier method, to solve the relaxed convex problem. However, they treat only the case of capacity maximization.

However, perhaps the most important system performance metric is the bit-error-rate (BER) or the vector-error-rate (VER). To the best of our knowledge, no algorithms for antenna subset selection exist to directly optimize the union bound on the system error rate. In this letter, we propose a new approach to antenna selection based on the minimization of the union bound, which is the sum of the all pairwise error probabilities (PEPs). This can be expressed as a sum of Gaussian Q-functions. To reduce the complexity of their evaluation, the Gaussian Q-functions are replaced by accurate exponential approximations. Moreover, by relaxing the antenna selection variables from discrete to continuous, we arrive at a convex optimization problem. Due to its convexity, efficient numerical methods such as interior-point algorithms can be applied to solve this optimization problem with polynomial complexity [13].

This letter is organized as follows. In Section II, the system model and the union bound for receive subset antenna selection are presented. In Section III, we formulate antenna selection as a convex program to minimize the union bound. Experimental results via Monte Carlo simulations are given in Section IV to verify performance improvements of our proposed algorithm. Section V gives the conclusions.

Notation: Bold symbols denote matrices or vectors. $(\cdot)^T$, $(\cdot)^H$, and $(\cdot)^*$ denote transpose, conjugate transpose, and conjugate, respectively. The sets of real numbers, nonnegative real numbers, and complex numbers are $\mathbb{R}$, $\mathbb{R}_+$ and $\mathbb{C}$, respectively. The set of all complex $K \times 1$ vectors, $M \times N$ matrices are denoted by $\mathbb{C}^K$ and $\mathbb{C}^{M\times N}$ respectively. A circularly symmetric complex Gaussian variable with mean μ and variance σ^2 is denoted by $z \sim \mathcal{CN}(\mu, \sigma^2)$. An $N \times N$ identity matrix is denoted by $\mathbf{I}_N$.

II. System Model

We consider a MIMO system with total of N_t transmit and N_r receive antennas, where $N_r \geq N_t$. At each transmission epoch, $M < N_r$ receive antennas are picked for signal reception. The fading coefficient h_{ij} is the complex path gain form transmit antenna j to receive antenna i. The set of all these fading coefficients forms the channel matrix $\mathbf{H} = [h_{i,j}] \in \mathbb{C}^{N_r \times N_t}$, where $h_{i,j} \sim \mathcal{CN}(0, 1)$ are identically independent distributed (i.i.d.). $\mathbf{H}$ is known to the receiver, but not to the transmitter. A block of $N \times N_t$ symbols represented by a $N_t \times N$ matrix

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$\mathbf{X} \triangleq (\mathbf{x}_1, \ldots, \mathbf{x}_N)$, is transmitted through the channel. The entries x_{ij}, $i = 1, \ldots, N_t$, $j = 1, \ldots, N$ of $\mathbf{X}_m$ with normalization such that $\mathbb{E}\{|x_{ij}|^2\} = 1\ \forall i, j$ are the transmitted signal from antenna i at time j

$$\mathbf{Y} = \mathbf{H}\mathbf{X} + \mathbf{Z}. \tag{1}$$

The entries y_{ij}, $i = 1, \ldots, N_r$, $j = 1, \ldots, N$ of $\mathbf{Y} \in \mathbb{C}^{N_r \times N}$ are the signals received from antenna i at time j. The noise matrix $\mathbf{Z} \in \mathbb{C}^{N_r \times N}$ consists of $\mathcal{CN}(0, (N_t/\rho))$ variables so that $\mathbb{E}[\mathbf{Z}\mathbf{Z}^\dagger] = (NN_t/\rho)\mathbf{I}_{N_r}$ where ρ is the SNR per receive antenna, regardless of the number of transmit antennas. This model includes MIMO spatial multiplexing as its specific case where $\mathbf{X}$ is a column vector of size N_t, i.e., $N = 1$.

