Capacity Analysis for Transmit Antenna Selection Using Orthogonal Space-Time Block Codes

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***Abstract*— Antenna selection for multiple-input multiple-output (MIMO) where only a subset of antennas at the transmitter and/or receiver are activated for signal transmission is a practical technique for the realization of full diversity. Despite extensive research, closed-form capacity expressions for MIMO systems employing transmit antenna selection (TAS) and orthogonal space-time block codes (OSTBCs) are not available. We thus derive the exact closed-form capacity expressions when an OSTBC is employed and N transmit antennas out of total L_t antennas are selected for transmission. The expressions are valid for a frequency-flat Rayleigh fading MIMO channel and avoid numerical integration methods.**

***Index Terms*— Antenna selection, closed-form, capacity, multiple-input multiple-output (MIMO), space-time codes.**

I. Introduction

MULTIPLE-INPUT multiple-output (MIMO) technologies mitigate the impact of fading. Orthogonal space-time block codes (OSTBCs), which are used with MIMO systems, achieve full diversity order and low-complexity maximum likelihood detection [1]. However, multiple radio frequency chains contribute to high costs and complexity of MIMO systems. Transmit antenna selection (TAS), where OSTBC signal matrices are transmitted over a selected subset of transmit antennas, is a practical technique for the realization of full diversity [2]. Note that receive antenna selection is a traditionally well-researched topic and the number of references are too numerous to mention here [2]–[4]. Various performance analysis for TAS has been reported in [5]–[8]. Information theoretic limits of TAS for MIMO spatial multiplexing systems, in the limit of large transmit antennas, for both low and high SNR are derived in [9].

The main contribution of this letter is therefore to provide closed-form capacity expressions for TAS with OSTBC over an independent Rayleigh fading channel. To the best of our knowledge, no such capacity expression is available. The resulting formulas avoid the need for lengthy Monte Carlo simulation

In Section 2, the system model with TAS is presented. Closed-form capacity expressions are derived in Section 3. Section 4 provides simulation results and conclusion[1].

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[1]*Notation*: Bold symbols denote matrices or vectors. $(\cdot)^T$, $(\cdot)^H$ and $(\cdot)^*$ denote transpose, conjugate transpose and conjugate, respectively. The set of complex numbers is $\mathbb{C}$, and the set of all complex $K \times 1$ vectors, $M \times N$ matrices are denoted by $\mathbb{C}^K$, $\mathbb{C}^{M\times N}$ respectively. A circularly symmetric complex Gaussian variable with mean μ and variance σ^2 is denoted by $z \sim \mathcal{CN}(\mu, \sigma^2)$. $\|.\|_F^2$ stands for the Frobenius norm.

II. System Model

We consider a MIMO system in a Rayleigh fading environment with L_t transmit and L_r receive antennas. Channel state information (CSI) is perfectly available at the receiver. The receiver selects N best transmit antennas out of L_t and this decision is sent back the transmitter via a rate-limited feedback channel that is assumed to be instantaneous and error-free. Let the full channel $\mathbf{H} = [h_{ij}]$ where $h_{ij} \sim \mathcal{CN}(0,1)$ is the channel gain between the ith transmit and jth receive antenna. $\widetilde{\mathbf{H}} \in \mathbb{C}^{L_r\times N}$ consists of the channel gains for the N selected transmit antennas and L_r received antennas which is a submatrix of the channel matrix $\mathbf{H} \in \mathbb{C}^{L_r\times L_t}$. Denote $\mathbf{h}_j$ $(j = 1, 2, ..., L_t)$ the columns of the channel matrix $\mathbf{H}$. The columns are sorted according to their norms, $\|\mathbf{h}_{i_1}\| \geq ... \geq \|\mathbf{h}_{i_{L_t}}\|$ where $i_k \in \{1, 2, ..., L_t\}$ and the order of the indexes $i_1, i_2, \ldots, i_{L_t}$ is, in general, different from the order of the indexes $1, 2, \ldots, L_t$. Thus, $\widetilde{\mathbf{H}}$ can be obtained from $\mathbf{H}$ by removing the columns with indexes $i_{N+1}, \ldots, i_{L_t}$. Based on this selection criterion the total received signal power is maximized. The received signals are expressed as

$$\mathbf{Y} = \sqrt{\frac{E_s}{N}}\widetilde{\mathbf{H}}\mathbf{X} + \mathbf{V} \tag{1}$$

where $\mathbf{Y} \in \mathbb{C}^{L_r\times T}$ is the complex received signal matrix and $\mathbf{X} \in \mathbb{C}^{N\times T}$ is the complex transmitted signal matrix, which is a member of an OSTBC [1]. $\mathbf{V} \in \mathbb{C}^{L_r\times T}$ is the additive noise matrix with independent and identical distributed entries of $\mathcal{CN}(0, N_0)$. The coefficient $\sqrt{E_s/N}$ ensures that the total transmitted power in each channel use is E_s and independent of the number of transmit antennas.

