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# Receive antenna selection for spatial multiplexing systems based on union bound minimization

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***Abstract*— Despite their high spectral efficiencies, multiple-input multiple-output (MIMO) systems suffer from high cost and complexity due to multiple radio frequency chains at both link ends. A possible solution is to select a subset of the available antennas at transmitter and/or receiver based on maximal capacity or minimal error rates. In this paper, we propose a receive antenna selection algorithm to minimize the union bound on the vector error rate. By relaxing the antenna selection variables from discrete to continuous, we formulate the problem as a convex optimization problem. An efficient iterative method can be used to obtain the solution.**

## I. Introduction

Wireless communication systems, employing multiple antennas at both the transmitter and the receiver, achieve remarkably high spectral efficiencies in rich-scattering multipath environments. A well-known example of a such system is the BLAST (Bell Laboratories layered space time) architecture [1]. However, MIMO systems suffer from high cost and complexity due to multiple radio frequency chains at both link ends.

Antenna selection technologies for MIMO systems have thus been extensively studied. Since the MIMO paradigm includes a wide range of techniques such as space-time codes that extract full diversity [2], [3], uncoded transmissions that achieve full spatial multiplexing [1] and schemes that exploit the diversity-multiplexing tradeoff [4], there are several antenna selection criteria. For full-diversity space-time codes, a subset of available antennas can be selected to maximize the channel norm [5]. For spatial-multiplexing systems, antennas can be selected to minimize the error rates [6]. Comprehensive tutorial papers on antenna selection can be found in [7], [8].

Exhaustive search based on maximum output SNR is proposed in [6] and [9] when the system implements linear receivers. Since exhaustive search is computationally expensive for large MIMO systems, several sub-optimal algorithms with lower complexity are derived at the expense of efficiency. A selection algorithm based on accurate approximation for the conditional probability on quasi-static MIMO systems is derived in [10].

In [11], Dua et.al. formulate the receive antenna selection problem as a combinatorial optimization problem and then relax it to a convex optimization problem. They employ an interior point algorithm, i.e. the barrier method, to solve the relaxed convex problem. However, they treat only the case of capacity maximization.

However, perhaps the most important system performance metric is the bit-error-rate (BER) or the vector-error-rate (VER). To the best of our knowledge, no algorithms for antenna subset selection exist to directly optimize the union bound on the system error rate. In this paper, we propose a new approach to antenna selection to minimize the union bound, which is the sum of the all pairwise error probabilities (PEPs). This can be expressed as a sum of Gaussian Q-functions. To reduce the complexity to evaluate the Gaussian Q-function, we choose to minimize accurate approximations for the Q-function instead. By relaxing the antenna selection variables from discrete to continuous, we formulate the problem as a convex optimization problem. Due to the convexity of our derived problem, efficient numerical methods such as interior-point algorithms can be applied to solve it with polynomial complexity [12].

The paper is organized as follows. In the next section, the system model and the union bound when implementing receive subset antenna selection are presented. In Section 3, we formulate antenna selection as a convex programming problem to minimize the union bound. Experimental results via Monte Carlo simulations are given in Section 4 to verify performance improvements of our proposed algorithm, followed by the conclusion.

*Notation*: Bold symbols denote matrices or vectors. $(\cdot)^T$, $(\cdot)^H$ and $(\cdot)^*$ denote transpose, conjugate transpose and conjugate, respectively. The sets of real numbers, nonnegative real numbers and complex numbers are $\mathbb{R}$, $\mathbb{R}_+$ and $\mathbb{C}$ respectively. The set of all complex $K \times 1$ vectors, $M \times N$ matrices are denoted by $\mathbb{C}^K$, $\mathbb{C}^{M\times N}$ respectively. A circularly symmetric complex Gaussian variable with mean $\mu$ and variance $\sigma^2$ is denoted by $z \sim \mathcal{CN}(\mu, \sigma^2)$. An $N \times N$ identity matrix is denoted by $\mathbf{I}_N$. A diagonal matrix with diagonal entries from vector $\mathbf{a}$ is denoted by $\mathrm{Diag}(\mathbf{a})$. $\mathbf{A} \geq \mathbf{B}$, $\mathbf{A} > \mathbf{B}$ denotes $\mathbf{A} - \mathbf{B}$ is semi-positive definite matrix and positive definite matrix. $\mathbf{e}$ denotes column vector with all entries of 1.

