

Amount of Fading Analysis for Transmit Antenna Selection in MIMO Systems

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Abstract—The amount of fading (AF) is a simple measure for the performance of a diversity system. This paper provides approximations and bounds for AF as well as the methods to derive the exact AF calculations for transmit antenna selection (TAS) on Rayleigh fading channels. We also derive a simple approximate formula for the relationship between the AF and the coding gain in a TAS system. Simulation results are provided to verify the results.

I. INTRODUCTION

In evaluating the performance of diversity combiners over fading channels, commonly used performance measures include the average symbol error rate (SER), the average bit error rate (BER), the diversity order, and the coding gain. Closed-form expressions for these measures may not always be available since statistical analysis requires averaging the instantaneous results over the fading distribution. The amount of fading (AF), a unified measure for the severity of fading, directly utilizes the moments of the fading distribution itself. Thus it is a simple but effective way to quantify fading in both single-input single-output (SISO) and multiple-input multiple-output (MIMO) systems.

For a SISO channel model, AF is defined by [1, eq. (2)],

$$AF = \frac{\text{Var}\{\alpha^2\}}{(\mathcal{E}\{\alpha^2\})^2}, \quad (1)$$

where α is the instantaneous fading amplitude of a complex fading channel, $\mathcal{E}\{\cdot\}$ and $\text{Var}\{\cdot\}$ are the statistical average and variance respectively. For a SISO Rayleigh fading channel, $AF = 1$. And in a generalized Nakagami- m fading channel, $AF = 1/m$ [2].

However, an important practical problem associated with multiple antennas is that the hardware cost and complexity will increase with every additional antenna under use due to the requirement of dedicated radio frequency chains. One promising solution is to apply transmit antenna selection (TAS) where actual transmission is performed through the selected subset [3] [4]. If the transmitter always utilizes a set of optimally selected antennas, i.e., the antennas that yield the largest received signal power, it is guaranteed to get the same diversity order as the system using all antennas. Importantly, the system complexity reduces as a result of antenna selection.

The AF has been used to quantify the severity of fading experienced at the output of a MIMO system. In [5], the

AF is obtained for the output of equal gain combiner (EGC) in equally correlated fading channels. In [6], closed-form expressions for the AF are given on identically-distributed spatially-correlated Nakagami- m fading channels. But to the best of our knowledge, no paper has thus far provided an AF analysis for MIMO systems with antenna selection.

In this paper, we analyze the AF of MIMO systems with TAS in Rayleigh fading channels. Rigorous derivation of the AF generally leads to cumbersome results. Therefore, besides providing methods of getting exact values for AF, we present the lower bounds, upper bounds and approximate values of AF. It is of interest to show AF is related to other performance measures. It turns out that the AF and the coding gain, which is related to the symbol error rate (SER), have a simple relationship at high SNR.

This paper is organized as follows. Section II introduces the system and channel model. In Section III, the exact derivations, bounds and approximations for AF are analyzed for MIMO systems with TAS on independent and identically distributed (i.i.d.) and receiver correlation (RC) channels. Section IV derives the relationship between AF and SER. Simulation results are provided to verify the analysis of AF in Section V. The main conclusions are summarized in Section VI.

II. SYSTEM AND CHANNEL MODEL

Consider a MIMO system with n_T transmit and n_R receive antennas. We assume that the correlation properties at the transmitter are independent of those at the receiver. Consequently, the overall channel correlation can be expressed by the Kronecker product of the transmitter correlation and the receiver correlation matrices [7]. The $n_R \times n_T$ channel matrix $\mathbf{H} = [h_{ij}]$ where $h_{ij} \sim \mathcal{CN}(0, 1)$ can thus be written as

$$\mathbf{H} = \mathbf{R}_r^{1/2} \mathbf{H}_w \mathbf{R}_t^{1/2} \quad (2)$$

where \mathbf{R}_r and \mathbf{R}_t are the non-negative definite matrices of receive and transmit correlation, respectively. \mathbf{H}_w is the spatially white matrix for all the $n_R \times n_T$ i.i.d. Rayleigh fading channels.

