Precoder Design for Space-Time Coded MIMO Systems with Correlated Rayleigh Fading Channels

Khoa T. Phan, Sergiy A.Vorobyov and Chintha Tellambura

Department of Electrical and Computer Engineering
University of Alberta
Edmonton, AB, Canada T6G 2V4
Email: {khoa, vorobyov, chintha}@ece.ualberta.ca

***Abstract*— A general framework for precoder design for multiple-input multiple-output (MIMO) systems is presented in the paper for the case of correlated Rayleigh fading channels. The transmitter exploits the knowledge of the transmit and receive correlation matrices, while the instantaneous channel state information (CSI) is assumed to be unknown. The latter assumption is especially important for the case of time-varying channels, for which the instantaneous CSI is typically unavailable at the transmitter. It is also assumed that the receiver has a precise knowledge of the instantaneous CSI. The precoder operates along with an orthogonal space-time block code (OSTBC) and aims at minimizing the Chernoff bound on the symbol error rate (SER). We show that for some particular correlation scenarios, the closed-form precoder designs can be obtained. Our simulations demonstrate the advantages of the proposed precoding techniques.**

I. Introduction

Multi-antenna systems have recently gained a significant attention due to their ability to mitigate fading. Moreover, multi-antenna systems offer significant channel capacity gains. Space-time block codes (STBCs) have been developed for such systems [1]. Among different STBCs the orthogonal STBCs (OSTBCs) are of a special interest because the corresponding maximum-likelihood (ML) decoder boils down to a simple match filter [2].

STBCs are design based on the assumption that the channel state information (CSI) is unavailable at the transmitter. However, if the CSI is available at the transmitter, the combination of STBC with precoding techniques can be used in order to adopt the system to the current channel conditions without changing the fixed structure of the transmitter and the receiver [3]–[5].

Typically, the full instantaneous CSI is assumed to be available at the transmitter for linear precoder design [3], [6]. The CSI can be estimated at the transmitter when a time-division duplex (TDD) mode is employed. However, in frequency-division duplex (FDD) systems, the instantaneous CSI has to be estimated at the receiver and feed back to the transmitter. It can significantly consume the bandwidth of the system, especially in the case of time-varying cannel. Hence, it is more reasonable to assume that the transmitter only has partial channel knowledge such as transmit and receive correlation matrices. Note that these correlation matrices vary at a much slower rate than the instantaneous CSI and can be obtained reliably at the transmitter.

In this paper, we consider the problem of precoder design over a jointly transmit-receive correlated Rayleigh fading MIMO channel with OSTBC. We assume that only the transmit and receive correlation matrices are available at the transmitter, while the precise instantaneous CSI is known at the receiver. Our precoder aims to minimize the Chernoff bound on the exact symbol error rate (SER). We show that for some popular correlation models, the closed-form precoder designs which have very low computational complexity can be obtained. The authors in [7], [8] have considered the precoder design for correlated channel but with some variants, i.e. in correlation model and design criteria.

The paper is organized as follows. In Section 2, the system model is presented. The precoder design problem formulation is given in Section 3, while the specific precoder designs for different correlation scenarios are summarized in Section 4. Section 5 contains our simulation results, and Section 6 concludes the paper.

II. System model

The received signal for MIMO system which uses a combination of STBC and precoding can be written as

$$\mathbf{Y} = \sqrt{\frac{E_s}{N_t}}\mathbf{HFC} + \mathbf{Z} \tag{1}$$

where N_t and N_r are the numbers of transmit and receive antennas, respectively, E_s is the total transmitted power, $\mathbf{Y}$ is the $N_r \times T$ matrix of received signal, $\mathbf{C}$ is the $N_t \times T$ STBC matrix, $\mathbf{F}$ is the $N_t \times N_t$ precoding matrix, $\mathbf{Z}$ is the $N_t \times T$ matrix of additive white Gaussian noise (AWGN) with zero mean and variance N_o, $\mathbf{H}$ is the $N_r \times N_t$ channel matrix.

