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Precoding for Multiuser Orthogonal Space-Time Block-Coded OFDM Downlink over Spatially-Correlated Channels

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***Abstract*— This paper considers precoding for closed-loop multiuser orthogonal space-time block-coded (OSTBC) orthogonal frequency-division multiplexing (OFDM) downlink over multiple-input multiple-output (MIMO) channels. We present a general transmit-antenna-correlated, frequency-selective fading MIMO channel model with imperfect channel estimates and feedback delay, and derive the conditional means of the channel response. Exploiting the channel statistics, we develop new linear precoding and non-linear Tomlinson-Harashima precoding (THP) to maximize the signal-to-noise power ratio (SNR). By considering the conditional mean as the equivalent channel matrix, our precoding takes into account the estimation errors and channel time variations over feedback delay. We confirm the intuition that when channel state information at the transmitter (CSIT) becomes accurate, the full-CSIT precoder outperforms the covariance-feedback precoder. The proposed precoder has a bit error rate (BER) gain over unprecoded systems. As well, non-linear THP is shown to outperform linear precoding.**

***Index Terms*— precoding, closed-loop, MIMO, multiuser OFDM, OSTBC, antenna correlations, frequency-selective fading**

I. Introduction

Orthogonal frequency-division multiplexing (OFDM) is a popular modulation technique for high-rate data transmission over frequency-selective wireless channels. It can be deployed in multiuser systems where the base station (BS) communicates with multiple users simultaneously, and multiple access is enabled by allowing several users to share an OFDM symbol. Multiuser OFDM has already been adopted in several wireless standards and is now a strong candidate technique for fourth-generation (4G) systems. Multiple-input multiple-output (MIMO) techniques, using multiple antennas at both the transmitter and the receiver, can introduce spatial diversity and enable high link capacity and throughput. In a multiuser MIMO OFDM system, transmit diversity is an attractive feature because multiple antennas can be located at the BS and their benefits shared by all users. Orthogonal space-time block coding (OSTBC)[1] [1], [2] is a powerful transmit-diversity scheme, which can achieve full diversity with low-complexity optimal decoding, and hence has been adopted in the third-generation cellular standards.

[1]OSTBC here stands for orthogonal space-time block-coded and orthogonal space-time block coding, depending on the context.

OSTBC has originally been designed for open-loop systems, where channel state information (CSI) is only known at the receiver but not at the transmitter. However, in a closed-loop system, transmitter precoding can exploit the channel conditions, simplify each user's receiver, and offer significant system error-rate reduction or capacity gains. OSTBC can thus be implemented jointly with precoding to optimize the MIMO transmission [3]–[6]. On the other hand, the performance of open-loop OSTBC is degraded by the insufficient antenna spacing at the BS, which leads to antenna correlations. These significantly reduce the system capacity [7] and increase the bit error rate (BER) [8]. Precoding which can offer OSTBC the flexibility of adapting to correlated MIMO channels is thus decsribed [9], [10]. In OFDM systems, precoding, which enables pre-processing of the signals at a subcarrier level, increases capacity in spatially-multiplexed OFDM, and reduces the error rate in OSTBC OFDM [11].

A typical precoding design needs full CSI at the transmitter (CSIT). CSI can be estimated at the BS by using the reciprocity of radio channels in time-division duplex (TDD) systems. For frequency-division duplex (FDD) systems, CSI must be estimated at the receiver and sent back to the BS. Full-CSIT-based precoders have been developed for OSTBC systems over a flat-fading MIMO channel in [3], [5], [6]. In [3] a pair-wise error probability (PEP) criterion is derived for precoding design when CSIT is imperfect. Reference [5] proposes limited-feedback precoding over an uncorrelated MIMO channel, assuming that CSI at the receiver (CSIR) is perfect. In [6], a linear precoder is proposed in an OSTBC MIMO system considering imperfect CSIT, but perfect CSIR. However, error-rate-minimizing precoding for a more general OSTBC system with channel estimation errors and feedback delay over a spatially-correlated frequency-selective MIMO fading channel has not been studied yet. In [9]–[11], precoders with only covariance feedback (the long-term/second-order statistical information) are derived for OSTBC systems over correlated MIMO channels to minimize the worst case PEP. Nevertheless, if full CSIT can be accurate, full-CSIT precoding under a proper performance criterion may outperform covariance-feedback precoding.

In this paper, we develop precoding to effectively exploit full CSIT for a multiuser OSTBC OFDM downlink

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on frequency-selective fading MIMO channels with transmit-antenna correlations. We consider a general system model, in which each user's receiver estimates its channel imperfectly and sends the imperfect channel estimates back to the BS via a feedback channel, which introduces delay. We derive the conditional mean and variance of the channel matrix. Exploiting the channel statistics, we propose new robust precoding, which takes into account the CSIT uncertainty to maximize the received signal-to-noise power ratio (SNR). Both linear precoding and non-linear Tomlinson-Harashima precoding (THP) are considered. Compared to [3], [5], [6], [12], we consider a more general case with erroneous channel estimates, multiplicative time-varying effects, and transmit antenna correlations. Evidently, the error rate of a precoded system depends on the quantity and quality of CSIT. We confirm the intuition of the fundamental trade-offs between performance, feedback requirements and CSIT accuracy. As CSIT approaches perfect CSI, our precoding achieves a lower BER than covariance-feedback precoding in [11], although the latter requires a lower feedback rate. Non-linear precoding is shown to outperform linear precoding.

