On the Decoding and Optimizing Performance of Four-Group Decodable Space-Time Block Codes

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***Abstract*— A class of rate-one space-time block codes (STBC) allowing the decoding of transmitted symbols into four groups is recently proposed by Yuen, Guan and Tjhung. This code is called four-group decodable STBC (4Gp-STBC). In this paper, the equivalent channel of 4Gp-STBC is derived and a new method to decode 4Gp-STBC based on sphere decoders is presented. Furthermore, the performance of 4Gp-STBC is analyzed. A New signal rotation method is proposed, which performs better than the existing one.**

Keywords-Space-time block codes, quasi-orthogonal, performance analysis, low decoding complexity.

I. Introduction

The demand of high data rate communications over wireless systems has been increasing tremendously [1]. Wireless systems, as a counterpart of wireline networks, are required to provide the services for bandwidth consuming applications such as video data. However, the time-varying nature of wireless channels is the major source of erroneous transmission. Therefore, the advanced transmission techniques must be designed to combat with the fading wireless channels.

One of the techniques to mitigate the effect of fading wireless channels is space-time coding (STC) [2]. The spatial diversity of multiple-input multiple-output (MIMO) channel is exploited to improve the reliability of the wireless communication links. Among the designs of STC, space-time block codes (STBC) attract significant interest of research since they have low decoding complexity and yet provide full-diversity of MIMO channels [1].

The first STBC has been proposed by Alamouti [3] for the systems with two transmit antennas and multiple receive antennas. This code is shown to be a special case of orthogonal STBC (OSTBC) employing orthogonal designs [4]. Thanks to the minimal decoding complexity, OSTBC have been integrated in the third generation (3G) mobile wireless standards [5]. However, the main disadvantage of OSTBC is their low code rate; the rate-one code for complex constellations exists for two antennas only.

In order to design the high-rate STBC, the orthogonal constraints must be relaxed leading to the designs of quasi-orthogonal STBC (QSTBC) (see, e.g. [6], [7]). QSTBC also have low-decoding complexity. However, the rate-one code can be obtained for four antennas only.

Keeping the low decoding complexity as the design principle, several rate-one STBC for any number of transmit antennas have been proposed (see, e.g. [8], [9], [10]). These codes allow the decoding of transmitted symbols into two-group; thus the decoding complexity is significantly reduced compared with other rate-one codes in [11], [12].

Recently, a class of rate-one STBC has been proposed by Yuen, Tjung and Guan [13]. This code has an even lower decoding complexity than that of the codes in [8], [9], [10] since the transmitted symbols can be separated into four group for maximum likelihood (ML) detection. We call this code four-group decodable STBC (4Gp-STBC) for short.

In this paper, we will analyze the performance of the 4Gp-STBC in [13] by deriving the pair-wise error probability (PEP). The union of the PEP is less than 0.1 dB from the simulated PEP. Therefore, this bound can be used to optimize the performance of the codes. By product, an equivalent channel of the code is presented so that the low-complexity sphere decoders [14] can be exploited.

The paper is organized as follows. Section II introduces the system model and basic notations. Section III presents the construction of the 4Gp-STBC and their PEP calculation. Finally, conclusions and remarks are summarized in Section V.

II. System Model

A. Notation

We first set the common notations to be used throughout the paper. Superscripts T, $*$, and H denote matrix transpose, conjugate, and transpose conjugate, respectively. The identity and all-zero square matrices of proper size are denoted by $\boldsymbol{I}$ and $\boldsymbol{0}$. Unless otherwise stated, all the vectors are column and denoted by underlined lowercase letters. The diagonal matrix with elements of vector $\underline{x}$ on the main diagonal is denoted by diag($\underline{x}$). X_{F} stands for Frobenius norm of matrix X and $\otimes$ denotes Kronecker product [15]. $E[x]$ denotes average of x. A mean-m and variance-σ^2 circularly complex Gaussian random variable is written by $CN(m, \sigma^2)$.

We will deal with different types of signals. For clarity, we use the small letters a and b to solely represent the real and imaginary parts of the input information symbol s. Hence

$$s = a + jb, \quad j = \sqrt{-1}\,. \tag{1}$$

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In general, $\Re(X)$ and $\Im(X)$ denote the real and imaginary parts of matrix X, respectively.