¹Notation: Bold symbols denote matrices and vectors. $(\cdot)^T$ denotes transpose. $\|\mathbf{A}\|_F^2$ is the Frobenius norm of \mathbf{A} . A circularly complex Gaussian variable with mean μ and variance σ^2 is denoted by $z \sim \mathcal{CN}(\mu, \sigma^2)$. And $j^2 = -1$.

To perform TAS, L antennas are selected out of n_T transmit antennas. Define the column vectors $\mathbf{h}_j = (h_{1j}, h_{2j}, \dots, h_{n_R j})^T, 1 \leq j \leq n_T$. The square norm of \mathbf{h}_j is then denoted as $g_j = \|\mathbf{h}_j\|_F^2 = \sum_{i=1}^{n_R} |h_{ij}|^2$. The gains g_j are sorted in descend order and denoted by $g_{(1)} \geq g_{(2)} \geq \dots \geq g_{(n_T)}$, where $g_{(j)}$ is the j th largest. The L selected transmit antennas correspond to the 1 to L th largest $g_{(1)}, \dots, g_{(L)}$. Let $\mathbf{H}_t = (\mathbf{h}_{(1)}, \mathbf{h}_{(2)}, \dots, \mathbf{h}_{(L)})$ represent the selected L columns of \mathbf{H} . The received signals can be expressed by

$$\mathbf{Y} = \sqrt{\frac{E_s}{L}} \mathbf{H}_t \mathbf{X} + \mathbf{N}, \quad (3)$$

where E_s is the energy of the transmitted symbol, \mathbf{Y} is the $L \times T$ received signal matrix and \mathbf{X} represents the $n_R \times T$ transmitted signal matrix. T is the symbol period. The elements of \mathbf{N} are i.i.d. additive white Gaussian noise terms.

Assume that orthogonal space-time block codes (OSTBC) are used for the transmission over the L transmit antennas, the output SNR may be expressed as

$$\eta = \frac{E_s}{N_0 L R_s} \|\mathbf{H}_t\|_F^2 = \frac{E_s}{N_0 L R_s} C, \quad (4)$$

where $C = g_{(1)} + \dots + g_{(L)}$, N_0 and R_s are the one-sided power spectral density of white Gaussian noise and the symbol rate respectively.

According to the definition of AF in (1) and (4), the AF is independent of $\frac{E_s}{N_0 L R_s}$ and can be written as

$$\text{AF} = \frac{\text{Var}\{\eta\}}{(\mathcal{E}\{\eta\})^2} = \frac{\text{Var}\{C\}}{(\mathcal{E}\{C\})^2}. \quad (5)$$

In the remainder of this paper, $(n_T, L; n_R)$ denotes a TAS system with L ($L \leq n_T$) transmit antennas selected. By contrast, $(n_T; n_R)$ denotes a system where all the n_T transmit and n_R receive antennas are used. Systems with TAS use maximal ratio combining (MRC) at the receiver side.

III. AF FOR TAS SYSTEM ON RAYLEIGH FADING CHANNELS

A. Exact AF Derivation

The exact AF in TAS system AF_{TAS} can be expressed as

$$\text{AF}_{\text{TAS}} = \frac{\text{Var}\{g_{(1)} + \dots + g_{(L)}\}}{(\mathcal{E}\{g_{(1)} + \dots + g_{(L)}\})^2}. \quad (6)$$

If h_{ij} are independent with each other, g_j are i.i.d. chi-squared random variables with $2n_R$ degrees of freedom. Their probability density function (pdf) and cumulative distribution function (cdf) are available [8]. Using the theory of order statistics [9], the joint pdf of $g_{(1)}, g_{(2)}, \dots, g_{(L)}$ is

$$\begin{aligned} & f_{g_{(1)}, \dots, g_{(L)}}(g_1, \dots, g_L) \\ &= \frac{n_T!}{(n_T - L)! \{(n_R - 1)!\}^L} \left(\prod_{j=1}^L g_j \right)^{n_R - 1} (e^{-\sum_{j=1}^L g_j}) \\ & \times (1 - e^{-g_L} \sum_{k=0}^{n_R - 1} \frac{g_L^k}{k!})^{n_T - L}, \end{aligned} \quad (7)$$

where $g_1 \geq g_2 \geq \dots \geq g_L \geq 0$. The general moments of $\mathcal{E}\{g_1^{a_1} \dots g_L^{a_L}\}$ (a_1, \dots, a_L are arbitrary numbers) can then be calculated by finite sums, details can be seen in [10]. The exact AF expression can be derived using (6).

