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Signal-to-interference-plus-noise ratio Analysis for MIMO-OFDM with Carrier Frequency Offset and Channel Estimation Errors

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Abstract— **In this paper, we derive the signal-to-interference-plus-noise ratio (SINR) for multiple-input multiple-output (MIMO) orthogonal frequency-division multiplexing (OFDM) systems in the presence of frequency offset and channel estimation errors. The channel is assumed to be frequency-selective Rayleigh fading. Our analysis of the demodulated signal shows that the interference can be decomposed into two independent components: Inter-Carrier Interference and interference contributed by other transmit antennas. The SINR for MIMO-OFDM systems with equal gain combining (EGC) and maximal ratio combining (MRC) are also derived.**

I. Introduction

Orthogonal frequency-division multiplexing (OFDM) has been widely used for wireless applications such as standards of high data rate wireless local area networks, mainly due to its remarkable resistance to frequency-selective fading, achieved by dividing the available bandwidth into many narrow parallel overlapping subchannels [1]. The multiple-input multiple-output (MIMO) technique [2] can be used together with OFDM to operate at a high-throughput mode, the diversity mode or the combination of both [3]. Such systems achieve high spectral efficiencies and/or a large coverage area that are critical for future-generation wireless networks.

However, MIMO-OFDM faces several technical challenges. One is that, as in single-input single-output (SISO)-OFDM, MIMO-OFDM systems are highly sensitive to the frequency offset, which introduces intercarrier interference and thereby significantly degrades the system performance [4]. The other major problem for MIMO-OFDM is that channel estimation becomes increasingly difficult with the increase in the number of antennas. Several estimators of channel, frequency offset or both for SISO-OFDM have already been developed [5]–[7]. Optimal training signal design for MIMO-OFDM channel estimation has been considered in [9] and preamble training for MIMO-OFDM has been proposed in [10]. Recursive estimation of channel and frequency offset in MIMO-OFDM systems is discussed in [11].

Previously, the signal-to-interference-plus-noise ratio (SINR) for SISO-OFDM has been evaluated as a measure of the performance degradation due to frequency offset [4], [5], [12], [13]. In [13], the SINR on a single subcarrier is evaluated accurately in Rayleigh or Rician fading channels. However, [13] assumes that the channel is perfectly known at the receiver side and that the frequency offset is negligible. Although some estimators are highly accurate, we know that their performance is ultimately limited by the Cramér-Rao Lower Bound (CRLB) [14], i.e., the residual frequency offset and channel estimation errors after frequency offset correction and channel equalization degrade the system performance.

In this paper, we extend the SINR derived in [13] to MIMO-OFDM systems, taking into consideration the residual frequency offset and channel estimation errors. We assume that the frequency offsets between different transmit and receive antenna pairs are different, a model that is suitable for a slow or high mobility of mobile user. We also derived the SINR for equal gain combining (EGC) and maximal ratio combining (MRC), two well known diversity reception techniques.

This paper is organized as follows. The MIMO-OFDM system model is given in Section II. In section III, the SINR degradation in MIMO-OFDM systems due to frequency offset and channel estimation errors is analyzed for three different reception cases: no diversity combining, EGC and MRC. Numerical results are given in Section IV, followed by conclusions in Section V.

II. System and Signal Model

We introduce some notation. Bold symbols denote matrices or vectors. $(\cdot)^H$ is Hermitian (complex conjugate transpose). diag$\{\mathbf{x}\}$ stands for the diagonal matrix with the entries of column vector $\mathbf{x}$ on its diagonal. The imaginary unit is $\jmath = \sqrt{-1}$. A circularly symmetric complex Gaussian random variable (RV) w with mean μ and variance σ^2 is denoted by $w \sim \mathcal{CN}(\mu, \sigma^2)$. $\mathbf{a}[i]$ is the ith entry of vector $\mathbf{a}$ and $[\mathbf{B}]_{k,i}$ is the (k,i)th entry of matrix $\mathbf{B}$.

Let us consider a MIMO-OFDM system with N_t transmit antennas and N_r receive antennas and N subcarriers are used at each transmit antenna. For the ith transmit antenna $(1 \leq i \leq N_t)$, the frequency domain complex data symbols $\mathbf{x}_i$ is a $N \times 1$ vector, the elements of which are drawn from an M-ary square Quadrature Amplitude Modulation (QAM) or Phase-Shift Keying (PSK) constellation. The OFDM symbol is generated by taking Inverse Discrete Fourier Transform (IDFT) of $\mathbf{x}_i$. A Cyclic Prefix (CP) is inserted for each OFDM symbol, whose duration should be longer than the channel impulse duration to avoid the inter symbol interference.