Following the approach of [12], we define diagonal matrix Δ of size $N_r \times N_r$ with diagonal entries

$$\Delta_i = \begin{cases} 1, & \text{if } i\text{th receive antenna selected} \\ 0, & \text{otherwise.} \end{cases} \tag{2}$$

The MIMO channel can then be rewritten incorporating receive antenna selection as the following:

$$\mathbf{Y} = \Delta\mathbf{H}\mathbf{X} + \mathbf{Z} \tag{3}$$

with the new effective channel $\Delta\mathbf{H}$.

The receiver performs maximum-likelihood (ML) detection over all possible codewords $\mathbf{X}$ to obtain

$$\tilde{\mathbf{X}} = \arg\min_{\mathbf{X}\in\mathbf{C}} \|\mathbf{Y} - \Delta\mathbf{H}\mathbf{X}\|_F^2 \tag{4}$$

where $\mathbf{C}$ is the codebook and $\|.\|_F$ denotes the Frobenius norm of the matrix.

With ML detection in (4), the probability that at least one of the entries of $\mathbf{X}$ is in error (PEP) conditioned on the channel matrix $\mathbf{H}$ is given by [14]

$$P_{\text{pep}}(\mathbf{X}_m \rightarrow \mathbf{X}_n|\mathbf{H}) = Q\left(\sqrt{\frac{\rho}{2}\|\Delta\mathbf{H}(\mathbf{X}_m - \mathbf{X}_n)\|_F^2}\right) \tag{5}$$

where $Q(.)$ denotes the Gaussian tail probability $Q(x) = (1/\sqrt{2\pi})\int_x^\infty \exp^{-(1/2)t^2}\,\mathrm{d}t$. The PEP depends on the specific codeword pair $(\mathbf{X}_m, \mathbf{X}_n)$, the instantaneous channel realization $\mathbf{H}$, and the receive antennas selected.

III. Receive Antenna Selection as a Convex Optimization

In this section, using matrix manipulation, we rewrite (5) as a convex function with variables as entries of Δ. Evaluating the Gaussian Q-function possibly may require high computational time, a drawback for online applications. Using an approximation to the Gaussian Q-function in the form of a sum of exponentials [15], we reduce the computational complexity.

First, we denote $\Gamma_{m,n} = \mathbf{X}_m - \mathbf{X}_n$ as the difference between codewords. Using $\Delta^H\Delta = \Delta$ and the fact that $\text{Trace}(\mathbf{AB}) = \text{Trace}(\mathbf{BA})$, we have

$$\begin{aligned}\|\Delta\mathbf{H}(\mathbf{X}_m - \mathbf{X}_n)\|_F^2 &= \|\Delta\mathbf{H}\Gamma_{m,n}\|_F^2 \\ &= \text{Trace}\left(\Gamma_{m,n}^H\mathbf{H}^H\Delta^H\Delta\mathbf{H}\Gamma_{m,n}\right) \\ &= \text{Trace}\left(\mathbf{H}\Gamma_{m,n}\Gamma_{m,n}^H\mathbf{H}^H\Delta\right) \\ &= \tilde{\mathbf{h}}_{m,n}^H\mathbf{u} \end{aligned} \tag{6}$$

where $\tilde{\mathbf{h}}_{m,n}$ and $\mathbf{u}$ are the column vectors of diagonal elements of matrices $\mathbf{H}\Gamma_{m,n}\Gamma_{m,n}^H\mathbf{H}^H$, Δ respectively. Note that all the elements of $\tilde{\mathbf{h}}_{m,n}$ are nonnegative and $\text{Trace}(\Delta) = \sum_i u_i$ where $\mathbf{u} = [u_1, \ldots, u_{N_r}]^T = [\Delta_1, \ldots, \Delta_{N_r}]^T$. The PEP can then be re-expressed with variable $\mathbf{u}$

$$P_{\text{pep}}(\mathbf{X}_m \rightarrow \mathbf{X}_n|\mathbf{H}) = Q\left(\sqrt{\frac{\rho}{2}\tilde{\mathbf{h}}_{m,n}^H\mathbf{u}}\right). \tag{7}$$