B. System Model

We consider data transmission over a quasi-static Rayleigh flat fading channel, i.e. the channel is fixed for the duration of a codeword, but can vary from one codeword to another codeword. The transmitter and receiver are equipped with M transmit and N receive antennas. The channel gain h_{ik} (i = 1, 2, …, M; k = 1, 2, …, N) between the (i, k)-th transmit-receive antenna pair is assumed $CN(0, 1)$. We assume no spatial correlation at either transmit or receive array, and the receiver, but not the transmitter, completely knows the channel gains.

The ST encoder parses data symbols into a T-by-M code matrix X as follows:

$$X = \left[c_{t,i} \right]_{\substack{i=1,\ldots,M \\ t=1,\ldots,T}} \tag{2}$$

where c_{ti} is the symbol transmitted from antenna i at time t. The average energy of code matrices is constrained such that

$$E[trace(X^H X)] = E\left[\|X\|_F^2\right] = \sum_{i=1}^{M}\sum_{t=1}^{T} E\left[\left|c_{t,i}\right|^2\right] = T. \tag{3}$$

The received signals y_{tk} of the k-th antenna at time t can be arranged in a matrix Y of size T–by-N. Thus, one can represent the transmit-receive signal relation as

$$Y = \sqrt{\rho} XH + Z \tag{4}$$

where $H = [h_{ik}]$, and $Z = [z_{ik}]$ of size T-by-N, and z_{ik} are independently, identically distributed (i.i.d.) $CN(0, 1)$. The transmit power is scaled by ρ so that the average signal-to-noise ratio (SNR) at each receive antenna is ρ, independent of the number of transmit antennas. However, ρ is sometimes omitted for notational brevity.

The mapping of a block of K data symbols (s_1, s_2, …, s_K) into a T-by-M code matrix can be represented in a general dispersion form [16] as follows:

$$X = \sum_{k=1}^{K}\left(a_k A_k + b_k B_k\right) \tag{5}$$

where A_k and B_k, (k = 1, 2, …, K) are T-by-M complex-valued constant matrices; they are commonly called dispersion matrices.

The code rate $R_{X,M}$ of an ST code X designed for M transmit antennas is the ratio of data symbols transmitted in an ST code matrix and the number of channel uses T: $R_{X,M} = K/T$.

III. Performance of Four-Group Decodable STBC

In general, the transmitted symbols of a STBC are jointly ML decoded [16]. This approach leads to high decoding complexity for the high-rate STBC. Therefore, it is desirable to reduce the decoding complexity by separating transmitted symbols into subgroups. This design philosophy has been implemented for OSTBC and QSTBC by imposing stringent orthogonal constraints on the dispersion matrices [4],[6],[7]. Due to the orthogonal constraints, the code rate of OSTBC and QSTBC are less than 1 for more than 4 antennas. Therefore, the orthogonality constraints must be further relaxed for the designs of high-rate STBC with low complexity.

A. Encoding

The sufficient condition so that the transmitted symbols can be separated at the receiver is specified in [17, Theorem 1]. If two symbols s_p and s_q are separable at the receiver, their dispersion matrices must be satisfied:

$$A_p^H A_q + A_q^H A_p = 0 \tag{6a}$$

$$B_p^H B_q + B_q^H B_p = 0 \tag{6b}$$

$$A_p^H B_q + B_q^H A_p = 0 \tag{6c}$$

Furthermore, Yuen *et al.* identify a class of four-group decodable STBC (4Gp-STBC) in [13] by providing sufficient conditions to make a STBC become four group decodable.

Proposition 1: Given a 4Gp-STBC for M transmit antennas, with code length T, and K sets of dispersion matrices denoted as $\{A_q, B_q\}$, $1 \le q \le K$. A 4Gp-STBC with code length $2T$ for $2M$ transmit antennas, which consists of $2K$ sets of dispersion matrices denoted as $\{\underline{A}_q, \underline{B}_q\}$, $1 \le q \le 2K$, can be constructed using the following four mapping rules:

$$\underline{A}_{2k-1} = \begin{bmatrix} A_k & 0 \\ 0 & A_k \end{bmatrix}, \quad \underline{A}_{2k} = \begin{bmatrix} jB_k & 0 \\ 0 & jB_k \end{bmatrix},$$

$$\underline{B}_{2k-1} = \begin{bmatrix} 0 & -jA_k \\ -jA_k & 0 \end{bmatrix}, \quad \underline{B}_{2k} = \begin{bmatrix} 0 & B_k \\ B_k & 0 \end{bmatrix}. \tag{7}$$