When an OSTBC is used, the ML decoder for an OSTBC modulated signal decomposes the MIMO system to independent single input single output (SISO) additive white Gaussian noise channels defined as [1]

$$\tilde{s}_q = \sqrt{\frac{E_s}{N}}\left(\frac{1}{R_s}\|\widetilde{\mathbf{H}}\|_F^2\right)s_q + \nu_q, \quad q = 1, ..., Q \tag{2}$$

where $\nu_q \sim \mathcal{CN}\left(0, \frac{1}{R_s}\|\widetilde{\mathbf{H}}\|_F^2 N_0\right)$, R_s is the code rate, and s_q are information-bearing symbols.

We conclude that the achievable SNR per symbol for an M-ary constellation is

$$\gamma_s = \frac{E_s}{N_0}\frac{1}{R_s N}\|\widetilde{\mathbf{H}}\|_F^2 = c\rho\|\widetilde{\mathbf{H}}\|_F^2 \tag{3}$$

where $\rho = \frac{E_s}{N_0}$ is the SNR per channel use and $c = 1/(R_s N)$. Let $\gamma_k = c\rho\|\mathbf{h}_k\|^2$, $k = 1, 2, ..., L_t$, are the scaled norms of the columns of $\mathbf{H}$. Therefore, γ_k is a chi-squared i.i.d. random variable. In transmit antenna selection, the best N antennas

$$f_{\gamma_s}(\lambda) = N\binom{L_t}{N}\frac{(c\rho)^{-L_rN}}{\Gamma(L_r)^N}\sum_{i_1,\ldots,i_N} a(L_r;i_1,\ldots,i_N)\prod_{k=1}^{N-1}\frac{i_k!}{k^{i_k}}\sum_{j=0}^{L_t-N}\binom{L_t-N}{j}(-1)^j \times\left\{\sum_{n\in B}\binom{j}{n_0,\ldots,n_{L_t-1}}\frac{(c_{nj}+i_N)!(c\rho)^{r+N+i_N}}{(N+j)^{c_{nj}+i_N+1}A_{nj}}\left\{\sum_{m=1}^{r+N-1}P_m\frac{\lambda^{m-1}}{(m-1)!}e^{-\frac{\lambda}{c\rho}}+\sum_{l=1}^{c_{nj}+i_N+1}Q_l\frac{\lambda^{l-1}e^{-\frac{\lambda}{\eta_j}}}{(l-1)!}\right\}\right\} \quad (7)$$

with the largest γ_k are selected. Thus, the received SNR per symbol (3) can be written as

$$\gamma_s = \sum_{k=1}^{N}\gamma_{(k)} \quad (4)$$

where $\gamma_{(k)} = c\rho\|\mathbf{h}_{i_k}\|^2$. Note that this SNR is a sum of order statistics. As such, its distribution appears to be available only if the channel is independent fading. For our analysis, we use the approach via the moment generating function (MGF) of the effective SNR in (4).

III. Closed-Form Capacity Expressions

A. MIMO systems

The most difficult task is to obtain the MGF of the ordered statistics γ_s in (4). Fortunately, the MGF of the random variable γ_s, which is the Laplace transform of probability density function (pdf) $f_{\gamma_s}(\lambda)$, is given in [10]

$$\Phi_{\gamma_s}(s) = N\binom{L_t}{N}\frac{(c\rho)^{-L_rN}}{\Gamma(L_r)^N}\sum_{i_1,\ldots,i_N} a(L_r;i_1,\ldots,i_N) \prod_{k=1}^{N-1}\frac{i_k!}{k^{i_k}}\sum_{j=0}^{L_t-N}\binom{L_t-N}{j}(-1)^j \times\left\{\sum_{n\in B}\binom{j}{n_0,\ldots,n_{L_r-1}}\frac{(c_{nj}+i_N)!(c\rho)^{r+N+i_N}}{(N+j)^{c_{nj}+i_N+1}A_{nj}} \frac{1}{(1+c\rho s)^{r+N-1}}\cdot\frac{1}{(1+\eta_j s)^{c_{nj}+i_N+1}}\right\} \quad (5)$$

where $\binom{j}{n_0,\ldots,n_{L_r-1}} = \frac{j!}{n_0!\cdots n_{L_r-1}!}$, $\eta_j = \frac{Nc\rho}{N+j}$, $a(L_r;i_1,\ldots,i_N)$ is the coefficient of $x_1^{i_1}\ldots x_N^{i_N}$ in expression

$$(x_1+x_2+\cdots+x_N)^{L_r-1}(x_2+\cdots+x_N)^{L_r-1}\cdots x_N^{L_r-1}$$

and B is the set of all combinations of nonnegative integers of $n_0,n_1,\ldots,n_{L_r-1}$ such that $\sum_{k=0}^{L_r-1}n_k = j$, $c_{nj} = \sum_{k=1}^{L_r-1}kn_k$, $A_{nj} = \prod_{k=1}^{L_r-1}(k!)^{n_k}$ and $r = \sum_{k=1}^{N-1}i_k$.