There are several ways to approximate the Q-function with high accuracy. One well-known way to upper bound the Gaussian Q-function is to use the Chernoff upper bound [14]; that is

$$P_{\text{pep}}(\mathbf{X}_m \rightarrow \mathbf{X}_n|\mathbf{H}) \leq \underbrace{\frac{1}{2}\exp\left\{-\frac{\rho\tilde{\mathbf{h}}_{m,n}^H\mathbf{u}}{4}\right\}}_{\tilde{g}_{m,n}^1(\mathbf{u})}. \tag{8}$$

More accurate approximations than the Chernoff bound can be readily derived as a sum of weighted exponentials. Using the optimal approximation of Q-function with two exponentials, we denote the following Chiani bound [15]:

$$Q\left(\sqrt{\frac{\rho}{2}\tilde{\mathbf{h}}_{m,n}^H\mathbf{u}}\right) \leq \underbrace{\frac{1}{12}\exp\left\{-\frac{\rho\tilde{\mathbf{h}}_{m,n}^H\mathbf{u}}{4}\right\} + \frac{1}{4}\exp\left\{-\frac{\rho\tilde{\mathbf{h}}_{m,n}^H\mathbf{u}}{3}\right\}}_{\tilde{g}_{m,n}^2(\mathbf{u})}. \tag{9}$$

Clearly, a sum of more exponentials can be used to approximate Q-function even better [15]. However, in our optimization problem, there is a tradeoff between approximation accuracy and computational complexity.

We next assume that there are L possible code matrices $\mathbf{X}$, e.g., for spatial multiplexing systems, $L = |\mathcal{S}|^{N_t}$ with $|\mathcal{S}|$ being the size of the input modulation constellation $\mathcal{S}$. The union bound closely relates to the VER is given by

$$P_e \leq P_{UB} = \frac{2}{L}\sum_{m=1}^{L-1}\sum_{n=m+1}^{L} P_{\text{pep}}(\mathbf{X}_m \rightarrow \mathbf{X}_n|\mathbf{H}). \tag{10}$$

Using different expressions for the Q-function, we have the exact form of union bound as

$$P_{UB} = \frac{2}{L}\sum_{m=1}^{L-1}\sum_{n=m+1}^{L} Q\left(\sqrt{\frac{\rho}{2}\tilde{\mathbf{h}}_{m,n}^H\mathbf{u}}\right) \tag{11}$$

or the upper bounds

$$P_{UB} \leq \frac{2}{L}\sum_{m=1}^{L-1}\sum_{n=m+1}^{L} \tilde{g}_{m,n}^1(\mathbf{u}) = g_1(\mathbf{u}) \tag{12}$$

and

$$P_{UB} \leq \frac{2}{L}\sum_{m=1}^{L-1}\sum_{n=m+1}^{L} \tilde{g}_{m,n}^2(\mathbf{u}) = g_2(\mathbf{u}). \tag{13}$$

Given an instantaneous channel realization $\mathbf{H}$, the antenna selection problem is to pick the $M < N_r$ receive antennas such that they minimize the union bounds expressed in (11), (12), or (13). It is equivalent to find $\mathbf{u}$ such that

$$\mathbf{u} = \arg\min_{\substack{\mathbf{u}_i\in\{0,1\} \\ \sum_i u_i = M}} P_{UB}. \tag{14}$$

The binary variable vector $\mathbf{u} \in \{0,1\}^{N_r}$ makes the selection problem a NP-hard combinatorial optimization i.e., an exhaustive search to evaluate all the $\binom{N_r}{M}$ antenna subsets may be needed to pick the optimal solution $\mathbf{u}$. We will relax this binary constraint by allowing $u_i \in [0,1]$, $(i = 1,\ldots,N_r)$. Thus, the problem of receive subset selection for minimizing the union bound is approximated by the following optimization problem:

$$\begin{aligned} \min \quad & g_1(\mathbf{u}) \\ \text{subject to} \quad & 0 \le u_i \le 1, i = 1,\ldots,N_r \\ & \sum_{i=1}^{N_r} u_i = M. \end{aligned} \tag{15}$$

Similarly, we can minimize $g_2(\mathbf{u})$ as well.