## II. System Model

We consider a MIMO system with total of $N_t$ transmit and $N_r$ receive antennas, where $N_r \geq N_t$. At each transmission epoch, $M < N_r$ are picked receive antennas for signal reception. This paper considers the case $M \geq N_t$ only. For spatial multiplexing, the number of receive antennas should be at least the number of transmit antennas. If $M < N_t$, the system will be rank-deficient. The fading coefficient $h_{ij}$ is the complex path gain form transmit antenna $j$ to receive antenna $i$. We assume that the fading elements of the channel matrix $\mathbf{H} = [h_{i,j}] \in \mathbb{C}^{N_r \times N_t}$ are identically independent distributed (i.i.d.) with $h_{i,j} \sim \mathbb{CN}(0,1)$. $\mathbf{H}$ is known to the receiver, but not to the transmitter. A block of $N \times N_t$ symbols represented by a $N_t \times N$ matrix $\mathbf{X} \triangleq (\mathbf{x_1}, \ldots, \mathbf{x_N})$, is transmitted through the channel. The entries $x_{ij}, i = 1, \ldots, N_t, j = 1, \ldots, N$ of $\mathbf{X}_m$ with normalization such that $\mathbb{E}\{|x_{ij}|^2\} = 1 \; \forall i, \, j$ are the transmitted signal from antenna $i$ at time $j$

$$\mathbf{Y} = \mathbf{H}\mathbf{X} + \mathbf{Z}. \tag{1}$$

The entries $y_{ij}, i = 1, \ldots, N_r, j = 1, \ldots, N$ of $\mathbf{Y} \in \mathbb{C}^{N_r \times N}$ are the signals received from antenna $i$ at time $j$. The Gaussian noise matrix $\mathbf{Z} \in \mathbb{C}^{N_r \times N}$ consists of $\mathcal{CN}(0, \frac{N_t}{\rho})$ variables so that $\mathbb{E}[\mathbf{Z}\mathbf{Z}^\dagger] = \frac{N N_t}{\rho}\mathbf{I}_{N_r}$ where $\rho$ is the SNR per receive antenna, regardless of the number of transmit antennas. This model includes MIMO spatial multiplexing as its specific case where $\mathbf{X}$ is a column vector of size $N_t$, i.e. $N = 1$.

Following the approach of Dua et. al. [11], we define diagonal matrix $\Delta$ of size $N_r \times N_r$ with diagonal entries

$$\Delta_i = \begin{cases} 1, & \text{if } i^{th} \text{ receive antenna selected} \\ 0, & \text{otherwise.} \end{cases} \tag{2}$$

The MIMO channel can then be re-written incorporating receive antenna selection as the following

$$\mathbf{Y} = \Delta\mathbf{H}\mathbf{X} + \mathbf{Z} \tag{3}$$

with the new effective channel $\Delta\mathbf{H}$.

The receiver performs maximum-likelihood detection over all possible codewords $\mathbf{X}$ to obtain

$$\tilde{\mathbf{X}} = \arg\min_{\mathbf{X} \in \mathcal{C}} \|\mathbf{Y} - \Delta\mathbf{H}\mathbf{X}\|_F^2, \tag{4}$$

where $\mathcal{C}$ is the codebook and $\|.\|_F$ denotes the Frobenius norm of the matrix, that is

$$\|\mathbf{R}\|_F^2 = \sum_{i,j} |r_{ij}|^2 = \text{Trace}(\mathbf{R}\mathbf{R}^H) = \text{Trace}(\mathbf{R}^H\mathbf{R}).$$