When the channel is correlated at the receiver side, the elements within the columns of \mathbf{H} are correlated but uncorrelated between columns. Thus $g_j, 1 \leq j \leq n_T$ are i.i.d.. The pdf of $g_{\{j\}}, 0 \leq j \leq L$ can be derived using inverse Z transform. Follow the same procedure as above, the exact AF can be achieved by (6).

However, for gaining insight into system performance, in both cases the exact formulas for AF are too cumbersome for use. Therefore, we aim to simplify its calculation.

B. AF Bounds and Approximations for i.i.d. Channels

In a $(n_T; n_R)$ system, the AF is written as [6] (Here is the case when $m = 1$)

$$\text{AF} = \frac{\sum_{k=1}^{n_R n_T} \lambda_k^2}{(\sum_{k=1}^{n_R n_T} \lambda_k)^2}, \quad (8)$$

where $\{\lambda_k\}_{k=1}^{n_R n_T}$ are the eigenvalues of the $n_R n_T \times n_R n_T$ channel correlation matrix $\mathbf{R} = \mathcal{E}\{\text{vec}(\mathbf{H}) \text{vec}(\mathbf{H})^H\}$. With independent fading, the correlation matrix \mathbf{R} is an identity matrix and all the λ_k 's are equal to unity. Therefore, AF_{iid} reduces to

$$\text{AF}_{\text{iid}} = \frac{n_R n_T}{(n_R n_T)^2} = \frac{1}{n_R n_T}. \quad (9)$$

The AF is generally known as a measure of the severity of fading. It could also measure the randomness of a random variable, namely, the higher the AF, the larger the spread of the fading distribution [11] [12]. Therefore, the more i.i.d. $|g_{ij}|^2$ included in C , the smaller the AF will be. This is due to the multiplication of the denominator while the numerator remains largely unchanged in (6). As a result, the AF should decrease with the increase of the number of receive antennas n_R and the selected transmit antennas L .

Correspondingly, for a $(n_T, L; n_R)$ system with i.i.d Rayleigh fading channels, we derive an upper bound and a lower bound for the AF as (denoted by $\text{AF}_{\text{iid \& TAS}}$)

$$\frac{1}{n_R n_T} \leq \text{AF}_{\text{iid \& TAS}} \leq \frac{1}{n_R L}. \quad (10)$$

The approximate value for $\text{AF}_{\text{iid \& TAS}}$ is given by

$$\text{AF}_{\text{iid \& TAS}} \approx \frac{1}{n_R (L + (n_T - L)\alpha)}, \quad (11)$$

where $0 \leq \alpha \leq 1$. When $\alpha = 0$, $\text{AF}_{\text{iid \& TAS}}$ reaches the lower bound and when $\alpha = 1$ $\text{AF}_{\text{iid \& TAS}}$ reaches its upper bound.

C. AF Bounds and Approximation for Receiver Correlated Channels

Using the Kronecker model, the eigenvalues of the correlation matrix can be decomposed as $\lambda_k = \lambda_j \lambda_i$, where λ_j and λ_i are the eigenvalues for transmitter and receiver correlation

matrices, $k = 1, \dots, n_R n_T$; $i = 1, \dots, n_R$; $j = 1, \dots, n_T$. All λ_j equal 1. The $n_R n_T$ eigenvalues λ_k constitute of n_T λ_i , i.e., the multiplicity of each λ_k is n_T . From (8), the AF for $(n_T; n_R)$ system with RC AF_{RC} can be expressed as

$$\text{AF}_{\text{RC}} = \frac{1}{n_T} \frac{\sum_{i=1}^{n_R} \lambda_i^2}{\left(\sum_{i=1}^{n_R} \lambda_i\right)^2}. \quad (12)$$