From the fractional solution $\mathbf{u}$ of the problem, the receive antennas with indices corresponding to the M largest u_i are selected.

Proposition 1: The optimization problem in (15) is convex in $\mathbf{u} \in \mathbb{R}_+^{N_r}$.

Proof: Here, we show that $e^{-\beta u/2}$ where $\beta,\ u > 0$ is convex

$$\frac{\partial e^{\frac{-\beta u}{2}}}{\partial u} = -\frac{\beta}{2} e^{\frac{-\beta u}{2}} < 0, \quad 0 < u < \infty \tag{16}$$

$$\frac{\partial^2 e^{\frac{-\beta u}{2}}}{\partial u^2} = \left(\frac{\beta}{2}\right)^2 e^{\frac{-\beta u}{2}} > 0, \quad 0 < u < \infty. \tag{17}$$

The convexity is preserved under an affine transformation [13] and note that $\tilde{\mathbf{h}}_{m,n}$ has all its elements being real nonnegative. Thus, $\exp\{-(\rho\tilde{\mathbf{h}}_{m,n}^H \mathbf{u}/4)\}$ is convex w.r.t. variable $\mathbf{u} \in \mathbb{R}_+^{N_r}$. Finally, noting that the sum of convex functions is convex, we conclude that $g_1(\mathbf{u})$ is itself convex. The two constraints are clearly convex. The convexity of the problem suggests that there is no local minima but only one global minima. The convexity of $g_2(\mathbf{u})$ follows since it is the sum of two convex exponentials.

Remark: The exact union bound expression P_{UB} in (11) is in fact also a convex funtion w.r.t. $\mathbf{u} \in \mathbb{R}_+^{N_r}$, which follows from the convexity of $Q(\sqrt{\beta u})$ w.r.t. $u > 0$ [16]. However, the minimization of (11) is beyond the scope of this letter.

It is well known that a convex optimization problem can be solved by using interior point methods which require polynomial complexity. We derive a log barrier method to solve (15) in the Appendix. More details of this method are found in [13].

IV. Simulation Results

In this section, we study the VER of systems which implements antenna selection for different antenna configurations (varying N_t, N_r and M) through Monte Carlo simulations. For simulation, each Rayleigh fading channel realization is constant for 20 frames to produce more accuarate results. ML signal detection is employed in all cases. For comparison, the performance curve of a popular selection criteria, i.e., eigenvalue-based [3] is plotted. There are $\binom{N_r}{M}$ selections of submatrix $\hat{\mathbb{H}}$ of size $M \times N_t$. Let $\mathcal{H}$ be the collection of all the submatrices $\hat{\mathbf{H}}$. The eigenvalue-based selection is done by exhaustive search over all possibilities $\hat{\mathbf{H}}$, equivalently

$$\mathbf{H}_{sel} = \arg\max_{\hat{\mathbf{H}} \in \mathcal{H}} \lambda_{\min}(\hat{\mathbf{H}}^H \hat{\mathbf{H}}) \tag{18}$$

Moreover, the optimal exact union bound minimization antenna selection is also plotted. This is done by searching all the antenna subsets, which has obviously high complexity for systems