For the multipath MIMO-OFDM channel between the ith transmit antenna and the kth receive antenna, $h_{k,i}(1), h_{k,i}(2), \cdots, h_{k,i}(L)$ represent the identical independent distributed (i.i.d.) time-domain taps and L stands for the maximum delay. We assume that $\sum_{d=1}^{L} |h_{k,i}(d)|^2 = 1$ is satisfied for each k, i. The frequency-domain channel response on the n subcarrier can be expressed by [15]

$$H_{k,i}^{(n)} = \sum_{d=0}^{L_{k,i}-1} h_{k,i}(d) e^{-\jmath \frac{2\pi n d}{N}}. \quad (1)$$

$H_{k,i}^{(n)} \sim \mathcal{CN}(0,1)$, i.e., the channel model is Rayleigh fading. We also assume that $H_{k,i}^{(n)}$ are independent to $H_{p,q}^{(l)}$ if $k \neq p$ and/or $i \neq q$, i.e., if either transmit antenna or receive antenna is different, the channel frequency responses of subcarriers are independent. The covariance of channel frequency response between the subcarriers between the ith transmit antenna and the kth receive antenna can be give by

$$R_{H_{k,i}^{(n)} H_{k,i}^{(l)}} = \sum_{d=0}^{L_{max}-1} \mathbb{E}\left\{ |h_{k,i}(d)|^2 \right\} e^{-\frac{\jmath 2\pi d (l-n)}{N}} \quad l \neq n, \quad (2)$$

where L_{max} is the maximum tap delay of all $N_r \times N_t$ channels.

At the receiver side, frequency offset may arise from: 1) mismatch between transmit and receive oscillators; 2) Doppler shift due to user mobility; and 3) carrier frequency offset compensation errors. We define $\delta_{k,i}$ to be the frequency offset with respect to the ith transmit antenna and the kth receive antenna. The normalized frequency offset $\varepsilon_{k,i} = \frac{\delta_{k,i}}{\Delta f}$, where Δf is the subcarrier bandwidth. Without loss of generality, we assume that $\varepsilon_{k,i}$ are i.i.d. RVs with zero mean for each (k,i) and the assumption that the frequency offsets between different transmit and receive antennas are identical can be seemed as a special case. The received $N \times 1$ signal vector on the kth receive antenna in time domain after removing the CP can be expressed as

$$\mathbf{y}_k = \sqrt{\frac{E_s}{N_t}} \sum_{i=1}^{N_t} \mathbf{E}_{k,i} \mathbf{F} \mathbf{H}_{k,i} \mathbf{x}_i + \mathbf{w}_k, \quad (3)$$

where

$$\mathbf{E}_{k,i} = \mathrm{diag}\left\{ e^{\jmath 0}, e^{\jmath \frac{2\pi \varepsilon_{k,i}}{N}}, \cdots, e^{\jmath \frac{2\pi \varepsilon_{k,i}(N-1)}{N}} \right\}, \quad (4)$$

$\mathbf{F}$ is the $N \times N$ IDFT matrix, $\mathbf{H}_{k,i} = \mathrm{diag}\left\{ H_{k,i}^{(0)}, H_{k,i}^{(1)}, \cdots, H_{k,i}^{(N-1)} \right\}$, and $\mathbf{w}_k$ is a $N \times 1$ vector of additive complex white Gaussian noise (AWGN), where $\mathbf{w}_k[n] \sim \mathcal{CN}(0, \sigma_w^2)$. Note that the transmit power is equally allocated at all transmit antennas.

III. SINR Analysis

In this section, we assume the subcarrier frequency offset and channel estimation errors are zero-mean RVs. After defining $\Delta\varepsilon_{k,i}$ and $\Delta\mathbf{H}_{k,i}$ to be the frequency offset estimation error and channel estimation error for the (k,i) antenna pair, the estimated frequency offset and channel estimation can be expressed as $\hat{\varepsilon}_{k,i} = \varepsilon_{k,i} + \Delta\varepsilon_{k,i}$ and $\hat{\mathbf{H}}_{k,i} = \mathbf{H}_{k,i} + \Delta\mathbf{H}_{k,i}$, where $\varepsilon_{k,i}$ and $\mathbf{H}_{k,i}$ are the exact frequency offset and channel response. The receiver, firstly, use the estimated $\hat{\varepsilon}_{k,j}$, $\hat{\mathbf{H}}_{k,j}$ ($j \neq i$) to eliminate the signals transmitted from the transmitted antennas other than the ith transmit antenna and then compensate with $\hat{\varepsilon}_{k,i}$ and $\hat{\mathbf{H}}_{k,i}$. After performing frequency offset pre-compensation and the zero-forcing equalization, we obtain