The worst AF occurs when one of λ_i equals n_R and the other λ_i equal 0, i.e., $\text{rank}(\mathbf{R}_r) = 1$. The upper bound gets to $1/n_T$. Using the Lagrange multiplier method, we can prove the lower bound of AF with correlation as $\frac{1}{n_R n_T}$. Furthermore, if the rank of the receiver correlation is given as $\text{rank}(\mathbf{R}_r) = r_r$, the lower bound can be tightened to

$$\text{AF}_{\text{RC}} \geq \frac{1}{n_T r_r}. \quad (13)$$

With TAS, the range for L is $[1, n_T]$. Thus, the upper bound for AF in a $(n_T, L; n_R)$ system with RC is found to be $\frac{1}{L + (n_T - L)\beta}$, where $0 \leq \beta \leq 1$. When $\beta = 1$, the inequality reduces to the bound for $(n_T; n_R)$ system with RC. When $\beta = 0$, the upper bound corresponds to the worst case as $\frac{1}{L}$. Now the AF in a TAS system with RC (denoted as AF_{RC&TAS}) can be bounded by

$$\frac{1}{n_R n_T} \leq \text{AF}_{\text{RC\&TAS}} \leq \frac{1}{L + (n_T - L)\beta}, \quad 0 \leq \beta \leq 1. \quad (14)$$

For the constant RC model, the AF can be approximated by substituting [6, eq. (14)] into the right side of (14)

$$\text{AF}_{\text{RC\&TAS}} \approx \frac{1}{L + (n_T - L)\alpha} \frac{1 + |\rho|^2(n_R - 1)}{n_R}. \quad (15)$$

D. AF Bounds for General Correlated Channels

In the most general case, the channel is correlated both at the transmitter and the receiver. The AF can be calculated from (5). The AF may increase more than the RC case since more correlation may be involved. If we denote AF in $(n_T; n_R)$ and in $(n_T, L; n_R)$ with general correlation (GC) as AF_{GC} and AF_{GC&TAS}, respectively, the bounds are given by

$$\frac{1}{n_R n_T} \leq \text{AF}_{\text{GC}} \leq \text{AF}_{\text{GC\&TAS}} \leq 1. \quad (16)$$

If $\text{rank}(\mathbf{R}) = r$ is given, the lower bound can be tightened,

$$\text{AF}_{\text{GC\&TAS}} \geq \frac{1}{r}. \quad (17)$$

IV. RELATIONSHIP BETWEEN AF AND SER

Being parameters of quantifying channel performance, AF and SER have a certain relationship. This section shows that for a $(n_T, L; n_R)$ system operating on Rayleigh fading channels, when OSTBC are used for the transmission over the $L \times n_R$ link, the average SER at high SNR relates to the AF by a simple expression.

In [13], the average SER P_e of an uncoded (or coded) MIMO system at high SNR is approximated by the expression

$$P_e \approx c(G_c \cdot \bar{\gamma})^{-G_d}, \quad (18)$$

where c is a scaling constant depending on the modulation employed and the nature of the channel, G_c represents the coding gain and G_d represents the diversity order.

Assuming Maximum Likelihood (ML) detection is used at the receiver, the corresponding instantaneous SER is given by [8]

$$P_e \approx \bar{N}_e Q\left(\sqrt{\eta \frac{d_{\min}^2}{2}}\right), \quad (19)$$

where \bar{N}_e and d_{\min} are the number of the nearest neighbors and the minimum distance of the underlying scaler constellation separation respectively, η is given by (4) assuming $\mathbf{H}_t = \mathbf{H}$. Applying the Chernoff bound $Q(x) \leq e^{-\frac{x^2}{2}}$, P_e can be expressed as

$$P_e \leq \bar{N}_e e^{-\frac{1}{4n_T R_s} \frac{E_s}{N_0} \|\mathbf{H}\|_F^2}. \quad (20)$$