The Rayleigh fading channel $\mathbf{H}$ can be modeled in the case of transmit and receive correlation as

$$\mathbf{H} = \mathbf{R}_{\mathrm{Rx}}^{\frac{1}{2}}\tilde{\mathbf{H}}\mathbf{R}_{\mathrm{Tx}}^{\frac{1}{2}} \tag{2}$$

where $\mathbf{R}_{\mathrm{Tx}}$ and $\mathbf{R}_{\mathrm{Rx}}$ are the transmit and receive side correlation matrices of sizes $N_t \times N_t$ and $N_r \times N_r$, respectively, and $\tilde{\mathbf{H}}$ composes of independent identically distributed (i.i.d.) elements with zero-mean and unit variance. Note that the correlation between different MIMO channel links is modeled

0840-7789/07/$25.00 ©2007 IEEE

under the assumption that the correlation among receive antennas is independent of the correlation among transmit antennas (and vice versa). Writing (2) using the vectorizing operator vec(.) as $\text{vec}(\mathbf{H}) = \mathbf{R}^{1/2}\text{vec}(\tilde{\mathbf{H}})$, it can be verified that the matrix $\mathbf{R}$ can be written in terms of the Kronecker product of transmit and receive correlation matrices, i.e.,

$$\mathbf{R} = \mathbf{R}_{\text{Tx}}^T \otimes \mathbf{R}_{\text{Rx}} \tag{3}$$

where $\text{vec}(\cdot)$ denotes the vectorization operation.

We denote the $Q \times 1$ vector consisting of complex information-bearing symbols prior to space-time encoding as $\mathbf{s} = [s_1, s_2, \ldots, s_Q]^T$, where $(\cdot)^T$ denotes the transpose. Though different signal modulation schemes such as M-PAM, M-QAM, or M-PSK can be used, hereafter we assumed that M-PAM is applied and $\mathbb{E}(|s_i|^2) = 1$, where $\mathbb{E}(\cdot)$ stands for the mathematical expectation. An $N_t \times T$ OSTBC codematrix $\mathbf{X}(\mathbf{s})$ is used to encode the input vector $\mathbf{s}$. Note that this codematrix satisfies the following properties [2]

- All the elements in $\mathbf{X}(\mathbf{s})$ are linear functions of $s_1, s_2, \ldots, s_Q$ and their complex conjugates.
- $\mathbf{X}^H(\mathbf{s})\mathbf{X}(\mathbf{s}) = \|\mathbf{s}\|_F^2 \mathbf{I}$ for all $\mathbf{s} \in \mathbb{C}^Q$, where $\|\cdot\|^2$ and $\mathbb{C}^Q$ denote the Euclidian norm of vector and the set of complex $Q \times 1$ vectors, correspondingly.

It has been shown in [2] that if OSTBCs are used, the ML decoder can be simplified to a symbol-by-symbol decoder of the following form

$$\tilde{s}_q = \sqrt{\frac{E_s}{N_t}}\left(\frac{1}{R_s}\|\mathbf{HF}\|_F^2\right) s_q + \nu_q, \quad q = 1, ..., Q \tag{4}$$

where $\nu_q \sim \mathcal{CN}\left(0, \frac{1}{R_s}\|\mathbf{HF}\|_F^2 N_0\right)$ is the complex Gaussian noise with zero-mean and variance $\frac{1}{R_s}\|\mathbf{HF}\|_F^2 N_0$, $R_s = Q/T$ is the code rate, and $\|\cdot\|_F^2$ denotes the Frobenius norm of matrix. Using (4) the effective instantaneous signal-to-noise ratio (SNR) per symbol in M-ary constellation can be expressed as

$$\gamma = \frac{|\tilde{s}_q|^2}{|\nu_q|^2} = \frac{E_s}{N_0 R_s N_t}\alpha \tag{5}$$

where $\alpha = \|\mathbf{HF}\|_F^2$.

III. Problem formulation

In this section, we develop an optimization problem for designing the precoding matrix $\mathbf{F}$ which minimizes the Chernoff bound on the exact SER.

Given the receive instantaneous SNR γ, the SERs in the case of M-PAM modulation can be evaluated as [9]

$$\text{SER} = \frac{2}{\pi}\frac{M-1}{M}\int_0^{\frac{\pi}{2}} \exp\left(-\frac{g_{\text{PAM}}\gamma}{\sin^2\phi}\right) d\phi \tag{6}$$

where $g_{\text{PAM}} = 3/(M^2-1)$.