With ML detection in (4), the pair-wise error probability conditioned on the channel matrix $\mathbf{H}$ is given by

$$P_{\text{pep}}\left(\mathbf{X}_m \to \mathbf{X}_n | \, \mathbf{H}\right) = Q\left(\sqrt{\frac{\rho}{2}\|\Delta\mathbf{H}(\mathbf{X}_m - \mathbf{X}_n)\|_F^2}\right), \tag{5}$$

where $Q(.)$ denotes the Gaussian tail probability $Q(x) = \frac{1}{\sqrt{2\pi}}\int_x^\infty \exp^{-\frac{1}{2}t^2} \, \mathrm{d}t$. The PEP depends on the specific codeword pair $(\mathbf{X}_m, \mathbf{X}_n)$, the instantaneous channel realization $\mathbf{H}$ and the receive antennas selected.

## III. Receive antenna selection as a convex optimization

In this section, by using matrix manipulation, we rewrite (5) as a convex function with variables as entries of $\Delta$. Evaluating the Gaussian Q-function possibly may require high computational time, a drawback for online applications. Using an approximation to the Gaussian Q-function as a sum of exponentials [13], we reduce the computational complexity.

First, we denote $\Gamma_{m,n} = \mathbf{X}_m - \mathbf{X}_n$ as the difference between codewords. Using the fact that $\Delta^H\Delta = \Delta$ and $\text{Trace}(\mathbf{AB}) = \text{Trace}(\mathbf{BA})$, we have

$$\begin{aligned} \|\Delta\mathbf{H}(\mathbf{X}_m - \mathbf{X}_n)\|_F^2 &= \|\Delta\mathbf{H}\Gamma_{m,n}\|_F^2 \\ &= \text{Trace}\Big(\Gamma_{m,n}^H\mathbf{H}^H\Delta^H\Delta\mathbf{H}\Gamma_{m,n}\Big) \\ &= \text{Trace}\Big(\mathbf{H}\Gamma_{m,n}\Gamma_{m,n}^H\mathbf{H}^H\Delta\Big) \\ &= \tilde{\mathbf{h}}_{m,n}^H\mathbf{u} \end{aligned} \tag{6}$$

where $\tilde{\mathbf{h}}_{m,n}$ and $\mathbf{u}$ are the column vectors of diagonal elements of matrices $\mathbf{H}\Gamma_{m,n}\Gamma_{m,n}^H\mathbf{H}^H$, $\Delta$ respectively. Note that all the elements of $\tilde{\mathbf{h}}_{m,n}$ are nonnegative and $\text{Trace}(\Delta) = \sum_i u_i$ where $\mathbf{u} = [u_1, \ldots, u_{N_r}]^T = [\Delta_1, \ldots, \Delta_{N_r}]^T$.
The PEP can then be re-expressed with variable $\mathbf{u}$

$$P_{\text{pep}}\left(\mathbf{X}_m \to \mathbf{X}_n | \, \mathbf{H}\right) = Q\left(\sqrt{\frac{\rho}{2}\tilde{\mathbf{h}}_{m,n}^H\mathbf{u}}\right). \tag{7}$$

The Gaussian Q-function can be evaluated using a lookup table or a polynomial approximation [14] and often available in most mathematical software. However, it is still an issue for systems with large dimension. There are several ways to approximate the Q-function with high accuracy. The authors in [13] proposed a general formula to approximate the function $\text{erfc}(x)$ where $Q(x) = \frac{1}{2}\text{erfc}(\frac{x}{\sqrt{2}})$

$$\begin{aligned} \text{erfc}(x) &\leq \frac{2}{\pi}\sum_{i=1}^{N}\int_{\theta_{i-1}}^{\theta_i} \exp(-\frac{x^2}{\sin^2\theta_i})\mathrm{d}\theta \\ &= \sum_{i=1}^{N} a_i \exp(-b_i x^2) \end{aligned} \tag{8}$$

where

$$a_i = \frac{2(\theta_i - \theta_{i-1})}{\pi}, \qquad b_i = \frac{1}{\sin^2\theta_i} \tag{9}$$