Given the correlation matrix \mathbf{R} , the moment generating function (MGF) of the random variable $\|\mathbf{H}\|_F^2$, denoted as $\psi_{\|\mathbf{H}\|_F^2}(s)$, is given by [14, eq. (3.44)]

$$\begin{aligned} \psi_{\|\mathbf{H}\|_F^2}(s) &= \mathcal{E}\{e^{-s\|\mathbf{H}\|_F^2}\} \\ &= \prod_{k=1}^r (1 + s\lambda_k)^{-1}, \end{aligned} \quad (21)$$

Averaging the instantaneous SER over the statistics of fading, the \bar{P}_e can be upper bounded by setting $s = \frac{d_{\min}^2}{4n_T R_s} \frac{E_s}{N_0}$ in (21). That is,

$$\bar{P}_e \leq \bar{N}_e \prod_{k=1}^r \left(1 + \frac{d_{\min}^2}{4n_T R_s} \frac{E_s}{N_0} \lambda_k\right)^{-1}. \quad (22)$$

When channels are i.i.d., all λ_k equal 1. Thus, at the high SNR, the $\bar{P}_{e\text{iid}}$ can be simplified to

$$\bar{P}_{e\text{iid}} \leq \bar{N}_e \left(\frac{d_{\min}^2}{4n_T R_s} \frac{E_s}{N_0}\right)^{-n_R n_T}. \quad (23)$$

It is clear that the diversity order $G_d = n_R n_T$ and the coding gain $G_c = \frac{d_{\min}^2}{4n_T R_s}$. Compared with (9), in the i.i.d. case, the AF equals the inverse of the diversity order

$$\text{AF}_{\text{iid}} = \frac{1}{G_d}. \quad (24)$$

In a $(n_T, L; n_R)$ system, since the largest L of g_j 's are selected, the following inequality holds

$$\frac{\|\mathbf{H}\|_F^2}{n_T} \leq \frac{\|\mathbf{H}_t\|_F^2}{L} \leq \frac{\|\mathbf{H}\|_F^2}{L}. \quad (25)$$

Combining the definition of η in (4), we have

$$\frac{\|\mathbf{H}\|_F^2}{n_T R_s} \frac{E_s}{N_0} \leq \eta \leq \frac{\|\mathbf{H}\|_F^2}{L R_s} \frac{E_s}{N_0}. \quad (26)$$

With the average SER in (23), we find

$$\bar{N}_e \left(\frac{d_{\min}^2 E_s}{4LR_s N_0} \right)^{-n_T n_R} \leq \bar{P}_{eTAS} \leq \bar{N}_e \left(\frac{d_{\min}^2 E_s}{4n_T R_s N_0} \right)^{-n_T n_R}. \quad (27)$$

As $1 \leq L \leq n_T$, \bar{P}_{eTAS} can be approximated as

$$\bar{P}_{eTAS} \approx \bar{N}_e \left(\frac{d_{\min}^2 E_s}{4R_s(L + (n_T - L)\alpha) N_0} \right)^{-n_T n_R}, \quad (28)$$

where $0 \leq \alpha \leq 1$. That is, the coding gain $G_c \approx \frac{1}{L + (n_T - L)\alpha} \frac{d_{\min}^2}{4R_s}$. Compared to the $AF_{iid \& TAS}$ for $(n_T, L; n_R)$ system with i.i.d. fading channels in (11), the coding gain can thus be approximated as a function of AF,

$$G_c \approx \frac{d_{\min}^2}{4R_s n_R} AF_{iid \& TAS}. \quad (29)$$

With the increasing of n_T and n_R , the upper bound and the lower bound will be closer and the approximate G_c is more accurate. When $L = n_T$, the upper bound and lower bound converge to the same value.