II. SYSTEM MODEL

This section will introduce the system model of an N-subcarrier OFDM downlink with M_T transmit antennas and U simultaneously active users in the presence of transmit antenna correlations. The u-th user has M_u receive antennas, and the total number of receive antennas is $M_R = \sum_{u=1}^{U} M_u$. The U users share one OFDM symbol and the u-th user is assigned a subset $\mathbb{K}_u$ containing $N_u = \lfloor N/U \rfloor$ subcarriers, where $\lfloor \cdot \rfloor$ is the floor function. At the BS, a subcarrier allocation algorithm maps the user data to the corresponding subcarriers, and this algorithm is known at both the BS and the mobile stations (MS). The receive antennas at the MS are uncorrelated if antenna spacing is greater than half wavelength [13]. This condition is satisfied by practical systems that use carrier frequencies in the order of GHz.

A. Multiuser OFDM with Transmit-Antenna Correlations

The channel between the m-th transmit antenna and the u-th user's n-th receive antenna is a wideband frequency-selective fading channel with L resolvable paths. The l-th path gain is a zero-mean complex Gaussian random variable (Rayleigh fading) with variance σ_l^2. At the time i, the set of the l-th path gains between the BS and the u-th user can be represented by an $M_u \times M_T$ matrix $\mathbf{H}_{u,l}[i]$ with entries $h_{u_{m,n},l}[i]$. We assume that the channel gains remain constant over several OFDM symbol intervals. As in [8], the $M_u \times M_T$ channel on the k-th subcarrier of the u-th user can be represented as

$$\mathbf{H}_u[k,i] = \sum_{l=0}^{L-1} \mathbf{H}_{u,l}[i] e^{-\jmath \frac{2\pi}{N} kl} \mathbf{r}_T = \breve{\mathbf{H}}_u[k,i]\mathbf{r}_T, \quad k \in \mathbb{K}_u. \tag{1}$$

Different users have the same transmit antenna correlation matrix $\mathbf{R}_T = \mathbf{r}_T^H \mathbf{r}_T$ with entries [7]

$$R_T(p,q) = \mathcal{J}_0 \left(2\pi |p-q| \zeta_T\right), \tag{2}$$

where $\mathcal{J}_0$ is zero-order Bessel function of the first kind; $\zeta_T = \Delta \frac{d_T}{\lambda_c}$, $\lambda_c = c/f_c$ is the wavelength at the carrier frequency f_c, Δ is the angle of arrival spread, and the transmit antennas are spaced by d_T.

At the receiver, the k-th received signal vector is

$$\mathbf{Y}_u[k,i] = \mathbf{H}_u[k,i]\mathbf{X}_u[k,i] + \mathbf{W}_u[k,i], \quad k \in \mathbb{K}_u, \tag{3}$$

where $\mathbf{Y}_u[k,i]$ is an M_u-dimensional vector and $\mathbf{W}_u[k,i]$ is the noise vector where the entries $W_{u_n}[k,i]$ are i.i.d. additive white Gaussian noise (AWGN) samples with zero mean and variance σ_W^2. The input data vector is $\mathbf{X}_u[k,i] = \left[X_{u_1}[k,i] \dots X_{u_{M_T}}[k,i]\right]^T$; $X_{u_m}[k,i]$ denotes an M-ary QAM symbol on the k-th subcarrier sent to the u-th user by the m-th transmit antenna at time i. In a multiuser OSTBC OFDM system, $\mathbf{X}_u[k,i]$ will be an OSTBC matrix instead of a data vector. The u-th user's $N_u M_u \times N_u M_T$ channel matrix $\mathbf{H}_u[i]$ is given by

$$\mathbf{H}_u[i] = \text{diag}\left[\mathbf{H}_u[j,i] \dots \mathbf{H}_u[k,i]\right], \quad j,k \in \mathbb{K}_u. \tag{4}$$

We consider the structure described by (3), in which precoding can be designed individually for each subcarrier.

B. OSTBC OFDM

A $T \times M_T$ code matrix $\mathbf{C}$ for OSTBC satisfies

$$\mathbf{C}^H \mathbf{C} = \left(\sum_{t=1}^{P} |c_t|^2\right) \mathbf{I}_{M_T}. \tag{5}$$

The code rate in this case is $R = P/T$, where P represents the number of symbols transmitted over the T time slots. The full-rate codes transmit an average of one symbol per symbol period, i.e., $R = 1$. OSTBC can be directly applied to OFDM at a subcarrier level to offer full spatial diversity gain, if there is no correlation between transmit antennas or different paths. For example, the full-rate transmission matrix of Alamouti-coded OFDM is to transmit $\begin{pmatrix} X_1[k] & -X_2^*[k] \\ X_2[k] & X_1^*[k] \end{pmatrix}$ onto the subcarrier k, i.e., $X_1[k]$ and $X_2[k]$ are transmitted over the 1-st and 2-nd antenna at the first time slot, respectively; the $-X_2^*[k]$ and $X_1^*[k]$ are transmitted in the following slots. Full-rate complex orthogonal designs do not exist for more than two transmit antennas. The transmission rate of OSTBC is thus less than or equal to 1, i.e., $R \leq 1$.