By choosing $N = 1$, from (7) and (8), we obtain the well-known Chernoff upper bound

$$P_{\text{pep}}\left(\mathbf{X}_m \to \mathbf{X}_n | \, \mathbf{H}\right) \leq \underbrace{\frac{1}{2}\exp\left\{-\frac{\rho\tilde{\mathbf{h}}_{m,n}^H\mathbf{u}}{4}\right\}}_{\tilde{g}_{m,n}^1(\mathbf{u})}. \tag{10}$$

More accurate approximations than the Chernoff bound can be readily extended as a sum of more than two exponential terms. The following is the optimal approximation of Q-function with $N = 2$ in (8), termed 'Chiani bound' [13]

$$Q(x) \leq \frac{1}{12}e^{-\frac{x^2}{2}} + \frac{1}{4}e^{-\frac{2x^2}{3}}. \tag{11}$$

Therefore,

$$Q\left(\sqrt{\frac{\rho}{2}\tilde{\mathbf{h}}_{m,n}^H\mathbf{u}}\right) \leq \underbrace{\frac{1}{12}\exp\left\{-\frac{\rho\tilde{\mathbf{h}}_{m,n}^H\mathbf{u}}{4}\right\}+\frac{1}{4}\exp\left\{-\frac{\rho\tilde{\mathbf{h}}_{m,n}^H\mathbf{u}}{3}\right\}}_{\tilde{g}_{m,n}^2(\mathbf{u})}. \quad (12)$$

Clearly, sum of more exponentials can be used to approximate Q-function (8). However, in our optimization problem, there is a tradeoff between approximation accuracy and computational complexity.

We next assume that there are $L$ possible code matrices $\mathbf{X}$, e.g. for spatial multiplexing systems, $L = |\mathcal{S}|^{N_t}$ with $|\mathcal{S}|$ being the size of the input modulation constellation $\mathcal{S}$. The union bound on the error probability is given by

$$P_e \leq P_{UB} = \frac{2}{L}\sum_{m=1}^{L-1}\sum_{n=m+1}^{L} P_{\text{pep}}\left(\mathbf{X}_m \to \mathbf{X}_n | \, \mathbf{H}\right). \quad (13)$$

Using different expressions for the Q-function, we have the exact form of union bound as

$$P_{UB} = \frac{2}{L}\sum_{m=1}^{L-1}\sum_{n=m+1}^{L} Q\left(\sqrt{\frac{\rho}{2}\tilde{\mathbf{h}}_{m,n}^H\mathbf{u}}\right), \quad (14)$$

or the upper bounds

$$P_{UB} \leq \frac{2}{L}\sum_{m=1}^{L-1}\sum_{n=m+1}^{L} \tilde{g}_{m,n}^1(\mathbf{u}) = g_1(\mathbf{u}), \quad (15)$$

and

$$P_{UB} \leq \frac{2}{L}\sum_{m=1}^{L-1}\sum_{n=m+1}^{L} \tilde{g}_{m,n}^2(\mathbf{u}) = g_2(\mathbf{u}). \quad (16)$$

Given an instantaneous channel realization $\mathbf{H}$, the antenna selection problem is to pick the $M < N_r$ receive antennas such that they will minimize the union bound in (14), (15), or (16). It is equivalent to find $\mathbf{u}$ such that

$$\mathbf{u} = \arg \min_{\substack{\mathbf{u}_i \in \{0,1\} \\ \sum_i u_i = M}} P_{UB}. \quad (17)$$

The binary variable vector $\mathbf{u} \in \{0,1\}^{N_r}$ makes the selection problem a NP-hard combinatorial optimization i.e. an exhaustive search to evaluate all the $\binom{N_r}{M}$ antenna subsets may be needed to pick the optimal solution $\mathbf{u}$. We will relax this binary constraint by allowing $u_i \in [0,1], (i = 1, \ldots, N_r)$. Thus, the problem of receive subset selection for minimizing the union bound is approximated by the following optimization problem:

$$\begin{aligned} \min \quad & g_1(\mathbf{u}) & (18)\\ \text{subject to} \quad & 0 \leq u_i \leq 1, i = 1, ..., N_r, \\ & \sum_{i=1}^{N_r} u_i = M. \end{aligned}$$

Similarly, we can minimize $g_2(\mathbf{u})$ as well.