$$\begin{aligned} \mathbf{r}_{k,i} &= \mathbf{F}^H \hat{\mathbf{E}}_{k,i}^H \left(\mathbf{y}_k - \sqrt{\frac{E_s}{N_t}} \sum_{j=1, j\neq i}^{N_t} \hat{\mathbf{E}}_{k,j} \mathbf{F} \hat{\mathbf{H}}_{k,j} \mathbf{x}_j \right) \\ &= \sqrt{\frac{E_s}{N_t}} \underbrace{\mathbf{F}^H \hat{\mathbf{E}}_{k,i}^H \mathbf{E}_{k,i} \mathbf{F} \mathbf{H}_{k,i} \mathbf{x}_i}_{\mathbf{s}_{k,i}} + \mathbf{\Phi}_{k,i} + \tilde{\mathbf{w}}_{k,i}. \end{aligned} \quad (5)$$

where $\hat{\mathbf{E}}_{k,j}$ is derived from $\mathbf{E}_{k,j}$ by replacing $\varepsilon_{k,j}$ with $\hat{\varepsilon}_{k,j}$. In (5), $\mathbf{\Phi}_{k,i}$ is the interference component contributed by signals transmitted by transmit antennas other than ith transmit antenna and can be given as

$$\mathbf{\Phi}_{k,i} = \sqrt{\frac{E_s}{N_t}} \sum_{j=1, j\neq i}^{N_t} \mathbf{F}^H \hat{\mathbf{E}}_{k,i}^H \left(\mathbf{E}_{k,j} \mathbf{F} \mathbf{H}_{k,j} - \hat{\mathbf{E}}_{k,j} \mathbf{F} \hat{\mathbf{H}}_{k,j} \right) \mathbf{x}_j, \quad (6)$$

In (5), $\tilde{\mathbf{w}}_{k,i}$ is the AWGN component of $\mathbf{r}_{k,i}$ given by

$$\tilde{\mathbf{w}}_{k,i} = \mathbf{F}^H \hat{\mathbf{E}}_{k,i}^H \mathbf{w}_k. \quad (7)$$

A. SINR Analysis without Receiver Combining

We assume that the $N_r \times N_t$ channels are fading independently and transmitted symbols are i.i.d. RVs with zero mean and unit variance. In this subsection, we will derive the average SINR for the channel between ith transmit antenna and kth receive antenna, denoted as $\bar{\gamma}_{k,i}$, when there is no combining at the receiver side.

In MIMO-OFDM systems, the average SINR $\bar{\gamma}_{k,i}$ respects to the OFDM symbol transmitted on N subcarriers and can be obtained by

$$\bar{\gamma}_{k,i} = \int_{\mathbf{H}_{k,i}} \gamma_{k,i}(\mathbf{H}_{k,i}) f_{\mathbf{H}_{k,i}}(\mathbf{H}_{k,i}) d\mathbf{H}_{k,i}, \quad (8)$$

where $f_{\mathbf{H}_{k,i}}(\mathbf{H}_{k,i})$ is the probability density function the frequency response $\mathbf{H}_{k,i}$. The main difficulty in evaluating (8) is that $\mathbf{H}_{k,i}$ is a $N \times 1$ random vector with correlation within elements $\mathbf{H}_{k,i}^{(n)}$ $(0 \leq n \leq N-1)$. Following the idea in [13], i.e., expressing the $\mathbf{H}_{k,i}$ as a function of channel frequency response on one subcarrier $\mathbf{H}_{k,i}^{(n)}$ and averaging over $\mathbf{H}_{k,i}^{(n)}$, (8) becomes

$$\bar{\gamma}_{k,i} = \int_{\mathbf{H}_{k,i}^{(n)}} \gamma_{k,i}(\mathbf{H}_{k,i}^{(n)}) f_{\mathbf{H}_{k,i}^{(n)}}(\mathbf{H}_{k,i}^{(n)}) d\mathbf{H}_{k,i}^{(n)}. \quad (9)$$