C. Statistical Channel Model

In this subsection, we present a general MIMO channel model accounting for imperfect channel estimates, multiplicative time-varying effects, and transmit-antenna correlations. We derive the conditional expectation and variance of the channel matrix exploiting the channel statistics.

In our case, each user's receiver has inaccurate channel estimates $\mathbf{H}_{R_u}[k,i]$ of the current actual, but unknown, channel $\mathbf{H}_u[k,i]$, $k \in \mathbb{K}_u$; the imperfect channel estimates are sent to the BS via a feedback channel which introduces delay τ_u, and $\tau_u \neq \tau_{u'}$, $\forall\, u \neq u'$. Consequently, the BS has the erroneous estimate $\mathbf{H}_{T_u}[k,i]$ of the actual (unknown) but outdated channel matrix $\mathbf{H}_u[k, i-\tau_u]$, in which the $\{m,n\}$th channel of the user u is τ_u seconds older than that in the current channel $\mathbf{H}[k,i]$. The actual channel matrices $\mathbf{H}_u[k,i]$ and $\mathbf{H}_u[k,i-\tau]$ are unknown at both the transmitter and the receiver, and $\mathbf{H}_u[k,i] \neq \mathbf{H}_u[k,i-\tau] \neq \mathbf{H}_{T_u}[k,i] \neq \mathbf{H}_{R_u}[k,i]$.

Without loss of generality, we model the frequency-selective fading channel as follows:

- The entries in a tap vector for the $\{m,n\}$th antenna pair of the u-th user $\mathbf{h}_{u_{m,n}}[i] = \left[h_{u_{m,n},0}[i], \ldots, h_{u_{m,n},L-1}[i]\right]^T$ are time-varying according to Clarke's 2-D isotropic scattering model with maximum Doppler shift f_{D_u} [13]. Since $\mathbf{h}_{u_{m,n}}[i-\tau_u]$ is a delayed version of $\mathbf{h}_{u_{m,n}}[i]$, they are jointly Gaussian with an auto-covariance matrix

$$\mathrm{E}\left[\mathbf{h}_{u_{m,n}}[i]\mathbf{h}^H_{u_{m,n}}[i-\tau_u]\right] = J_u \mathbf{R}_P, \tag{6}$$

  where $\mathbf{R}_P = \mathbf{r}_P^2 = \mathrm{diag}\left[\sigma_0^2, \ldots, \sigma_{L-1}^2\right]$, $J_u = \mathcal{J}_0(2\pi\epsilon_u)$, and $\epsilon_u = f_{D_u}\tau_u$ is the normalized maximum Doppler shift of the u-th user with delay τ_u. Different users have different Doppler shifts, i.e., $\epsilon_{u'} \neq \epsilon_u$.
- The channel estimates at the receiver are maximum likelihood (ML) estimates and can be expressed as

$$\mathbf{H}_{R_u}[k,i] = \mathbf{H}_u[k,i] + \mathbf{e}_u[k], \tag{7}$$

  where $\mathbf{e}_u[k]$ is the estimation error vector with entries $e_{m,n}[k] \sim \mathcal{CN}(0,\, \Omega_{e_u})$, $\forall\, i,k$, and $\mathbf{e}_u[k]$ is independent of all other stochastic processes, $\forall\, u$. The auto-covariance of $\mathbf{H}_{R_u}[k,i]$ is

$$\mathbf{C}_{\mathbf{H}_R\mathbf{H}_R} = \mathrm{E}\left[\mathbf{H}^H_{R_u}[k,i]\mathbf{H}_{R_u}[k,i]\right] = \mathbf{R}_T + \Omega_{e_u}\mathbf{I}_{M_T}. \tag{8}$$

  The proof of (8) is given in Appendix. Similarly, the cross-covariance of $\mathbf{H}_u[k,i]$ and $\mathbf{H}_{R_u}[k,i]$ is

$$\mathbf{C}_{\mathbf{H}_u\mathbf{H}_{R_u}} = \mathrm{E}\left[\mathbf{H}^H_u[k,i]\mathbf{H}_{R_u}[k,i]\right] = \mathbf{R}_T. \tag{9}$$

- We assume the transmitter channel matrix $\mathbf{H}_{T_u}[k,i]$ is an imperfect estimate of the actual but unknown $\mathbf{H}_u[k,i-\tau_u]$, which can be modeled by

$$\mathbf{H}_{T_u}[k,i] = \mathbf{H}_{R_u}[k,i-\tau_u] = \mathbf{H}_u[k,i-\tau_u] + \mathbf{e}_u[k]. \tag{10}$$

  The derivations are included in our full journal paper version. Combined with (6), we can show the covariance

$$\mathbf{C}_{\tau_\mathbf{u}} = \mathrm{E}\left[\mathbf{H}^H_u[k,i-\tau]\mathbf{H}_u[k,i]\right] = J_u\mathbf{R}_T. \tag{11}$$