Typically, the solution $\mathbf{u}$ of (18) is a set of fractional values. The receive antennas with indices corresponding to the $M$ largest $u_i$ are selected.

*Proposition 1:* The above optimization problem is convex in $\mathbf{u} \in \mathbb{R}_+^{N_r}$.

**Proof**: Here, we show that $e^{\frac{-\beta u}{2}}$ where $\beta, u > 0$ is convex

$$\frac{\partial \, e^{\frac{-\beta u}{2}}}{\partial u} = -\frac{\beta}{2}e^{\frac{-\beta u}{2}} < 0, \qquad 0 < u < \infty \quad (19)$$

$$\frac{\partial^2 \, e^{\frac{-\beta u}{2}}}{\partial u^2} = \left(\frac{\beta}{2}\right)^2 e^{\frac{-\beta u}{2}} > 0, \qquad 0 < u < \infty. \quad (20)$$

The convexity is preserved under an affine transformation [12] and note that $\tilde{\mathbf{h}}_{m,n}$ has all its elements being real nonnegative. Thus, $\exp\left\{-\frac{\rho\tilde{\mathbf{h}}_{m,n}^H\mathbf{u}}{4}\right\}$ is convex w.r.t. variable $\mathbf{u} \in \mathbb{R}_+^{N_r}$. Finally, noting that the sum of convex functions is convex, we conclude that $g_1(\mathbf{u})$ is itself convex. The two constraints are clearly convex.

It is well-known that a convex optimization problem can be solve either in closed form or by using interior point methods which require polynomial complexity. We employ a log barrier method [12] to solve (18). More details on the gradient derivation of the objective function and the algorithm can be found in [15].

## IV. Simulation results

This section studies the VER of systems which implements antenna selection for different antenna configurations (varying $N_t$, $N_r$ and $M$) through Monte Carlo simulations. For simulation, each Rayleigh fading channel realization is constant for 20 frames to produce more accuarate results. The ML detection is employed in all cases. For comparison, we also plot the performance curves of the eigenvalue-based [6] and the optimal exact union bound minimization antenna selection. This involves searching over $\binom{N_r}{M}$ possible submatrices of size $M \times N_t$. We plot the VER vs SNR at receiver of different selection schemes. In Fig. 1, the MIMO system has $N_t = 2$, $N_r = 3$, and $M = 2$. We next test our proposed algorithm when implementing MIMO system with larger dimension, $N_t = 3$, $N_r = 5$, and $M = 3$. There are discrepancies between the optimal union bound selection and Chernoff bound (or Chiani approximation) convex-based selection, especially in the high SNR region. Fig. 3 displays the gaps between the exhaustive search (17) and our convex relaxation method. The performance loss may arise from the rounding operation applied on the optimal solution of our optimization problem to pick corresponding receive antennas. However, our proposed algorithm outperforms the optimal eigenvalue-based criteria based on exhaustive search. Fig. 1 and Fig. 2 show that the Chernoff-based and Chiani-based optimization perform almost the same. This is because the performance of our proposed algorithm (or equivalently the antennas selected for each channel realization) does not depend on the optimal value (or the optimal solution $\mathbf{x}$) of our optimization problem (18) but the order of elements $x_i, i = 1, \ldots, N_r$ of the optimal $\mathbf{x}$. For the simualtion examples, the complexity of the proposed method is comparable to that of the optimal search