Using (5) and the moment generating function (MGF) for the random variable α, the SER in (6) can be rewritten as

$$\text{SER} = \frac{2}{\pi}\frac{M-1}{M}\int_0^{\frac{\pi}{2}} \Phi\left(-\frac{\tilde{g}}{\sin^2\phi}\right) d\phi \tag{7}$$

where $\tilde{g} = g_{\text{PAM}}E_s/(N_0 R_s N_t)$, and $\Phi(.)$ denotes the MGF.

Exploiting the property that $\text{Tr}(\mathbf{ABA}^H) = \text{vec}(\mathbf{A}^H)^H(\mathbf{I} \otimes \mathbf{B})\text{vec}(\mathbf{A}^H)$, we can write that

$$\begin{aligned}\alpha &= \|\mathbf{HF}\|_F^2 = \text{Tr}(\mathbf{HFF}^H\mathbf{H}^H) \\ &= \text{vec}(\mathbf{H}^H)^H(\mathbf{I} \otimes \mathbf{FF}^H)\text{vec}(\mathbf{H}^H)\end{aligned} \tag{8}$$

where $\text{Tr}(\cdot)$ denotes the trace operation. Furthermore, by denoting $\mathbf{R}_s = \mathbf{R}_{\text{Rx}} \otimes \mathbf{R}_{\text{Tx}}$ as the covariance matrix of $\text{vec}(\mathbf{H}^H)$, we can rewrite (8) as

$$\alpha = \text{vec}(\tilde{\mathbf{H}}^H)^H \mathbf{R}_s^{1/2}(\mathbf{I} \otimes \mathbf{FF}^H)\mathbf{R}_s^{1/2}\text{vec}(\tilde{\mathbf{H}}^H). \tag{9}$$

Thus, α is a positive-definite quadratic form of zero-mean unit-variance Gaussian vector $\text{vec}(\tilde{\mathbf{H}}^H)$.

The following theorem is in order [10].

Theorem 1: The MGF of the quadratic form $y = \mathbf{x}^H\mathbf{A}\mathbf{x}$ where $\mathbf{A}$ is Hermitian matrix and $\mathbf{x}$ is circularly symmetric complex Gaussian vector with mean $\bar{\mathbf{x}}$ and variance $\mathbf{R}_\mathbf{x}$ is given by

$$\Phi(s) = \int_0^\infty e^{sy}p_Y(y)dy = \frac{\exp(-\bar{\mathbf{x}}^H\mathbf{A}(\mathbf{I} - s\mathbf{R}_\mathbf{x}\mathbf{A})^{-1}\bar{\mathbf{x}})}{\det(\mathbf{I} - s\mathbf{R}_\mathbf{x}\mathbf{A})}. \tag{10}$$

where $p_Y(y)$ stands for the probability density function of random variable y.

Using theorem 1 and denoting $\bar{\mathbf{x}} = \mathbf{0}$, $\mathbf{A} = \mathbf{R}_s^{1/2}(\mathbf{I} \otimes \mathbf{FF}^H)\mathbf{R}_s^{1/2}$, and $\mathbf{R}_\mathbf{x} = \mathbf{I}$ the MGF for α in (9) can be written as

$$\begin{aligned}\Phi(s) &= \det\Big(\mathbf{I} - s\mathbf{R}_s^{1/2}(\mathbf{I} \otimes \mathbf{FF}^H)\mathbf{R}_s^{1/2}\Big)^{-1} \\ &= \det\Big(\mathbf{I} - s(\mathbf{I} \otimes \mathbf{FF}^H)\mathbf{R}_s\Big)^{-1} \\ &= \det\Big(\mathbf{I} - s(\mathbf{I} \otimes \mathbf{FF}^H)(\mathbf{R}_{\text{Rx}} \otimes \mathbf{R}_{\text{Tx}})\Big)^{-1} \\ &= \det\Big(\mathbf{I} - s\mathbf{R}_{\text{Rx}} \otimes (\mathbf{FF}^H\mathbf{R}_{\text{Tx}})\Big)^{-1}.\end{aligned} \tag{11}$$