Based on (5), the demodulated signal which is transmitted on the (k,i,n) subcarrier can be rewritten as

$$\begin{aligned}\mathbf{r}_{k,i}[n] =& \sqrt{\frac{E_s}{N_t}}\mathbf{s}_{k,i}[n] + \mathbf{\Phi}_{k,i}[n] + \tilde{\mathbf{w}}_{k,i}[n] \\ =& \sqrt{\frac{E_s}{N_t}} m_{k,i}^{(n)} H_{k,i}^{(n)} \mathbf{x}_i[n] + \eta_{k,i}^{(n)} \\ &+ \lambda_{k,i}^{(n)} - \hat{\lambda}_{k,i}^{(n)} + \xi_{k,i}^{(n)} - \hat{\xi}_{k,i}^{(n)} + \tilde{\mathbf{w}}_{k,i}[n],\end{aligned} \tag{10}$$

where $\sqrt{\frac{E_s}{N_t}} m_{k,i}^{(n)} H_{k,i}^{(n)} \mathbf{x}_i[n]$ is the useful signal component and $m_{k,i}^{(n)} = m_{k,i}^{(l)}|_{l=n} = \frac{\sin[\pi(l-n-\Delta\varepsilon_{k,i})]}{N\sin\left[\frac{\pi(l-n-\Delta\varepsilon_{k,i})}{N}\right]} e^{\frac{\jmath\pi(N-1)(l-n)}{N}}$. In (10), $\eta_{k,i}^{(n)}$ is the intercarrier interference contributed by subcarriers other than n with the same (k,i) transmit and receive antenna pair and $\eta_{k,i}^{(n)} = \sqrt{\frac{E_s}{N_t}} \sum_{l\neq n} m_{k,i}^{(l)} H_{k,i}^{(l)} \mathbf{x}_i[l]$. Based on (10), We define $\Delta\lambda_{k,i}^{(n)} = \lambda_{k,i}^{(n)} - \hat{\lambda}_{k,i}^{(n)}$ as the interference contributed by the same nth subcarrier but transmit antennas other than i and $\lambda_{k,i}^{(n)} = \sqrt{\frac{E_s}{N_t}} \sum_{j=1,j\neq i}^{N_t} m_{k,j}^{(n)} H_{k,j}^{(n)} \mathbf{x}_j[n]$, $\hat{\lambda}_{k,i}^{(n)} = \sqrt{\frac{E_s}{N_t}} \sum_{j=1,j\neq i}^{N_t} \hat{m}_{k,j}^{(n)} \hat{H}_{k,j}^{(n)} \mathbf{x}_j[n]$, where

$$\begin{aligned} m_{k,j\neq i}^{(l)} =& \frac{\sin[\pi(l-n+\varepsilon_{k,j}-\hat{\varepsilon}_{k,i})]}{N\sin\left[\frac{\pi(l-n+\varepsilon_{k,j}-\hat{\varepsilon}_{k,i})}{N}\right]} e^{\frac{\jmath\pi(N-1)(l-n)}{N}} \\ \hat{m}_{k,j\neq i}^{(l)} =& \frac{\sin[\pi(l-n+\hat{\varepsilon}_{k,j}-\hat{\varepsilon}_{k,i})]}{N\sin\left[\frac{\pi(l-n+\hat{\varepsilon}_{k,j}-\hat{\varepsilon}_{k,i})}{N}\right]} e^{\frac{\jmath\pi(N-1)(l-n)}{N}}. \end{aligned} \tag{11}$$

Based on (10), we also define $\Delta\xi_{k,i}^{(n)} = \xi_{k,i}^{(n)} - \hat{\xi}_{k,i}^{(n)}$ as the interference contributed by the subcarriers other than n and transmit antennas other than i, $\xi_{k,i}^{(n)} = \sqrt{\frac{E_s}{N_t}} \sum_{l\neq n} \sum_{j=1,j\neq i}^{N_t} m_{k,j}^{(l)} H_{k,j}^{(l)} \mathbf{x}_j[l]$, $\hat{\xi}_{k,i}^{(n)} = \sqrt{\frac{E_s}{N_t}} \sum_{l\neq n} \sum_{j=1,j\neq i}^{N_t} \hat{m}_{k,j}^{(l)} \hat{H}_{k,j}^{(l)} \mathbf{x}_j[l]$.

From the analysis above, the SINR without combining at the receiver side and conditioned on $H_{k,i}^{(n)}$ (denoted as $\gamma_{k,i}^{\text{W/O-C}}(H_{k,i}^{(n)})$) can be expressed by

$$\gamma_{k,i}^{\text{W/O-C}}(H_{k,i}^{(n)}) = \frac{\frac{E_s}{N_t}\left|m_{k,i}^{(n)}\right|^2 \left|H_{k,i}^{(n)}\right|^2}{\sigma^2_{\eta_{k,i}^{(n)}} + \sigma^2_{\Delta\lambda_{k,i}^{(n)}} + \sigma^2_{\Delta\xi_{k,i}^{(n)}} + \sigma^2_{\tilde{\mathbf{w}}_{k,i}}}, \tag{12}$$