  Similarly, we can obtain

$$\begin{aligned} \mathbf{C}_{\mathbf{H}_{T_u}\mathbf{H}_{R_u}} &= J_u\mathbf{R}_T + \Omega_{e_u}\mathbf{I}_{M_T} \\ \mathbf{C}_{\mathbf{H}_{T_u}\mathbf{H}_u} &= \mathbf{C}_{\mathbf{H}_u\mathbf{H}_{T_u}} = J_u\mathbf{R}_T. \end{aligned} \tag{12}$$

Given $\mathbf{H}_{T_u}[k,i]$, the BS can obtain the statistics of the user u's channel vector $\vec{\mathbf{H}}_u[k,i] = \mathrm{vec}(\mathbf{H}_u[k,i])$ with the conditional expectation and variance [14]

$$\begin{aligned} \mathbf{H}_{\vec{\mathbf{H}}_u|\vec{\mathbf{H}}_{T_u}}[k,i] &= \mathbf{C}_{\vec{\mathbf{H}}_u\vec{\mathbf{H}}_{T_u}}\mathbf{C}^{-1}_{\vec{\mathbf{H}}_{T_u}\vec{\mathbf{H}}_{T_u}}\vec{\mathbf{H}}_{T_u}[k,i] \\ \mathbf{C}_{\vec{\mathbf{H}}_u|\vec{\mathbf{H}}_{T_u}} &= \mathbf{C}_{\vec{\mathbf{H}}_u\vec{\mathbf{H}}_u} - \mathbf{C}_{\vec{\mathbf{H}}_u\vec{\mathbf{H}}_{T_u}}\mathbf{C}^{-1}_{\vec{\mathbf{H}}_{T_u}\vec{\mathbf{H}}_{T_u}}\mathbf{C}_{\vec{\mathbf{H}}_{T_u}\vec{\mathbf{H}}_u}. \end{aligned} \tag{13}$$

The conditional mean and variance of the actual channel matrix $\mathbf{H}_u[k,i]$ thus can be given by

$$\begin{aligned} \mathbf{H}_{u|T_u}[k,i] &= J_u\mathbf{H}_{T_u}[k,i]\mathbf{R}_T\left(\mathbf{R}_T + \Omega_{e_u}\mathbf{I}_{M_T}\right)^{-1}, \\ \mathbf{C}_{u|T_u} &= \mathbf{R}_T - J_u^2\mathbf{R}_T\left(\mathbf{R}_T + \Omega_{e_u}\mathbf{I}_{M_T}\right)^{-1}\mathbf{R}_T, \end{aligned} \tag{14}$$

where $\mathrm{vec}\left(\mathbf{H}_{u|T_u}[k,i]\right) = \mathbf{H}_{\vec{\mathbf{H}}_u|\vec{\mathbf{H}}_{T_u}}[k,i]$. The proof of (14) is given in Appendix. Since the receiver has the information $\mathbf{H}_{R_u}[k,i-\tau] = \mathbf{H}_{T_u}[k,i]$ (10), the conditional mean of the channel matrix at the BS $\mathbf{H}_{u|T_u}[k,i]$ can also be calculated at both the BS and each user's receiver given $\mathbf{H}_{T_u}[k,i]$ and $\mathbf{H}_{R_u}[k,i-\tau]$, and the variance is dependent on the correlation matrix. Similarly, at each user's receiver, we can also obtain

$$\begin{aligned} \mathbf{H}_{u|R_u}[k,i] &= \mathbf{H}_{R_u}[k,i]\mathbf{R}_T\left(\mathbf{R}_T + \Omega_{e_u}\mathbf{I}_{M_T}\right)^{-1}, \\ \mathbf{C}_{u|R_u} &= \Omega_{e_u}\mathbf{R}_T\left(\mathbf{R}_T + \Omega_{e_u}\mathbf{I}_{M_T}\right)^{-1}. \end{aligned} \tag{15}$$

The conditional means $\mathbf{H}_{u|T_u}[k,i]$ and $\mathbf{H}_{u|R_u}[k,i]$ can be described as equivalent channels exploiting the channel statistics and uncertainty structure to mitigate the impact of imperfect CSI at the BS and the receivers [15]. The variances $\mathbf{C}_{u|T_u}$ and $\mathbf{C}_{u|R_u}$ indicate the CSIT and CSIR uncertainty given by the equivalent channels, respectively. The CSIR uncertainty is determined by the channel correlation matrix and the estimation error. If CSIR is perfect, i.e., $\Omega_{e_u} = 0$, $\forall\, u$, we have $\mathbf{H}_{u|R_u}[k,i] = \mathbf{H}_{R_u}[k,i] = \mathbf{H}_u[k,i]$. At the BS, beside $\mathbf{R}_T$ and estimation errors, the uncertainty also depends on the autocovariance factor J_u, which is the function of the normalized maximum Doppler shift. As the maximum Doppler shift increases, which may be caused by rapidly growing mobility of the user, the CSIT uncertainty may become significant.

III. Precoding for Multiuser OSTBC OFDM

In this section, we propose linear precoding and non-linear THP to maximize the SNR in the general multiuser OSTBC OFDM system model. Exploiting the conditional channel mean and variance, our precoders take into account the channel uncertainty due to estimation errors and time variations.