Inserting (11) into (7), we can write the exact SER as

$$\begin{aligned}\text{SER} &= \frac{2}{\pi}\frac{M-1}{M} \\ &\times \int_0^{\frac{\pi}{2}} \det\Big(\mathbf{I} + \frac{\tilde{g}}{\sin^2\phi}\mathbf{R}_{\text{Rx}} \otimes \big(\mathbf{FF}^H\mathbf{R}_{\text{Tx}}\big)\Big)^{-1} d\phi\end{aligned} \tag{12}$$

The Chernoff bound on the exact SER can be obtained by substituting $\phi = \pi/2$ into (12), and is given by

$$\text{SER}_{\text{Chernoff}} = \frac{M-1}{M}\det\Big(\mathbf{I} + \tilde{g}\mathbf{R}_{\text{Rx}} \otimes \big(\mathbf{FF}^H\mathbf{R}_{\text{Tx}}\big)\Big)^{-1}. \tag{13}$$

Our task now is to design the precoding matrix $\mathbf{F}$ which maximizes the Chernoff bound on the exact SER (13) subject to the average power constraint $\|\mathbf{F}\|_F^2 = \text{Tr}(\mathbf{FF}^H) = 1$. Mathematically, this problem can be expressed as

$$\min_{\mathbf{F}} \text{SER}_{\text{Chernoff}} \quad \text{subject to} \quad \text{Tr}(\mathbf{F}^H\mathbf{F}) = 1. \tag{14}$$

IV. PRECODER DESIGNS FOR SCENARIOS WITH TRANSMIT AND/OR RECEIVE CORRELATION

Applying the singular value decomposition (SVD) to the transmit and receive correlation matrices $\mathbf{R}_{\rm Tx} = \mathbf{U}_{\rm Tx}\mathbf{\Sigma}_{\rm Tx}\mathbf{U}_{\rm Tx}^H$ and $\mathbf{R}_{\rm Rx} = \mathbf{U}_{\rm Rx}\mathbf{\Sigma}_{\rm Rx}\mathbf{U}_{\rm Rx}^H$, we can rewrite the Chernoff bound on the exact SER as

$$\mathrm{SER}_{\rm Chernoff} = \frac{M-1}{M}\det\Big(\mathbf{I} + \tilde{g}\mathbf{\Sigma}_{\rm Rx}\otimes\big(\mathbf{\Sigma}_{\rm Tx}^{1/2}\tilde{\mathbf{F}}\mathbf{\Sigma}_{\rm Tx}^{1/2}\big)\Big)^{-1} \quad (15)$$

where $\tilde{\mathbf{F}} = \mathbf{U}_{\rm Tx}^H\mathbf{F}\mathbf{F}^H\mathbf{U}_{\rm Tx}$, $\mathbf{U}_{\rm Tx}, \mathbf{U}_{\rm Rx}$ are the matrices of eigenvectors, and $\mathbf{\Sigma}_{\rm Tx}, \mathbf{\Sigma}_{\rm Rx}$ are the diagonal eigenvalue matrices of the transmit and receive correlation matrices, respectively. Then, the optimization problem (14) can be rewritten as

$$\min_{\mathrm{Tr}(\tilde{\mathbf{F}})=1,\, \tilde{\mathbf{F}}\succcurlyeq 0} \det\Big(\mathbf{I} + \tilde{g}\mathbf{\Sigma}_{\rm Rx}\otimes\big(\mathbf{\Sigma}_{\rm Tx}^{1/2}\tilde{\mathbf{F}}\mathbf{\Sigma}_{\rm Tx}^{1/2}\big)\Big)^{-1}. \quad (16)$$

We now state the Hadamard inequality [11]:

Theorem 2: If $\mathbf{A} = [a_{ij}]$ is positive definite matrix of size n, then

$$\det(\mathbf{A}) \le \prod_{i=1}^{n} a_{ii}. \quad (17)$$

Furthermore, equality holds in (17) if and only if $\mathbf{A}$ is diagonal. Since the matrices $\mathbf{\Sigma}_{\rm Tx}$ and $\mathbf{\Sigma}_{\rm Rx}$ in (16) are diagonal, $\det\Big(\mathbf{I} + \tilde{g}\mathbf{\Sigma}_{\rm Rx}\otimes\big(\mathbf{\Sigma}_{\rm Tx}^{1/2}\tilde{\mathbf{F}}\mathbf{\Sigma}_{\rm Tx}^{1/2}\big)\Big)^{-1}$ will be minimized if $\Big(\mathbf{I} + \tilde{g}\mathbf{\Sigma}_{\rm Rx}\otimes\big(\mathbf{\Sigma}_{\rm Tx}^{1/2}\tilde{\mathbf{F}}\mathbf{\Sigma}_{\rm Tx}^{1/2}\big)\Big)$ is diagonal, that implies the optimal $\tilde{\mathbf{F}}$ for (16) is also diagonal. Moreover, based on the property that the logarithmic function $\log(x)$ is monotonic increasing for nonnegative x, we can rewrite the optimization problem (16) as the following equivalent problem

$$\min_{\tilde{F}(i,i)\ge 0,\, \mathrm{Tr}(\tilde{\mathbf{F}})\le 1} -\log\det\Big(\mathbf{I} + \tilde{g}\mathbf{\Sigma}_{\rm Rx}\otimes\big(\mathbf{\Sigma}_{\rm Tx}^{1/2}\tilde{\mathbf{F}}\mathbf{\Sigma}_{\rm Tx}^{1/2}\big)\Big) \quad (18)$$

where the constraint $\mathrm{Tr}(\tilde{\mathbf{F}}) = 1$ is replaced by its inequality equivalent that does not affect the optimal solution as will be shown later.

A. Transmit side correlation only

In the case of of no receive side correlation, $\mathbf{R}_{\rm Rx} = \mathbf{I}$, or equivalently $\mathbf{\Sigma}_{\rm Rx} = \mathbf{I}$. Thus, substituting $\mathbf{\Sigma}_{\rm Rx} = \mathbf{I}$ into (18) the precoder design problem can be simplified to the following problem

$$\min_{\tilde{F}(i,i)\ge 0,\, \mathrm{Tr}(\tilde{\mathbf{F}})\le 1} -\log\Big[\det\big(\mathbf{I} + \tilde{g}\mathbf{\Sigma}_{\rm Tx}^{1/2}\tilde{\mathbf{F}}\mathbf{\Sigma}_{\rm Tx}^{1/2}\big)\Big]^{N_r} \quad (19)$$

or equivalently to

$$\min_{\tilde{F}(i,i)\ge 0,\, \mathrm{Tr}(\tilde{\mathbf{F}})\le 1} -\log\det\big(\mathbf{I} + \tilde{g}\mathbf{\Sigma}_{\rm Tx}^{1/2}\tilde{\mathbf{F}}\mathbf{\Sigma}_{\rm Tx}^{1/2}\big). \quad (20)$$

The Langrangian for the problem (20) is given by

$$\begin{aligned}\mathcal{L}(\tilde{\mathbf{F}},\alpha,\mu) &= -\log\det\big(\mathbf{I} + \tilde{g}\mathbf{\Sigma}_{\rm Tx}^{1/2}\tilde{\mathbf{F}}\mathbf{\Sigma}_{\rm Tx}^{1/2}\big) - \mathrm{Tr}(\tilde{\mathbf{F}}\mathbf{D}_\alpha)\\ &+ \mu(\mathrm{Tr}(\tilde{\mathbf{F}}) - 1), \quad (21)\\ &\mu \ge 0,\ \alpha_i \ge 0,\ i = 1,\ldots,N_t,\ \mathbf{D}_\alpha = \mathrm{diag}(\alpha)\end{aligned}$$

where μ and α_i, $i = 1,\ldots,N_t$ are the Lagrange multipliers.