where $\sigma^2_{\eta_{k,i}^{(n)}}$, $\sigma^2_{\Delta\lambda_{k,i}^{(n)}}$, $\sigma^2_{\Delta\xi_{k,i}^{(n)}}$ and $\sigma^2_{\tilde{\mathbf{w}}_{k,i}}$ denote the variance of $\eta_{k,i}^{(n)}$, $\Delta\lambda_{k,i}^{(n)}$, $\Delta\xi_{k,i}^{(n)}$ and $\tilde{\mathbf{w}}_{k,i}$ respectively. To evaluating (12), we give the presuppositions that $\varepsilon_{k,i}$, $\Delta\varepsilon_{k,i}$ and $\Delta H_{k,i}^{(n)}$ are independent RVs with zero mean and variance of σ_ϵ^2, σ_{res}^2 and $\sigma_{\Delta H}^2$ respectively.

We rewritten the intercarrier interference $\eta_{k,i}^{(n)}$ in (10) as $\eta_{k,i}^{(n)} = H_{k,i}^{(n)}\alpha_{k,i}^{(n)} + \beta_{k,i}^{(n)}$, where $\alpha_{k,i}^{(n)}$ and $\beta_{k,i}^{(n)}$ are RVs of zero mean. The $H_{k,i}^{(n)}\alpha_{k,i}^{(n)}$ component in $\eta_{k,i}^{(n)}$ is proportional to the channel frequency response $H_{k,i}^{(n)}$ and represents the intercarrier interference part that fades synchronously with the useful signal. The $\beta_{k,i}^{(n)}$ part fades independent with the useful signal. Both $\alpha_{k,i}^{(n)}$ and $\beta_{k,i}^{(n)}$ depend on the frequency channel characterization and do not depend on the $H_{k,i}^{(n)}$ [13]. As $\alpha_{k,i}^{(n)}$ and $\beta_{k,i}^{(n)}$ are uncorrelated,

$$\sigma^2_{\eta_{k,i}^{(n)}} = \left|H_{k,i}^{(n)}\right|^2 \sigma^2_{\alpha_{k,i}^{(n)}} + \sigma^2_{\beta_{k,i}^{(n)}}. \tag{13}$$

We derive the $\sigma^2_{\alpha_{k,i}^{(n)}}$ and $\sigma^2_{\beta_{k,i}^{(n)}}$ as

$$\begin{aligned}\sigma^2_{\alpha_{k,i}^{(n)}} =& \frac{E_s}{N_t}\cdot\mathbb{E}\left\{\left|R^{-1}_{H_{k,i}^{(n)}H_{k,i}^{(n)}}\right|^2 \sum_{l\neq n}\left|m_{k,i}^{(l)} R_{H_{k,i}^{(l)}H_{k,i}^{(n)}}\right|^2\right\} \\ \cong& \frac{\pi^2\sigma_{res}^2 E_s}{N_t}\cdot\sum_{l\neq n}\frac{1}{N^2\sin^2\left[\frac{\pi(l-n)}{N}\right]} \\ &\cdot\left|\sum_{d=0}^{L_{max}-1}\mathbb{E}\left\{\left|h_{k,i}(d)\right|^2\right\} e^{-\frac{\jmath 2\pi d(l-n)}{N}}\right|^2,\end{aligned} \tag{14}$$

$$\begin{aligned}&\sigma^2_{\beta_{k,i}^{(n)}} \\ &= \frac{E_s}{N_t}\cdot\mathbb{E}\Big\{\sum_{l\neq n}\left|m_{k,i}^{(l)}\right|^2\big(R_{H_{k,i}^{(l)}H_{k,i}^{(l)}} - R^{-1}_{H_{k,i}^{(n)}H_{k,i}^{(n)}}\left|R_{H_{k,i}^{(l)}H_{k,i}^{(n)}}\right|^2\big)\Big\} \\ &= \frac{E_s}{N_t}\cdot\mathbb{E}\Big\{\sum_{l\neq n}\left|m_{k,i}^{(l)}\right|^2\Big\} - \sigma^2_{\alpha_{k,i}^{(n)}} \\ &\cong \frac{\pi^2\sigma_{res}^2 E_s}{3N_t} - \sigma^2_{\alpha_{k,i}^{(n)}},\end{aligned} \tag{15}$$

where $R_{H_{k,i}^{(l)}H_{k,i}^{(n)}}$ is given by (2). If the following conditions of