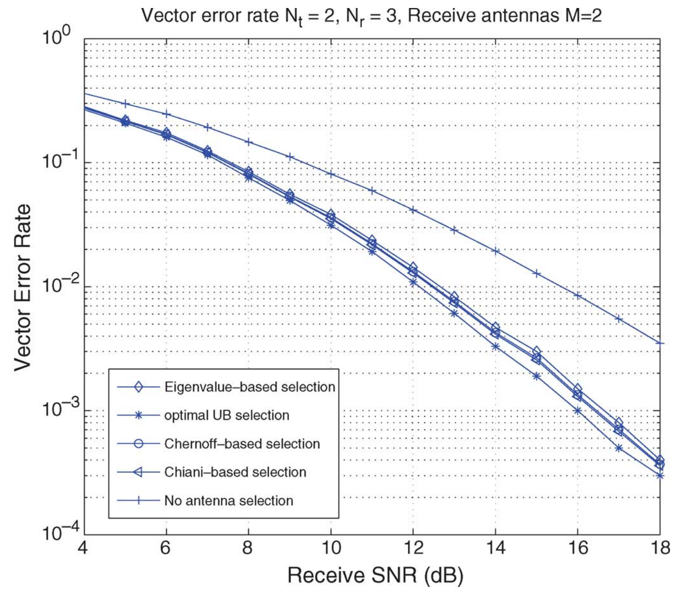

Fig. 1. VER with antenna selection, $N_t = 2, N_r = 3, M = 2$, 4 bp/Hz/s.

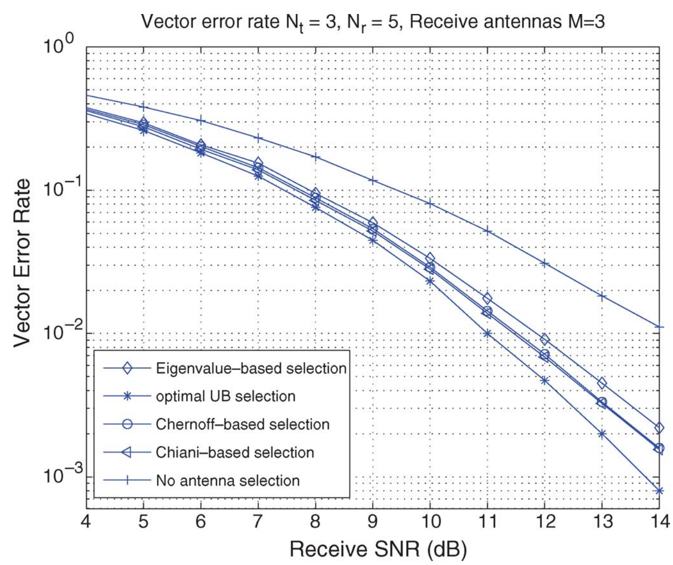

Fig. 2. VER with antenna selection, $N_t = 3, N_r = 5, M = 3$, 6 bp/Hz/s.

with large dimensions. We plot the VER versus SNR at receiver of different selection schemes using maximum-likelihood (ML) detection. ML detection can be done by using an efficient sphere decoding [17]. In Fig. 1, the MIMO system has $N_t = 2, N_r = 3$, and $M = 2$. We next test our proposed algorithm when implementing MIMO system with larger dimension, $N_t = 3, N_r = 5$, and $M = 3$. Fig. 3 displays the performance gaps between the exhaustive search (14) and our convex-based relaxation selection. Our proposed method has some loss, especially in the high SNR region in which the optimal union bound search provides the best performance. Moreover, the performance loss may arise from the rounding operation applied on the optimal solution of our optimization problem to pick corresponding receive antennas. Another related issue is that the union bound is a sum of a large number of PEPs. As a result, the rounding off may lead to a loss. However, our proposed algorithm outperforms the optimal eigenvalue-based criteria based on exhaustive search. Fig. 1 and

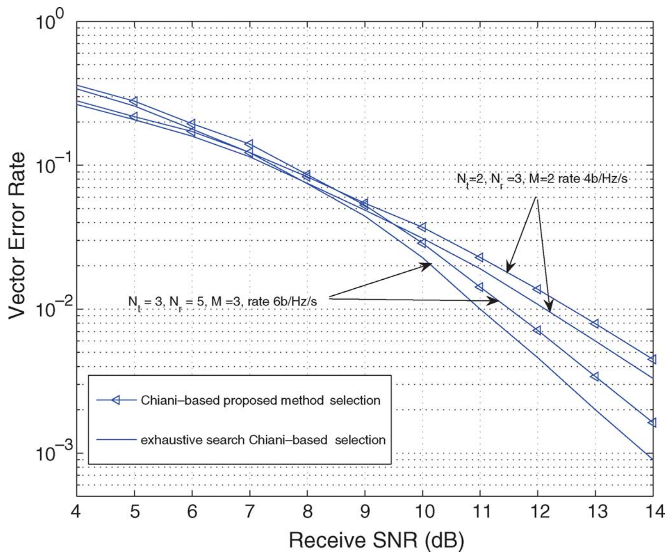

Fig. 3. Performance of exhaustive search and convex relaxation.