When there is correlation between receiver antennas, at high SNR the \bar{P}_{eRC} can be bounded using (23)

$$\bar{P}_{eRC} \leq \bar{N}_e \left(\frac{d_{\min}^2 E_s}{4n_T R_s N_0} \right)^{-r} \prod_{k=1}^r (\lambda_k)^{-1}. \quad (30)$$

The best situation reduces to no correlation at the receiver side. The worst situation will still be only one of the λ_i is n_R . Under the Kronecker model, we derive the lower bound and upper bound of \bar{P}_{eRC} as

$$\begin{aligned} \bar{N}_e \left(\frac{d_{\min}^2 E_s}{4n_T R_s N_0} \right)^{-n_T n_R} &\leq \bar{P}_{eRC} \\ &\leq \bar{N}_e \left(\frac{d_{\min}^2 E_s}{4n_T R_s N_0} \right)^{-n_T} n_R^{-n_T}. \end{aligned} \quad (31)$$

And corresponding to the analysis in Section III, the average SER for a $(n_T, L; n_R)$ system with RC can be bounded as

$$\begin{aligned} \bar{N}_e \left(\frac{d_{\min}^2 E_s}{4n_T R_s N_0} \right)^{-n_T n_R} &\leq \bar{P}_{eRC \& TAS} \\ &\leq \bar{N}_e \left(\frac{d_{\min}^2 E_s}{4LR_s N_0} \right)^{-n_T} n_R^{-n_T}. \end{aligned} \quad (32)$$

V. NUMERICAL RESULTS

This section illustrates our results in specific systems. To demonstrate results in TAS systems with i.i.d fading channels, Fig. 1 shows the upper bound, lower bound and the approximation value for the AF using (10) and (11) where setting $\alpha = 0.6$ in a (8;4) system. We determine α by setting the value that gives the best approximate AF with respect to simulation results. When increasing L the AF approaches the lower bound as we expect. Observe that the AF is closer to the lower bound compared to the upper bound, which shows that TAS doesn't deteriorate much of the system behavior in term of the AF. When $L \geq \frac{1}{3}n_T$, the lower bound can be viewed as a good approximation of the AF.

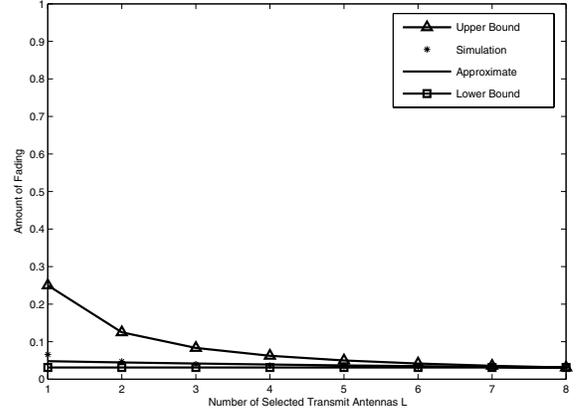


Fig. 1. $AF_{iid \& TAS}$ with different L in system (8;4) setting $\alpha = 0.6$

Assuming constant RC, in Fig. 2 we show results of $AF_{RC \& TAS}$ using (15) for different ρ s when $n_T = 4, n_R = 3$. When the correlation is large as $\rho = 1$, the approximation of AF is well bounded by the upper bound. Being a measure of the severity of fading, the AF increases with ρ when L is fixed.

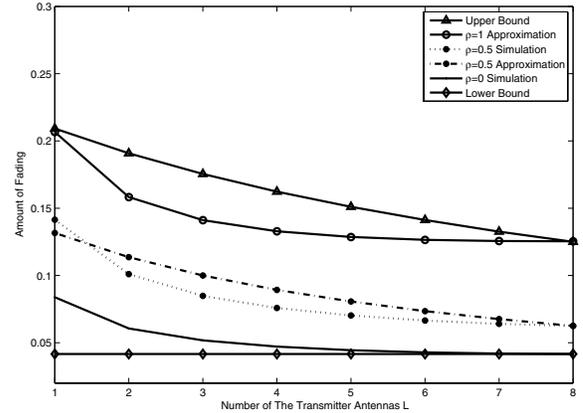


Fig. 2. $AF_{RC \& TAS}$ with different L and different ρ in system (4,L;3) with constant receiver correlation

Fig. 3 shows how AF changes with different L and different correlation coefficient ρ when both transmitter and receiver are modeled as constant correlation. When ρ is less than 0.5, the AF decreases with L . On the contrary, when ρ is larger than 0.5, an increase in L leads to the increase in AF. This implies that when the correlation is strong, the more transmit antennas are selected, the worse system will perform in terms of the AF.