We take the inverse Laplace transform of $\Phi_{\gamma_s}(s)$ to find $f_{\gamma_s}(\lambda)$. Using partial fractions, the linearity property of inverse Laplace transform and the fact that

$$\mathcal{L}^{-1}\{\Psi_\gamma^m(s)\} = \frac{1}{(m-1)!\gamma^m}\lambda^{m-1}e^{-\frac{\lambda}{\gamma}}, \qquad m>1 \quad (6)$$

where $\Psi_\gamma(s) = (1+\gamma s)^{-1}$, the pdf of $\gamma_s = c\rho\|\widetilde{\mathbf{H}}\|_F^2$ follows in (7), where the coefficients are given by

$$P_m = \frac{(c\rho)^{1-r-N}}{(r+N-1-m)!}\cdot\frac{\partial^{r+N-1-m}}{\partial s^{r+N-1-m}}\left[\Psi_{\eta_j}^{c_{nj}+i_N+1}(s)\right]_{s=-1/(c\rho)}$$
$$Q_l = \frac{(\eta_j)^{-c_{nj}-i_N-1}}{(c_{nj}+i_N+1-l)!}\cdot\frac{\partial^{c_{nj}+i_N+1-l}}{\partial s^{c_{nj}+i_N+1-l}}\left[\Psi_{c\rho}^{r+N-1}(s)\right]_{s=-1/\eta_j}.$$

These coefficients can be obtained in closed-form without any differentiation [11].

The exact average capacity in nats/s/Hz is given in (8) where we use the following result from [12, App. B]

$$\hat{C}_{m-1}(\nu) \triangleq \int_0^\infty \log(1+\lambda)\lambda^{m-1}e^{-\nu\lambda}d\lambda,\ \nu>0, n=1,2,\ldots = (m-1)!e^{\nu}\sum_{k=1}^{m}\frac{\Gamma(-m+k,\nu)}{\nu^k}$$

where $\Gamma(a,z) = \int_z^\infty e^{-x}x^{a-1}dx$ is the complementary incomplete gamma function.

B. MISO systems

For MISO systems, or equivalently $L_r = 1$, the MGF for γ_s in (5) can be reduced into simpler form, thus the capacity expression (8) is greatly simplified. To be more specific, it has been shown in [7] that

$$\Phi_{\gamma_s}(s) = \frac{L_t!}{N!}\frac{1}{(1+c\rho s)^N}\prod_{j=1}^{L_t-N}\frac{1}{N+j}\cdot\frac{1}{1+\eta_j s}. \quad (9)$$

As before, we derive the pdf of γ_s and the exact average capacity in nats/s/Hz as

$$f_{\gamma_s}(\lambda) = \tilde{C}_0\left\{\sum_{m=1}^{N}\frac{\tilde{P}_m\lambda^{m-1}e^{-\frac{\lambda}{c\rho}}}{(m-1)!}+\sum_{j=1}^{L_t-N}\frac{\tilde{Q}_je^{-\frac{\lambda}{\eta_j}}}{\eta_j}\right\} \quad (10)$$

and

$$C = C_0\left\{\sum_{m=1}^{N}\frac{\tilde{P}_m\hat{C}_{m-1}(1/c\rho)}{(m-1)!}+\sum_{j=1}^{L_t-N}\frac{\tilde{Q}_j}{\eta_j}\hat{C}_0(1/\eta_j)\right\} \quad (11)$$

where the coefficients are

$$C_0 = R_s\tilde{C}_0 = R_s\frac{L_t!}{N!}\prod_{j=1}^{L_t-N}\frac{1}{N+j},$$
$$\tilde{P}_m = \frac{(c\rho)^{-N}}{(N-m)!}\frac{\partial^{N-m}}{\partial s^{N-m}}\left[\prod_{j=1}^{L_t-N}\frac{1}{1+\eta_j s}\right]_{s=-1/(c\rho)}, 1\le m\le N,$$
$$\tilde{Q}_j = \left[\frac{1}{(1+c\rho s)^N}\prod_{\substack{i=1\\ i\ne j}}^{L_t-N}\frac{1}{1+\eta_i s}\right]_{s=-1/\eta_j}. \quad (12)$$