1. $|\varepsilon_{k,j}| \ll 1$ for each (k,j);
2. $|\hat{\varepsilon}_{k,i}| + |\varepsilon_{k,j}| < 1$ for each (k,i,j);
3. $|\hat{\varepsilon}_{k,i}| + |\hat{\varepsilon}_{k,j}| < 1$ for each (k,i,j)

are satisfied simultaneously, we can approximate

$$\begin{aligned}\Delta\lambda_{k,i}^{(n)} =& \sqrt{\frac{E_s}{N_t}}\sum_{j=1,j\neq i}^{N_t}\Bigg[\frac{\pi^2\left(\varepsilon_{k,j}-\hat{\varepsilon}_{k,i}+\frac{\Delta\varepsilon_{k,j}}{2}\right)H_{k,j}^{(n)}\Delta\varepsilon_{k,j}}{3} \\ &-\left(1-\frac{\pi^2\left(\hat{\varepsilon}_{k,j}-\hat{\varepsilon}_{k,i}\right)^2}{6}\right)\Delta H_{k,j}^{(n)}\Bigg]\mathbf{x}_j[n] \\ &+ o(\Delta\varepsilon_{k,j}, \Delta H_{k,j})\end{aligned} \tag{16}$$

and

$$\Delta\xi_{k,i}^{(n)} = \sqrt{\frac{E_s}{N_t}} \sum_{l\neq n} \sum_{j=1,j\neq i}^{N_t} \frac{(-1)^{l-n+1} e^{\frac{\jmath\pi(N-1)(l-n)}{N}}}{N \sin\left[\frac{\pi(l-n)}{N}\right]} \cdot \left[\pi \cos\left(\pi\left(\varepsilon_{k,j} - \hat{\varepsilon}_{k,i} + \frac{\Delta\varepsilon_{k,j}}{2}\right)\right) H_{k,j}^{(l)} \Delta\varepsilon_{k,j} + \sin\left(\pi\left(\hat{\varepsilon}_{k,j} - \hat{\varepsilon}_{k,i}\right)\right) \Delta H_{k,j}^{(l)}\right] \mathbf{x}_j[l] + o(\Delta\varepsilon_{k,j}, \Delta H_{k,j}) \tag{17}$$

with $o(\Delta\varepsilon_{k,j}, \Delta H_{k,j})$ represents the higher order item of $\Delta\varepsilon_{k,j}$ and $\Delta H_{k,j}$. It's easy to prove that $\Delta\lambda_{k,i}^{(n)}$ and $\Delta\xi_{k,i}^{(n)}$ are zero-mean RVs, and their variances are given by

$$\sigma^2_{\Delta\lambda_{k,i}^{(n)}} \cong \frac{(N_t-1)\pi^4 E_s}{9N_t}\left(2\sigma_\epsilon^2\sigma_{res}^2 + \sigma_{res}^4 + \frac{\mathbb{E}\left\{\Delta\varepsilon_{k,j}^4\right\}}{4}\right) + \frac{(N_t-1)E_s}{N_t} \cdot \sigma^2_{\Delta H} \cdot \left[1 + \frac{\pi^4\left(\mathbb{E}\left\{\varepsilon_{k,j}^4\right\} + 8\sigma_\epsilon^2\sigma_{res}^2 + 2\sigma_\epsilon^4 + 2\sigma_{res}^4\right)}{18} - \frac{2\pi^2\left(\sigma_\epsilon^2 + \sigma_{res}^2\right)}{3}\right] \tag{18}$$

and

$$\sigma^2_{\Delta\xi_{k,i}^{(n)}} \cong \frac{(N_t-1)E_s}{3N_t}\Big[\pi^2\sigma_{res}^2 - \pi^4\big(2\sigma_\epsilon^2\sigma_{res}^2 + \sigma_{res}^4 + \frac{\mathbb{E}\left\{\Delta\varepsilon_{k,j}^4\right\}}{4}\big)\Big] + \frac{2(N_t-1)\pi^2 E_s}{3N_t}\left(\sigma_\epsilon^2 + \sigma_{res}^2\right)\sigma^2_{\Delta H}. \tag{19}$$

Thus, the SINR conditioned on $H_{k,i}^{(n)}$ can obtained by substituting (13),(14),(15),(18) and (19) into (12) with $\sigma_m^2 = |m_{k,i}^{(n)}|^2 = 1 - \frac{\pi^2\sigma_{res}^2}{3} + \frac{\pi^4\mathbb{E}\left\{\Delta\varepsilon_{k,j}^4\right\}}{36}$. By average $|H_{k,i}^{(n)}|^2$, the average SINR on the (k,i) antenna pair can be obtained.