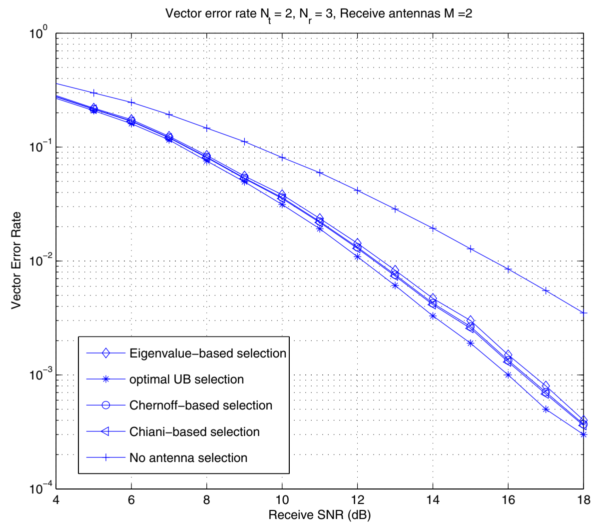


Fig. 1. Vector Error Rate with antenna selection, $N_t = 2$, $N_r = 3$, $M = 2$, 4bp/Hz/s.

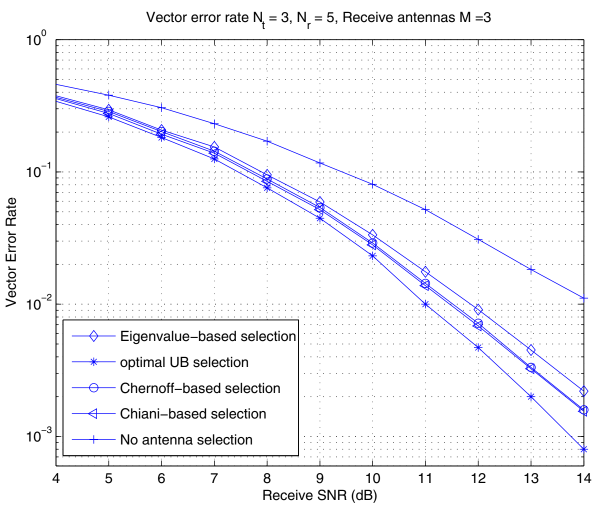


Fig. 2. Vector Error Rate with antenna selection, $N_t = 3$, $N_r = 5$, $M = 3$, 6bp/Hz/s.

due to a large number of codeword pairs in the union bound calculation. This has motivated us to investigate the bound over the codeword pairs with small distances only which is the subject of our current research.

## V. CONCLUSION

We have proposed a novel solution to the problem of receive antenna selection to minimize accurate approximations of the exact union bound. Since we are able to formulate the antenna selection as a convex programming problem, interior-point methods such as the log barrier method can be used efficiently with polynomial complexity. The proposed algorithm outperforms eigenvalue-based selection in terms of the vector error rate.

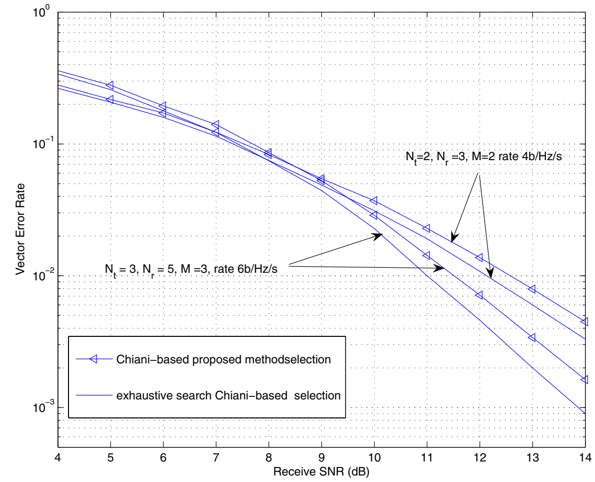


Fig. 3. Performance comparisions between exhaustive search and our convex relaxation method.

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