Fig. 2 show that the Chernoff-based and Chiani-based optimization perform almost the same. This is because the selected antennas depend only on the order of elements $x_i, i = 1, \ldots, N_r$ of the optimal $\mathbf{x}$ but the optimal value. Simulation examples suggest that the complexity of the proposed method is comparable to that of the optimal search due to a large number of codeword pairs in the union bound calculation. This suggests that the use of a truncated union bound which sums over a set of dominant PEPs may reduce the complexity.

V. Conclusions

We have proposed a novel solution to the problem of receive antenna selection to minimize accurate approximations of the exact union bound. Since a convex problem is obtained, interior-point methods can be used with polynomial complexity. The proposed algorithm outperforms eigenvalue-based selection in terms of the vector error rate.

Appendix

A. Gradient Derivation

We compute the gradient of $g_1(\mathbf{u})$ with respect to $\mathbf{u} \in \mathbb{R}_+^{N_r}$

$$\frac{\partial g_1(\mathbf{u})}{\partial \mathbf{u}} = \frac{\partial \left[\frac{2}{L} \sum_{m=1}^{L-1} \sum_{n=m+1}^{L} \frac{1}{2} \exp\left\{ -\frac{\rho \tilde{\mathbf{h}}_{m,n}^H \mathbf{u}}{4} \right\} \right]}{\partial \mathbf{u}}$$
$$= \frac{2}{L} \sum_{m=1}^{L-1} \sum_{n=m+1}^{L} \mathbf{dg}_{m,n} \quad (19)$$

where $\mathbf{dg}_{m,n} = [\mathrm{dg}_{m,n1}, \mathrm{dg}_{m,n2}, \ldots, \mathrm{dg}_{m,nN_r}]^T$, $\mathrm{dg}_{m,ni} = -(\rho/8)\tilde{h}_{m,ni} \exp\{-(\rho \tilde{h}_{m,ni} u_i/4)\}$, $i = 1, \ldots, N_r$.

B. Barrier Method for Function $f(\mathbf{u})$ [13]

Given strictly feasible $\mathbf{u} \in \mathbb{R}^{N_r}$, $t = t^{(0)} > 0$, $\mu > 1$ (update parameter), $\epsilon > 0$ (tolerance), repeat:

- eentering step: compute $\mathbf{u}^*(t)$ by minimizing function $tf(\mathbf{u}) + \phi(\mathbf{u})$, subject to $\sum_i u_i = M$ starting at current $\mathbf{x}$ using Newton method;
- update $\mathbf{u} := \mathbf{u}^*(t)$;
- stoping criterion: If $N_r/t < \epsilon$, then stop;
- Increase $t = \mu t$,

where $\phi(\mathbf{x}) = -\sum_{i=1}^{N_r} \log(u_i(1-u_i))$ is the log barrier function. A note on derivatives of the barrier function

$$\frac{\partial \phi(\mathbf{u})}{\partial \mathbf{u}} = \frac{\partial \sum_{i=1}^{N_r} - \log\left(u_i(1-u_i)\right)}{\partial \mathbf{u}} = [\mathrm{d}\phi_1, \ldots, \mathrm{d}\phi_{N_r}]^T \quad (20)$$

where $\mathrm{d}\phi_i = (2u_i - 1)/u_i(1-u_i)$, $i = 1, \ldots, N_r$.