The capacity expressions in (8) and (11) do not involve any integrations and can be computed easily with high accuracy using mathematical software such as MATLAB.

$$
\begin{aligned}
C &= \mathbb{E}\{R_s \log(1+\gamma_s)\} = R_s \int_0^\infty \log(1+\lambda) f_{\gamma_s}(\lambda) d\lambda \\
&= R_s N \binom{L_t}{N} \frac{(c\rho)^{-L_r N}}{\Gamma(L_r)^N} \sum_{i_1,\ldots,i_N} a(L_r; i_1, \ldots, i_N) \prod_{k=1}^{N-1} \frac{i_k!}{k^{i_k}} \sum_{j=0}^{L_t-N} \binom{L_t-N}{j} (-1)^j \\
&\times \left\{ \sum_{n \in B} \binom{j}{n_0, \ldots, n_{L_r-1}} \frac{(c_{nj}+i_N)!(c\rho)^{r+N+i_N}}{(N+j)^{c_{nj}+i_N+1} A_{nj}} \left\{ \sum_{m=1}^{r+N-1} \frac{P_m \hat{C}_{m-1}(1/(c\rho))}{(m-1)!} + \sum_{l=1}^{c_{nj}+i_N+1} \frac{Q_l \hat{C}_{l-1}(1/\eta_j)}{(l-1)!} \right\} \right\} \qquad (8)
\end{aligned}
$$

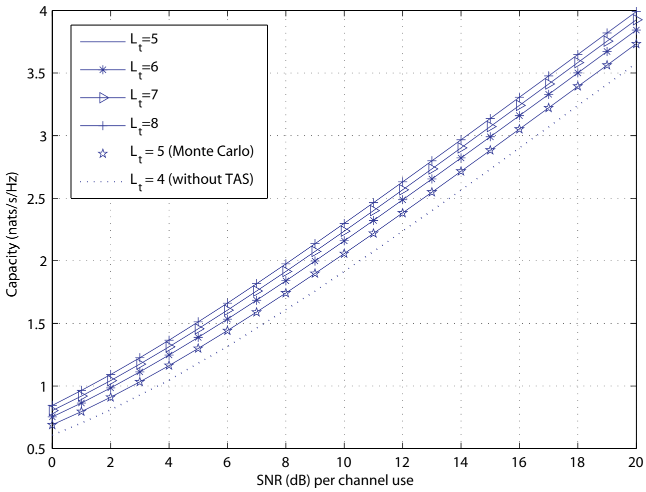

Fig. 1. Capacity of MISO with TAS, OSTBC rate 3/4, N=4.

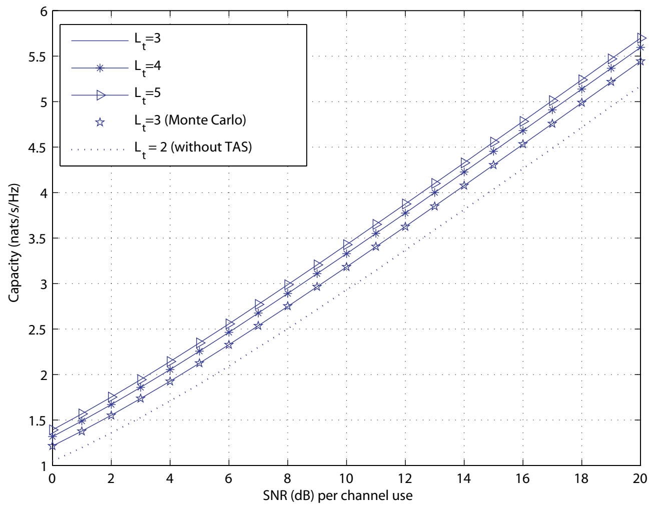

Fig. 2. Capacity of MIMO with TAS, Alamouti, N=2.

IV. Results and Conclusion

In this section, we plot capacity curves for different systems. Fig. 1 plots the capacity (11) for a MISO system using the OSTBC with 4 transmit antennas, rate 3/4 as defined in [1]. As there are more available transmit antennas to choose from, the system capacity increases, which is intuitive. The capacity gain is about 2.5 dB for most of operating SNRs when there are 4 more transmit antennas. For MIMO systems, full-rate Alamouti code [13] has been used ($N = 2$) with 2 receive antennas. The capacity gain is similar to that of the SIMO case. Monte Carlo simulation validates the correctness of the closed-form expressions (8) and (11).

In this letter, we have analyzed the capacity of TAS and OSTBCs for MIMO systems. The closed-form capacity expressions, which avoids the need for numerical integration methods, were derived. Our results are sufficiently general to handle systems with an arbitrary number of antennas at both ends of a MIMO link.

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