From [12], $-\log\det\big(\mathbf{I} + \tilde{g}\Sigma_{\rm Tx}^{1/2}\tilde{\mathbf{F}}\Sigma_{\rm Tx}^{1/2}\big)$ is convex w.r.t. $\tilde{\mathbf{F}} \succcurlyeq 0$, thus (20) is a convex program. Therefore, the globally optimal solution to this problem can be found by solving the following system of Karush-Kuhn-Tucker (KKT) conditions

$$\begin{aligned}-\tilde{g}\mathbf{\Sigma}_{\rm Tx}^{1/2}\big(\mathbf{I} + \tilde{g}\mathbf{\Sigma}_{\rm Tx}^{1/2}\tilde{\mathbf{F}}\mathbf{\Sigma}_{\rm Tx}^{1/2}\big)^{-1}\mathbf{\Sigma}_{\rm Tx}^{1/2} - \mathbf{D}_\alpha + \mu\mathbf{I} = 0,\\ \alpha_i\tilde{F}(i,i) = 0,\ i = 1,\ldots N_t, \quad \mu(\mathrm{Tr}(\tilde{\mathbf{F}}) - 1) = 0 \quad (22)\end{aligned}$$

where we have used the fact that $[\log\det(\mathbf{X})]' = \mathbf{X}^{-1}$ for positive definite matrix $\mathbf{X}$. Then the linear precoder in the case of no transmit side correlation is given by the following waterfilling-like solution

$$\tilde{F}(i,i) = \Big(\mu^{-1} - \big(\tilde{g}\mathbf{\Sigma}_{\rm Tx}(i,i)\big)^{-1}\Big)^{+}, \qquad i = 1,\ldots,N_t \quad (23)$$

where $x^+ = \max\{0, x\}$, and μ is chosen such that $\sum_i \tilde{F}(i,i) = 1$.

B. Receive side correlation only

Similarly, in the case of no receive side correlation, i.e., $\mathbf{R}_{\rm Rx} = \mathbf{I}_{N_r}$ or, equivalently, $\mathbf{\Sigma}_{\rm Rx} = \mathbf{I}_{N_r}$, the problem (18) can be simplified to the following problem

$$\min_{\hat{F}(i,i)\ge 0,\, \mathrm{Tr}(\hat{\mathbf{F}})\le 1} -\log\det\big(\mathbf{I} + \tilde{g}\mathbf{\Sigma}_{\rm Rx}\otimes\hat{\mathbf{F}}\big) \quad (24)$$

where $\hat{\mathbf{F}} = \mathbf{F}\mathbf{F}^H$. Furthermore, using some properties of Kronecker product and the fact that the matrix $\mathbf{\Sigma}_{\rm Rx}$ is diagonal, the above problem (24) can be rewritten as

$$\max_{\hat{F}(i,i)\ge 0,\, \mathrm{Tr}(\hat{\mathbf{F}})\le 1} \sum_{j=1}^{N_r} -\log\det\Big(\mathbf{I} + \tilde{g}\mathbf{\Sigma}_{\rm Rx}(j,j)\hat{\mathbf{F}}\Big). \quad (25)$$

Applying the Lagrange multiplier method for solving the problem (25), we find that the optimal $\hat{\mathbf{F}}$ is given by the solution of the following system of equations

$$\sum_{j=1}^{N_r} \tilde{g}\mathbf{\Sigma}_{\rm Rx}(j,j)\big(1 + \tilde{g}\mathbf{\Sigma}_{\rm Rx}(j,j)\hat{F}(i,i)\big)^{-1} = \mu,\ i = 1,\ldots,N_t. \quad (26)$$

Specifically, we can see that the optimal $\hat{\mathbf{F}}$ has equal diagonal elements $\hat{F}(i,i)$, $i = 1,\ldots,N_r$. Using the average power constraint $\mathrm{Tr}(\hat{\mathbf{F}}) = 1$, we can find that $\hat{\mathbf{F}} = \frac{1}{N_t}\mathbf{I}$. Therefore, the optimal precoding matrix in the case of no transmit side correlation is independent on the receive side correlation.

C. Both transmit and receive sides correlations

Finally, in the general case with both receive and transmit side correlation, the problem (18) can be rewritten as

$$\min_{\tilde{F}(i,i)\ge 0,\, \mathrm{Tr}(\tilde{\mathbf{F}})\le 1} \sum_{j=1}^{N_r} -\log\det\Big(\mathbf{I} + \tilde{g}\mathbf{\Sigma}_{\rm Rx}(j,j)\mathbf{\Sigma}_{\rm Tx}^{1/2}\tilde{\mathbf{F}}\mathbf{\Sigma}_{\rm Tx}^{1/2}\Big) \quad (27)$$

where we have used some properties of the Kronecker product and the fact that $\mathbf{\Sigma}_{\rm Rx}$ is diagonal.