A. SNR-Maximal Precoding

A linear precoder for flat-fading OSTBC MIMO channels is designed in [6], in which only additive estimation noise is considered and perfect CSIR is assumed. Here we consider the more general case in which multiplicative time-varying effects and imperfect CSIR are considered due to feedback delay and channel estimation errors.

The BS can calculate the u-th user's equivalent channel matrix $\mathbf{H}_{u|T_u}[k,i]$ (14), and operate singular value decomposition (SVD), which yields

$$\mathbf{H}_{u|T_u}[k] = \widetilde{\mathbf{U}}_u[k]\widetilde{\mathbf{\Gamma}}_u[k]\widetilde{\mathbf{V}}^H_u[k], \tag{16}$$

where $\widetilde{\mathbf{U}}_u[k]$ and $\widetilde{\mathbf{V}}_u[k]$ are $M_u \times M_u$ and $M_T \times M_T$ unitary matrices, and the diagonal singular value matrix $\widetilde{\mathbf{\Gamma}}_u[k]$ has real, non-negative entries $\tilde{\gamma}_{u_m}$. For simplicity, we omit the time index. As in [6], [16], [17], a general form of the linear precoding matrix $\mathbf{E}_u[k]$ can be given by

$$\mathbf{E}_u[k] = \widetilde{\mathbf{V}}_u[k]\mathbf{\Lambda}_u[k]\widetilde{\mathbf{V}}^H_u[k], \tag{17}$$

where $\mathbf{\Lambda}_u[k]$ is a positive semi-definite diagonal matrix representing the power distribution with the main diagonal entries λ_{u_m}. OSTBC can achieve the maximum possible SNR [18], which determines the system error probability. With precoding (17), the SNR on the k-th subcarrier in OSTBC OFDM can be given by

$$\mathrm{SNR}_u[k] = \frac{E_s}{\sigma_W^2}\mathrm{E}\left[\mathrm{tr}\left(\mathbf{\Lambda}_u^H[k]\widetilde{\mathbf{V}}_u^H[k]\mathbf{H}_u^H[k]\mathbf{H}_u[k]\widetilde{\mathbf{V}}_u[k]\mathbf{\Lambda}_u[k]\right)\right], \tag{18}$$

where $\mathrm{tr}(\cdot)$ denotes the trace of a matrix. Since the actual channel has the conditional mean $\mathbf{H}_{u|T_u}[k]$ and variance $\mathbf{C}_{u|T_u}$ as shown in (14), the error-rate minimization problem becomes

$$\mathbf{E}_u[k]_{\mathrm{opt}} = \mathfrak{L}(\mathbf{\Lambda}_u[k]) = \arg\max \mathrm{tr}\left[\widetilde{\mathbf{V}}_u^H[k]\left(\mathbf{H}_{u|T_u}^H[k]\mathbf{H}_{u|T_u}[k] + \mathbf{C}_{u|T_u}\right)\widetilde{\mathbf{V}}_u[k]\mathbf{\Lambda}_u^2[k]\right], \tag{19}$$

subject to $\mathrm{tr}\left(\mathbf{E}_u^H[k]\mathbf{E}_u[k]\right) = \mathrm{tr}\left(\mathbf{\Lambda}_u^2[k]\right) = M_T$. The function $\mathfrak{L}$ is linear and therefore it is concave in $\mathbf{\Lambda}_u[k]$. The objective of (19) is to look for the power allocation $\mathbf{\Lambda}_u[k]$ that optimizes the average SNR based on the quality of the transmitter equivalent channel matrix $\mathbf{H}_{u|T_u}[k]$. As $\mathbf{H}_{u|T_u}[k]$ approaches to the actual channel matrix $\mathbf{H}_u[k]$, $\widetilde{\mathbf{V}}_u[k]$ approaches $\mathbf{V}_u[k]$ and $\mathbf{\Lambda}_u[k]$ is primarily determined by the singular values of the channel matrix, where $\mathbf{V}_u[k]$ is the right unitary matrix of SVD of the actual channel $\mathbf{H}_u[k]$.

The problem of (19) can be solved numerically or analytically. Several software packages are available including *optimization toolbox* in MATLAB. For example, the function *fmincon* can be used, which offers efficient computations within a polynomial time. Furthermore, it has been shown that for some concrete uncertainty, the problem (19) simplifies to a quadratic form and in some cases, a closed-form solution exists [6]. At the receiver, the effective channel $\mathbf{H}_{u|R_u}[k,i]\mathbf{E}_u[k]_{\mathrm{opt}}$ is used for ML detection, where $\mathbf{H}_{u|R_u}[k,i]$ is the receiver equivalent channel matrix given in (15).

B. Non-Linear Tomlinson-Harashima Precoding

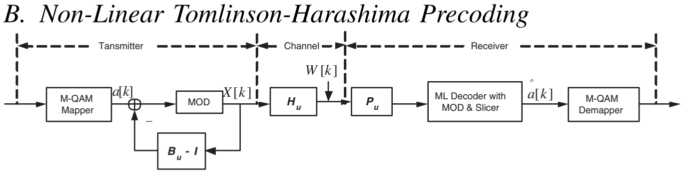

Fig. 1. Tomlinson-Harashima precoding in multiuser OSTBC OFDM downlink.