B. SINR Analysis with EGC at Receive Antennas

Not only does EGC increase the SINR, but also has lower implementation complexity than MRC. In EGC, the desired signals at all the N_r receive antennas are co-phased, equally weighted and summed to form the resultant desired signal. The EGC output may thus be expressed as

$$\mathbf{r}_i^{\mathrm{EGC}}[n] = \sum_{k=1}^{N_r} e^{-\jmath\theta_{k,i}^{(n)}} \mathbf{r}_{k,i}[n], \tag{20}$$

where $\theta_{k,i}^{(n)} = \arg\left\{m_{k,i}^{(n)} H_{k,i}^{(n)}\right\}$. After averaging out $\varepsilon_{k,i}$, $\Delta\varepsilon_{k,i}$ and $\Delta H_{k,i}^{(n)}$ for each (k,i), the SINR of $\mathbf{r}_i^{\mathrm{EGC}}[n]$ conditioned on the nth subcarrier is derived as

$$\gamma_i^{\mathrm{EGC}}\left(n\middle|H_{1,i}^{(n)}, \cdots, H_{N_r,i}^{(n)}\right) \cong \frac{\frac{E_s}{N_t}\cdot\sigma_m^2\cdot\left(\sum_{k=1}^{N_r}\left|H_{k,i}^{(n)}\right|^2 + \sum_{k\neq l}\left|H_{k,i}^{(n)}\right|\cdot\left|H_{l,i}^{(n)}\right|\right)}{\sum_{k=1}^{N_r}\left|H_{k,i}^{(n)}\right|^2\cdot\sigma^2_{\alpha_{k,i}^{(n)}} + N_r\kappa}, \tag{21}$$

where $\kappa = \sigma^2_{\beta_{k,i}^{(n)}} + \sigma^2_{\Delta\lambda_{k,i}^{(n)}} + \sigma^2_{\Delta\xi_{k,i}^{(n)}} + \sigma^2_{\tilde{\mathbf{w}}_{k,i}}$. When N_r is large enough (for example $N_r \geq 4$), (21) can be further simplified as

$$\gamma_i^{\mathrm{EGC}}\left(n\middle|H_{1,i}^{(n)}, \cdots, H_{N_r,i}^{(n)}\right) \cong \frac{\frac{E_s}{N_t}\cdot\sigma_m^2\cdot\left(\sum_{k=1}^{N_r}\left|H_{k,i}^{(n)}\right|^2 + \frac{N_r(N_r-1)\pi}{4}\right)}{\sum_{k=1}^{N_r}\left|H_{k,i}^{(n)}\right|^2\cdot\sigma^2_{\alpha_{k,i}^{(n)}} + N_r\kappa}. \tag{22}$$

The average SINR can be obtained by average $H_{k,i}^{(n)}$ $(1 \leq k \leq N_r)$.

C. SINR Analysis with MRC at Receive Antennas

MRC is a well-known optimal combining scheme. In MRC, different from EGC, the received signal at N_r receive antennas are multiplied with the Hermitian conjugates of the channel gains. Therefore, the signal at the output of MRC is

$$\mathbf{r}_i^{\mathrm{MRC}}[n] = \frac{\sum_{k=1}^{N_r}\omega_{k,i}\mathbf{r}_{k,i}[n]}{\sum_{k=1}^{N_r}|\omega_{k,i}|^2}, \tag{23}$$

where the complex combining coefficient is defined as $\omega_{k,i} = \left(\hat{H}_{k,i}^{(n)} m_{k,i}^{(n)}\right)^*$ for each k to maximize the resulting SINR. After averaging out $\varepsilon_{k,i}$, $\Delta\varepsilon_{k,i}$ and $\Delta H_{k,i}^{(n)}$ for each (k,i), the SINR of $\mathbf{r}_i^{\mathrm{MRC}}[n]$ conditioned on the nth subcarrier is derived as

$$\gamma_i^{\mathrm{MRC}}\left(n\middle|H_{1,i}^{(n)}, \cdots, H_{N_r,i}^{(n)}\right) \cong \frac{\frac{E_s}{N_t}\cdot\sigma_m^2\cdot\left(\sum_{k=1}^{N_r}\left|H_{k,i}^{(n)}\right|^2\right)^2}{\sum_{k=1}^{N_r}\left|H_{k,i}^{(n)}\right|^4\cdot\sigma^2_{\alpha_{k,i}^{(n)}} + \sum_{k=1}^{N_r}\left|H_{k,i}^{(n)}\right|^2\kappa' + N_r\cdot\kappa\cdot\sigma^2_{\Delta H}}, \tag{24}$$