The optimization problem (27) is convex, and can be efficiently solved using, for example, a simple *gradient descend* *method* [13]. Particularly, applying the Lagrange multiplier method for solving the problem (27), we find that the optimal $\hat{\mathbf{F}}$ is given by the solution of the following system of equations

$$\sum_{j=1}^{N_r} \frac{\tilde{g}\boldsymbol{\Sigma}_{\text{Rx}}(j,j)}{\boldsymbol{\Sigma}_{\text{Tx}}(i,i)^{-1} + \tilde{g}\boldsymbol{\Sigma}_{\text{Rx}}(j,j)\tilde{F}(i,i)} = \mu, \quad i = 1, \dots, N_t. \tag{28}$$

where μ is chosen such that $\sum_i \tilde{F}(i,i) = 1$.

It is worth noting that in the case of constant receive side correlation, i.e., $\mathbf{R}_{\text{Rx}} = \mathbf{I}(r)$ where $\mathbf{I}(r)$ is the matrix with the diagonal elements equal to 1 and off-diagonal elements equal to r, the problem (27) can be solved in closed form. In this case, $\mathbf{R}_{\text{Rx}}$ has one eigenvalue of order one equal to $\boldsymbol{\Sigma}_{\text{Rx}_1} = r(N_r - 1) + 1$ and one eigenvalue of order $N_r - 1$ equal to $\boldsymbol{\Sigma}_{\text{Rx}_2} = 1 - r$. Then, each equation in (28) can be simplified as

$$\frac{\tilde{g}_1}{\boldsymbol{\Sigma}_{\text{Tx}}(i,i)^{-1} + \tilde{g}_1\tilde{F}(i,i)} + \frac{(N_r - 1)\tilde{g}_2}{\boldsymbol{\Sigma}_{\text{Tx}}(i,i)^{-1} + \tilde{g}_2\tilde{F}(i,i)} = \mu, \, \forall i \tag{29}$$

where $\tilde{g}_1 = \tilde{g}\big[r(N_r - 1) + 1\big]$, $\tilde{g}_2 = \tilde{g}(1 - r)$. If the cross-correlation between different pairs of antennas is much smaller than 1, i.e., $\boldsymbol{\Sigma}_{\text{Rx}}(j,j) \approx N_r^{-1}\text{Tr}(\mathbf{R}_{\text{Rx}})\, \forall j$, each equation in (28) can be approximated as

$$\frac{N_r\hat{g}}{\boldsymbol{\Sigma}_{\text{Tx}}(i,i)^{-1} + \hat{g}\tilde{F}(i,i)} = \mu, \quad i = 1, \dots, N_t \tag{30}$$

where $\hat{g} = \tilde{g}N_r^{-1}\text{Tr}(\mathbf{R}_{\text{Rx}})$. Solving (30) we can derive the following approximate solution for the precoder

$$\tilde{F}(i,i) = \Big(N_r\mu^{-1} - (\boldsymbol{\Sigma}_{\text{Tx}}(i,i)\hat{g})^{-1}\Big)^+, \; i = 1, \dots, N_t. \tag{31}$$

V. Simulation Results

In this section, we investigate the performance of the proposed precoder. We simulate the system with four transmit and single receive antenna. 4-PAM modulation is used. Note that the 4-PAM modulation scheme is real. Therefore, the following full-rate real OSTBC can be adopted [2]

$$\mathbf{X}(\mathbf{s}) = \begin{bmatrix} s_1 & -s_2 & -s_3 & -s_4 \\ s_2 & s_1 & s_4 & -s_3 \\ s_3 & -s_4 & s_1 & s_2 \\ s_4 & s_3 & -s_2 & s_1 \end{bmatrix}.$$

The total available power at the transmitter is equal to 1, and the channel is assumed to be a correlated Rayleigh fading channel with constant transmit correlation. The correlation coefficient between different transmit antennas is equal to 0.4.

Fig. 1 displays the SER versus SNR for the system with and without precoding. We can see that the performace gain of the transmission scheme with precoding over the transmission scheme without precoding is about 2.5 dB.