In this subsection, we propose mean-feedback THP for SNR maximization in multiuser OSTBC OFDM downlink. The structure of the proposed precoder is illustrated in Fig. 1. The receiver side consists of a diagonal scaling matrix $\mathbf{P}_u[k]$, an ML decoder and a modulo arithmetic device. The transmitter side includes a modulo arithmetic feedback structure employing the matrix $\mathbf{B}_u[k]$, by which the transmitted symbols $X[k]$ are successively calculated for the data symbols $a[k]$ drawn from the initial M-ary QAM signal constellation. Ignoring the modulo device, the u-th user's feedback structure is equivalent to $\mathbf{B}_u^{-1}[k]$, which can be optimally designed as in (19), $\mathbf{B}_u[k]_{\mathrm{opt}} = \mathbf{E}_{\mathrm{MF}_u}^{-1}[k]_{\mathrm{opt}}$. The effective channel is $\mathbf{H}_u[k]\mathbf{E}_{\mathrm{MF}_u}[k]$ and ML decoding is used at the receiver. The overall precoding matrices can be written as $\mathbf{B}_u = \mathrm{diag}\left[\mathbf{B}_u[j]\ldots\mathbf{B}_u[k]\right]$ and $\mathbf{P}_u = \mathrm{diag}\left[\mathbf{P}_u[j]\ldots\mathbf{P}_u[k]\right]$, where $j,\, k \in \mathbb{K}_u$. The diagonal scaling matrix $\mathbf{P}_u$ is to keep the average transmit power constant.

THP employs modulo operation at both the transmitter and the receiver. The modulo $2\sqrt{M}$ reduction at the transmitter, which is applied separately to the real and imaginary parts of the input, is to restrict the transmitted signals into the boundary of $\Re\{X_u[k]\} \in \left(-\sqrt{M}, \sqrt{M}\right]$ and $\Im\{X_u[k]\} \in \left(-\sqrt{M}, \sqrt{M}\right]$. If the input sequence $a[k]$ is a sequence of i.i.d. samples, the output of the modulo device is also a sequence of i.i.d. random variables, and the real and imaginary parts are independent, i.e., we can assume $\mathrm{E}\left[\mathbf{X}_u[k]\mathbf{X}_u^H[k]\right] = E_s\mathbf{I}_{M_T},\ \forall\, k$, [19]. At the receiver, the filtered noise vector becomes $\mathbf{W}'_u = \mathbf{P}_u\mathbf{W}_u$, where the k-th entry $W'[k]$ has individual variance $\sigma^2_{W'_k}$. A slicer, which applies the same modulo operation as that at the transmitter, is used. After the ML decoding and discarding the modulo congruence, the unique estimates of the data symbols $\hat{a}[k]$ can be generated. The details of the THP operation are described in [19].

IV. Simulation Results

This section presents simulation results to show how our proposed precoding improves the error rate in a 64-subcarrier multiuser OSTBC OFDM system with transmit-antenna correlations. The vehicular B channel specified by ITU-R M. 1225 [20] is used where the channel taps are zero-mean complex Gaussian random processes with variances of $-4.9\,$dB, $-2.4\,$dB, $-15.2\,$dB, $-12.4\,$dB, $-27.6\,$dB, and $-18.4\,$dB relative to the total power gains. ML decoding is used at each user's receiver. The BS and the user terminals know the correlation matrix $\mathbf{R}_T$ with the correlation parameter $\zeta_T = \Delta\frac{d_T}{\lambda}$; the angle of arrival spread is assumed $12°$, i.e., $\Delta \approx 0.2$.

We consider the interval $\mathbb{I} = [0.9, 1]$ for the autocorrelation function J_u, and the values are assumed to be uniformly distributed in this interval. The maximum possible number of distinct Doppler shift values is U. In the interval $\mathbb{I} = [0.9, 1]$, the Doppler shifts $\epsilon_u = f_{D_u}\tau_u$ normalized with respect to feedback delays τ_u are in the range $[0, 0.1]$. For a wideband OFDM system with the carrier frequency $5\,$GHz, if the feedback delay is $100\,\mu s$, the users' mobile speeds are in the range from zero to $216\,$km/h.

Fig. 2 shows the BERs of proposed conditional-mean-based linear precoding (MFB-LP) in Alamouti-coded OFDM with perfect channel estimation. The cases of 2 users with $J_u \in \mathbb{I}$ and single user with the time-varying autocorrelation function J of 0.9 and 0.998 are considered. Each user has 2 receive antennas. The BER of covariance-feedback linear precoding (CFB-LP) proposed in [11] is shown as a reference. In the two-user case, at low SNR the proposed MFB-LP has a lower BER than CFB-LP, while has a higher BER at medium SNR due to high channel time variations. As the antenna correlations become severe, i.e., ζ_T decreases, the BERs of both mean-based and covariance-based precoding increase. For instance, the BER at ζ_T of 0.25 is higher than that at ζ_T of 0.4. However, the relationship among CFB-LP, MFB-LP and NoP does not change. In the single-user case, when the user is moving at the speed of 30 km/h ($J = 0.998$), proposed mean-based precoding offers 0.4 dB BER gain over covariance-feedback precoding at the BER of 10^{-3}. Clearly, we confirm the intuition that full-CSIT-based precoding outperforms covariance-feedback precoding as CSIT becomes accurate. The mobility of the user degrades the performance of the full-CSIT precoding. At the autocorrelation J of 0.9, the BER of MFB-LP remarkably increases, i.e., covariance feedback is more suitable in this condition.