References

[1] A. Gorokhov, D. Gore, and A. Paulraj, “Performance bounds for antenna selection in MIMO systems,” in *Proc. IEEE Int. Conf. Communications (ICC)*, Anchorage, AK, 2003, pp. 3021–3025.

[2] D. Gore and A. Paulraj, “Mimo antenna subset selection with space-time coding,” *IEEE Trans. Signal Process.*, vol. 50, no. 10, pp. 2580–2588, Oct. 2002.

[3] R. Heath and A. J. Paulraj, “Antenna selection for spatial multiplexing systems based on minimum error rate,” in *Proc. IEEE Int. Conf. Communications (ICC)*, Helsinki, Finland, 2001, vol. 7, pp. 2276–2280.

[4] R. Heath and D. Love, “Multimode antenna selection for spatial multiplexing systems with linear receivers,” *IEEE Trans. Signal Process.*, vol. 53, no. 8, pp. 3042–3056, Aug. 2005.

[5] J. Vicario, M. Lagunas, and C. Anton-Haro, “A cross-layer approach to transmit antenna selection,” *IEEE Trans. Wireless Commun.*, vol. 5, no. 8, pp. 1993–1997, Aug. 2006.

[6] A. Molisch and M. Win, “MIMO systems with antenna selection,” *IEEE Microwave Mag.*, vol. 5, no. 1, pp. 46–56, Mar. 2004.

[7] R. Heath, S. Sandhu, and A. Paulraj, “Antenna selection for spatial multiplexing systems with linear receivers,” *IEEE Commun. Lett.*, vol. 5, no. 4, pp. 142–144, Apr. 2001.

[8] F. Kammoun, S. Fontenelle, S. Rouquette, and J. Boutros, “Antenna selection for MIMO systems based on an accurate approximation of QAM error probability,” in *Proc. IEEE Vehicular Technology Conf. (VTC)*, 2005, vol. 1, pp. 206–210.

[9] A. Dhananjay, A. Gore, and A. Paulraj, “Receiver antenna selection for MIMO spatial multiplexing: Theory and algorithms,” *IEEE Trans. Signal Process.*, vol. 51, no. 11, pp. 2796–2807, Nov. 2003.

[10] M. Alkhansari and A. Gershman, “Fast antenna subset selection in wireless MIMO systems,” *IEEE Trans. Signal Process.*, vol. 52, no. 2, pp. 339–347, Feb. 2004.

[11] A. Molisch, M. Win, Y.-S. Choi, and J. Winters, “Capacity of MIMO systems with antenna selection,” *IEEE Trans. Wireless Commun.*, vol. 4, no. 4, pp. 1759–1772, Jul. 2005.

[12] A. Dua, K. Medepalli, and A. Paulraj, “Receive antenna selection in MIMO systems using convex optimization,” *IEEE Trans. Wireless Commun.*, vol. 5, no. 9, pp. 2353–2357, Sep. 2006.

[13] S. Boyd and L. Vandenberghe, *Convex Optimization*. Cambridge, U.K.: Cambridge Univ. Press, 2004.

[14] V. Tarokh, N. Seshadri, and A. R. Calderbank, “Space-time codes for high data rate wireless communication: Performance analysis and code construction,” *IEEE Trans. Inf. Theory*, vol. 44, no. 3, pp. 744–765, Mar. 1998.

[15] M. Chiani, D. Dardari, and M. Simon, “New exponential bounds and approximations for the computation of error probability in fading channels,” *IEEE Trans. Wireless Commun.*, vol. 2, no. 4, pp. 840–845, Jul. 2003.

[16] D. Palomar, J. M. Cioffi, and M. Lagunas, “Joint tx-rx beamforming design for multicarrier MIMO channels: A unified framework for convex optimization,” *IEEE Trans. Signal Process.*, vol. 51, no. 9, pp. 2381–2401, Sep. 2003.

[17] E. Agrell, T. Eriksson, A. Vardy, and K. Zeger, “Closest point search in lattices,” *IEEE Trans. Inf. Theory*, vol. 48, no. 8, pp. 2201–2214, Aug. 2002.