VI. Conclusion

The general framework for precoder design for OSTBC based MIMO systems is presented in the paper for the case of correlated Rayleigh fading channels. The optimal precoder

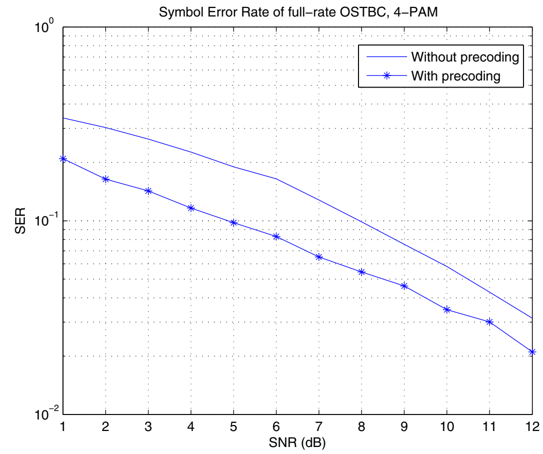

Fig. 1. SER performance of the transmission schemes with and without precoding.

exploits the knowledge of the transmit and receive correlation matrices at the transmitter and aims at minimizing the Chernoff bound on the exact SER. Three cases of transmit correlation only, receive correlation only, and both transmit and receive correlations are considered. The closed-form solutions with a very low computational complexity are derived where it is possible. The simulation example demonstrates the advantages of the precoded transmission scheme over the scheme without precoding.

References

[1] E. G. Larsson and P. Stoica, *Space-time Block Coding for Wireless Communications*. Cambridge University Press, 2003.

[2] V. Tarokh, H. Jafarkhani, and A. Calderbank, "Space-time block codes from orthogonal designs," *IEEE J. Select. Areas Commun.*, vol. 17, no. 3, pp. 451 – 460, Mar. 1999.

[3] H. Sampath, P. Stoica, and A. Paulraj, "Generalized linear precoder and decoder design for MIMO channels using the weighted MMSE criterion," *IEEE Trans. Commun.*, vol. 49, no. 12, pp. 2198 – 2206, Dec. 2001.

[4] E. Biglieri and et. al., *MIMO wireless communications*. Cambridge, UK: Cambridge University Press, 2007.

[5] G. Jongren, M. Skoglund, and B. Ottersten, "Combining beamforming and orthogonal space-time block coding," *IEEE Trans. Inform. Theory*, vol. 48, no. 3, pp. 611 – 627, Mar. 2002.

[6] A. Scaglione, P. Stoica, S. Barbarossa, G. Giannakis, and H. Sampath, "Optimal designs for space-time linear precoders and decoders," *IEEE Trans. Signal Processing*, vol. 50, no. 5, pp. 1051 – 1064, May 2002.

[7] M. Vu and A. Paulraj, "Optimal linear precoders for MIMO wireless correlated channels with non-zero mean in space-time coded systems," *IEEE Trans. Signal Processing*, vol. 54, no. 6, pp. 2318–2332, June 2006.

[8] H. R. Bahrami and T. Le-Ngoc, "Precoder design based on correlation matrices for MIMO systems," *IEEE Trans. Wireless Commun.*, vol. 5, no. 12, pp. 3579 – 3587, Dec. 2006.

[9] M. K. Simon and M.-S. Alouini, *Digital Communication over Fading Channels: A Unified approach to perfromance analysis*, 1st ed. New York: Wiley Series in Telecommunications and Signal Processing, 2000.

[10] G. L. Turin, "The characteristic function of hermitian quadratic forms in complex normal random variables," *Biometrika*, pp. 199–201, 1960.

[11] R. A. Horn and C. R. Johnson, *Matrix Analysis*. Cambridge, U.K.: Cambridge Univ. Press, 1985.

[12] L. Vandenberghe, S. Boyd, and S. Wu, "Determinant maximization with linear matrix inequality constraints," *SIAM J. Matrix Anal. Appl.*, vol. 19, pp. 499–533, 1998.

[13] S. Boyd and L. Vandenberghe, *Convex optimization*. Cambridge,U.K.: Cambridge Univ. Press, 2004.