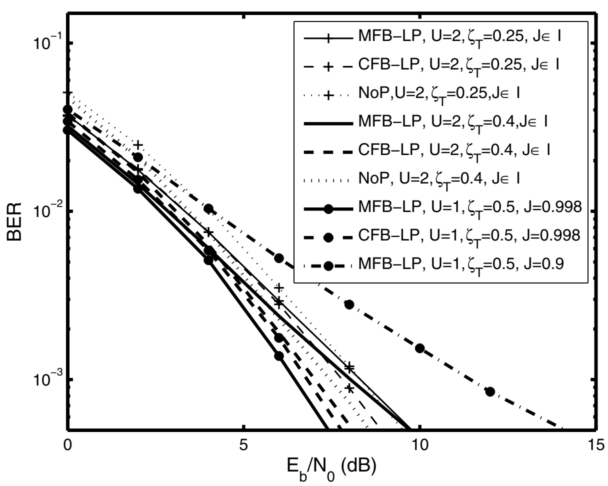

Fig. 2. BER as a function of SNR for mean-based linear precoding (MFB-LP), covariance-feedback linear precoding (CFB-LP) and no precoding (NoP) in 64-subcarrier 4-QAM Alamouti-coded OFDM systems with perfect feedback. $M_u = 2$, $U = 2$, $J_u \in \mathbb{I}$ and $U = 1$, $J = 0.9$ and $J = 0.998$.

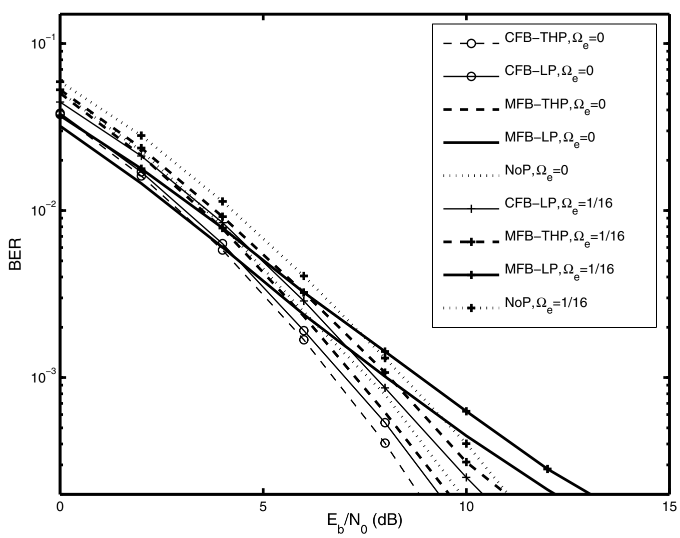

Fig. 3. BER as a function of SNR for mean-based linear precoding (MFB-LP), MFB-THP, covariance-feedback linear precoding (CFB-LP), MFB-THP and no precoding (NoP) in 64-subcarrier 4-QAM Alamouti-coded 4-user OFDM systems with perfect and imperfect estimation. $\zeta_T = 0.4$, $M_u = 2$, $U = 4$, $J_u \in \mathbb{I}$.

In Fig. 3, we show the performance of proposed mean-based linear precoding (MFB-LP), THP (MFB-THP) and the impact of imperfect channel estimation. 4-QAM Alamouti-coded 4-user OFDM is considered. When channel estimation is perfect, i.e, $\Omega_e = 0$, our proposed MFB-THP outperforms the no-precoding case even the mobility range is as high as $[0.9, 1]$, while our MFB-LP is more sensitive to the channel time variations. The variance of the channel estimation error Ω_e is assumed $1/16$, which leads to almost 1 dB and 0.8 dB BER loss in covariance-feedback precoding and proposed mean-based precoding. Clearly, non-linear precoding outperforms linear precoding. At a BER of 10^{-3}, $\Omega_e = 0$, CFB-THP has 0.6 dB gain over CFB-LP, and MFB-THP has about 1.2 dB gain over MFB-LP; when estimation is imperfect, MFB-THP achieves 0.8 dB gain over MFB-LP.

Fig. 4 shows the impact of the number of transmit antennas on our proposed mean-based precoding. We consider 16-QAM 2-user $1/2$-rate-OSTBC OFDM systems with 4 transmit antennas. The antenna correlation parameter ζ_T is set as 0.3. The BER of 4-QAM 2-user Alamouti-coded OFDM with 2 transmit antennas is given as a reference. The large number of transmit antennas improves the system performance at a high SNR region, and the system with the large number of transmit antennas is less sensitive to the estimation errors. Once again, non-linear THP outperforms linear precoding, in both perfect and imperfect estimation cases.