where we defined $\kappa' = \left[\kappa + \left(\frac{E_s}{N_t} + \sigma^2_{\alpha_{k,i}^{(n)}}\right)\sigma^2_{\Delta H}\right]$, which is also independent of (k,i,n). When N_r is large enough (for example $N_r \geq 4$), (24) can be further simplified as

$$\gamma_i^{\mathrm{MRC}}\left(n\middle|H_{1,i}^{(n)}, \cdots, H_{N_r,i}^{(n)}\right) \cong \frac{\frac{E_s}{N_t}\cdot\sigma_m^2\cdot\sum_{k=1}^{N_r}\left|H_{k,i}^{(n)}\right|^2}{\left(\sum_{k=1}^{N_r}\left|H_{k,i}^{(n)}\right|^2 - (N_r-1)\right)\sigma^2_{\alpha_{k,i}^{(n)}} + \kappa' + \kappa\cdot\sigma^2_{\Delta H}}. \tag{25}$$

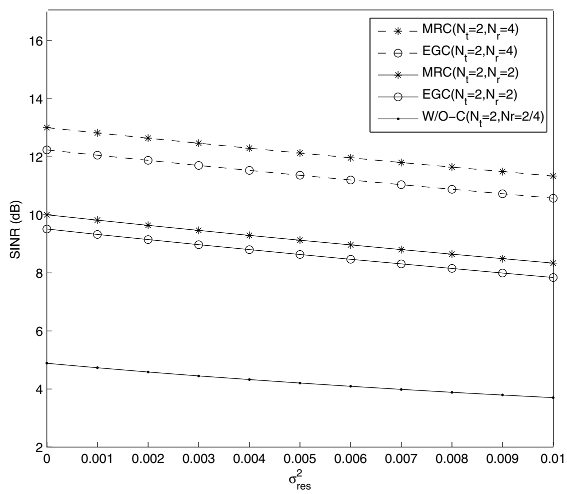

Fig. 1. SINR reduction by residual frequency offset in MIMO-OFDM.

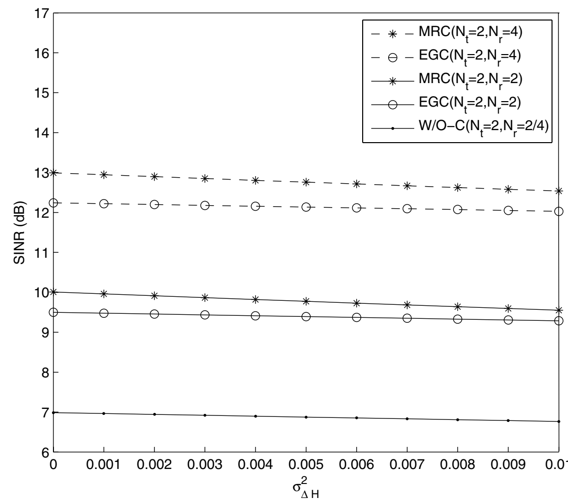

Fig. 2. SINR reduction by channel estimation error in MIMO-OFDM.

IV. Numerical Results

We now present numerical results to illustrate the average SINR impairment due to residual frequency offset and channel estimation errors. We consider a MIMO-OFDM system with $N = 128$ subcarriers and each subcarrier fades independently.

SINR degradation due to the residual frequency offsets is shown in Fig. 1, where the variance of channel estimation error $\sigma^2_{\triangle H} = 0$ and signal-to-noise ratio (SNR) is $10\,$dB. For larger σ^2_{res} (the variance of residual frequency offset), there is an increase in SINR degradation as expected. The SINR degradation due to the channel estimation error is also shown in Fig. 2, where $\sigma^2_{res} = 0$ and an SNR of $10\,$dB. The SINR also degrades as $\sigma^2_{\triangle H}$ increases. Comparison between these two figures indicates that the residual frequency offset degrades the SINR more seriously than channel estimation error with the same estimation error variance. In both figures, MRC outperforms EGC due to its maximization of the SINR. Both MRC and EGC benefit from increasing the number of receiver antennas. Without combining at the receiver side, the MIMO-OFDM system can not gain any diversity benefit by increasing the number of receiver antennas.

V. Conclusions

The SINR impairment due to frequency offset and channel estimation errors in MIMO-OFDM has been analyzed. Based on the analysis of the demodulated signal and interference, we derived the SINR for each receive antenna. The SINR for different receiver combining technologies, including EGC and MRC, for MIMO-OFDM has also been analyzed. The simple form of the derived expressions allows numerical evaluation of cases of practical interest.

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