We demonstrate the simple relations gained by (27) and (28) in Fig. 4 compared to simulation result in system (3,2;1) and $\alpha = 0.5$. Simulation employs the Alamouti scheme with 4-QAM signal constellation. Fig. 5 shows the average SER given by (28) in system (4,1;1) and (4,3;1) respectively. In both

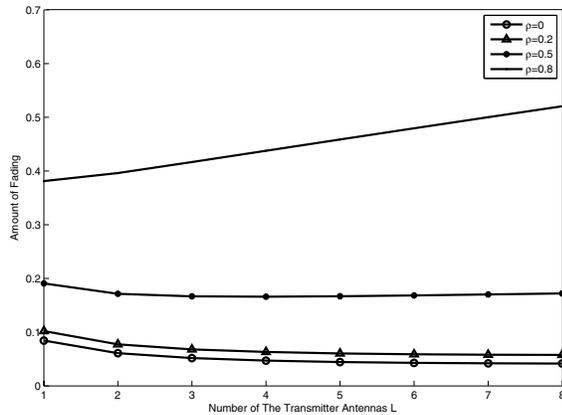


Fig. 3. AF_{GC&TAS} with different L and ρ in system $(4,L;3)$ with constant transmit and receiver correlation

figures we see that in the large SNR region, the approximations overlap with simulation results. Thus, the simple relationship between AF and coding gain (29) holds for a TAS system in the high SNR region.

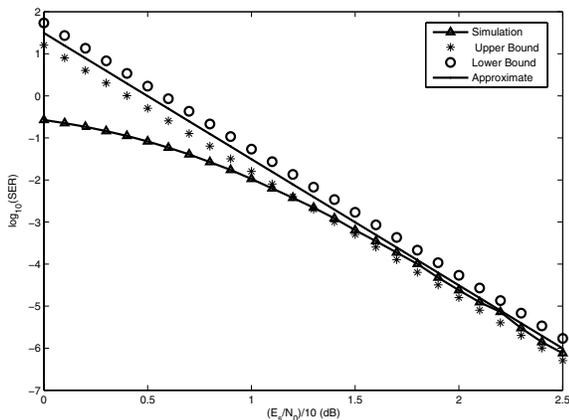


Fig. 4. The average SER of 4-QAM using Alamouti Scheme in system $(3,2;1)$

VI. CONCLUSIONS

AF is a simple measure to quantify severity of fading. However exact AF calculations usually involve tedious formulas in TAS systems. In this paper, we analyze the AF of TAS systems under both independent and correlated channels. We provide approximations and bounds for different cases and also find simplified relation between AF and coding gain. Simulation results show that in the high SNR region, the approximations are accurate.

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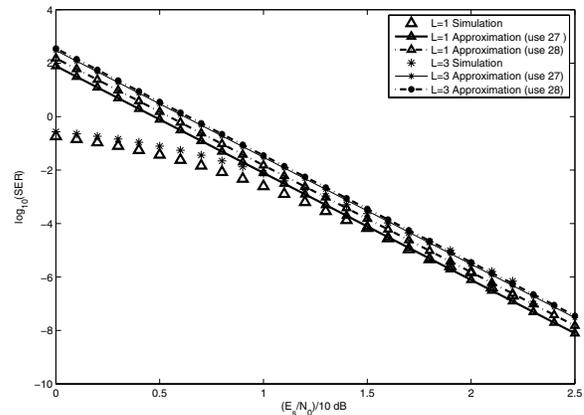


Fig. 5. The average SER of 4-QAM in system $(4,1;1)$ and $(4,3;1)$

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