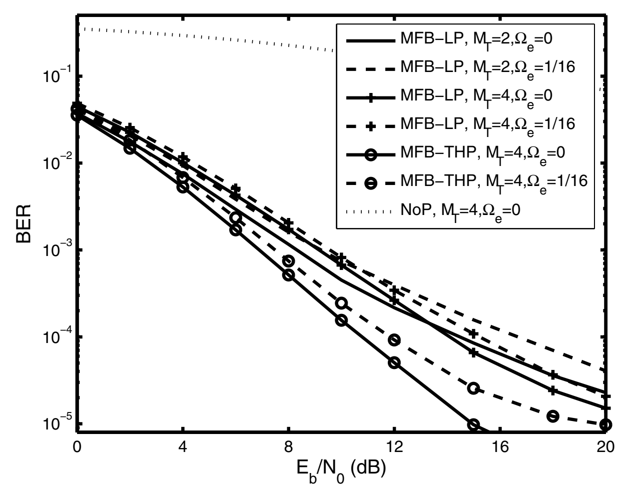

Fig. 4. BER as a function of SNR for mean-based linear precoding (MFB-LP), MFB-THP and no precoding (NoP) in 64-subcarrier 2-user 16-QAM $1/2$-rate-OSTBC OFDM systems and 4-QAM Alamouti-coded OFDM with perfect and imperfect estimation. $\zeta_T = 0.3$, $M_u = 2$, $J_u \in \mathbb{I}$.

V. Conclusions

We have considered a general case of a spatially-correlated, frequency-selective fading MIMO channel model with imperfect channel estimates and feedback delay, and derived the conditional means of the channel matrix. We have developed new linear precoding and non-linear THP to maximize the SNR in multiuser OSTBC OFDM systems. Our precoding takes into account the estimation errors and channel time variations over feedback delay. We have confirmed the intuition that when CSIT becomes accurate, the proposed full-CSIT precoder performs better than the covariance-feedback precoder. The proposed precoder has a BER gain over unprecoded systems. Non-linear THP has been shown to outperform linear precoding.

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VI. APPENDIX

Note that $\text{vec}(\mathbf{ABC}) = (\mathbf{C}^T \otimes \mathbf{A})\text{vec}(\mathbf{B})$. We omit the time index i in the following matrices.

Proof of (8). Since $\mathbf{H}_u[k] = \breve{\mathbf{H}}_u[k]\mathbf{r}_T$ given in (1), the covariance of the channel vector $\vec{\mathbf{H}}_{R_u}[k] = \text{vec}(\mathbf{H}_{R_u}[k])$ can be given by

$$\text{E}\left[\vec{\mathbf{H}}_{R_u}[k]\vec{\mathbf{H}}_{R_u}^H[k]\right] = \left(\mathbf{R}_T + \Omega_e \mathbf{I}_{M_T}\right) \otimes \mathbf{I}_{M_u}, \tag{20}$$

where the entries of $\breve{\mathbf{H}}_u[k]$ are i.i.d. Gaussian random variables with zero mean and variance normalized to unit. After expanding the left and right sides of (20), we can calculate the entries in the matrix $\text{E}\left[\mathbf{H}_{R_u}^H[k]\mathbf{H}_{R_u}[k]\right]$, and thus obtain (8).

Proof of (14). The vectorization of $\mathbf{H}_{u|T_u}[k]$ in (14) is $\mathbf{H}_{\vec{\mathbf{H}}_u|\vec{\mathbf{H}}_{T_u}}[k]$, which can be expressed by

$$\begin{aligned}\mathbf{H}_{\vec{\mathbf{H}}_u|\vec{\mathbf{H}}_{T_u}}[k] &= \mathbf{C}_{\vec{\mathbf{H}}_u\vec{\mathbf{H}}_{T_u}}\mathbf{C}^{-1}_{\vec{\mathbf{H}}_{T_u}\vec{\mathbf{H}}_{T_u}}\vec{\mathbf{H}}_{T_u}[k] \\ &= J_u\left(\left(\mathbf{R}_T^T\left(\mathbf{R}_T^T + \Omega_{e_u}\mathbf{I}_{M_T}\right)^{-1}\right) \otimes \mathbf{I}_{M_u}\right)\vec{\mathbf{H}}_{T_u}[k].\end{aligned} \tag{21}$$

We thus have

$$\mathbf{H}_{u|T_u}[k] = J_u \mathbf{H}_{T_u}[k]\mathbf{R}_T\left(\mathbf{R}_T + \Omega_{e_u}\mathbf{I}_{M_T}\right)^{-1}, \tag{22}$$

where $\text{vec}\left(\mathbf{H}_{u|T_u}[k]\right) = \mathbf{H}_{\vec{\mathbf{H}}_u|\vec{\mathbf{H}}_{T_u}}[k]$. Similarly, the conditional variance of $\mathbf{H}_u$ given by $\mathbf{H}_{u|T_u}[k]$ is

$$\mathbf{C}_{u|T_u} = \mathbf{R}_T - J_u^2\mathbf{R}_T\left(\mathbf{R}_T + \Omega_{e_u}\mathbf{I}_{M_T}\right)^{-1}\mathbf{R}_T. \tag{23}$$

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