The recursive construction of 4Gp-STBC specified in Proposition 1 suggests that we can start with the rate-one minimum decoding complexity QSTBC (MDC-QSTBC) for 4 transmit antennas proposed in [18] to construct 4Gp-STBC for 8, 16 transmit antennas and so on, because MDC-QSTBC is one of the STBC satisfying (6); the resulting STBC is thus called 4Gp-QSTBC. For practical interest, we will illustrate the encoding process of 4Gp-QSTBC for 8 transmit antennas from the MDC-QSTBC for 4 transmit antennas in the following.

Note that MDC-QSTBC in [18] is actually equivalent to the ABBA codes [19]. We can write the code matrix of MDC-QSTBC for 4 transmit antennas as

$$F_4 = \begin{bmatrix} a_1 + ja_3 & a_2 + ja_4 & b_1 + jb_3 & b_2 + jb_4 \\ -a_2 + ja_4 & a_1 - ja_3 & -b_2 + jb_4 & b_1 - jb_3 \\ b_1 + jb_3 & b_2 + jb_4 & a_1 + ja_3 & a_2 + ja_4 \\ -b_2 + jb_4 & b_1 - jb_3 & -a_2 + ja_4 & a_1 - ja_3 \end{bmatrix} \tag{8}$$
$$= \begin{bmatrix} x_1 & x_2 & x_3 & x_4 \\ -x_2^* & x_1^* & -x_4^* & x_3^* \\ x_3 & x_4 & x_1 & x_2 \\ -x_4^* & x_3^* & -x_2^* & x_1^* \end{bmatrix}$$

where $x_1 = a_1 + ja_3$, $x_2 = a_2 + ja_4$, $x_3 = b_1 + jb_3$, and $x_4 = b_2 + jb_4$ are intermediate symbols used to highlight the ABBA structure of MDC-QSTBC codes [7], [18]. The four transmitted symbols $s_i = a_i + jb_i$, ($i = 1, \ldots, 4$) in the code matrix F_4 can be separated at the receiver for ML detection. We now build the code matrix of 4Gp-QSTBC for 8 transmit antennas from F_4 using mapping rules in (7) below:

$$F_8 = \begin{bmatrix} x_1 & x_5 & x_2 & x_6 & x_3 & x_7 & x_4 & x_8 \\ -x_5^* & x_1^* & -x_6^* & x_2^* & -x_7^* & x_3^* & -x_8^* & x_4^* \\ x_2 & x_6 & x_1 & x_5 & x_4 & x_8 & x_3 & x_7 \\ -x_6^* & x_2^* & -x_5^* & x_1^* & -x_8^* & x_4^* & -x_7^* & x_3^* \\ x_3 & x_7 & x_4 & x_8 & x_1 & x_5 & x_2 & x_6 \\ -x_7^* & x_3^* & -x_8^* & x_4^* & -x_5^* & x_1^* & -x_6^* & x_2^* \\ x_4 & x_8 & x_3 & x_7 & x_2 & x_6 & x_1 & x_5 \\ -x_8^* & x_4^* & -x_7^* & x_3^* & -x_6^* & x_2^* & -x_5^* & x_1^* \end{bmatrix}. \quad (9)$$

where $x_1 = a_1 + ja_5$, $x_2 = a_2 + ja_6$, $x_3 = b_1 + jb_5$, $x_4 = b_2 + jb_6$, $x_5 = a_3 + ja_7$, $x_6 = a_4 + ja_8$, $x_7 = b_3 + jb_7$, and $x_8 = b_4 + jb_8$ are intermediate variables.

B. Decoding

We know that the four symbols $s_1, \ldots, s_4$ of F_4 can be separately detected. Therefore, from Proposition 1, the 4 groups of 8 symbols of F_8 can be detected independently. These 4 groups are (s_1, s_2), (s_3, s_4), (s_5, s_6), and (s_7, s_8). We will present the decoding of 4Gp-QSTBC for 8 transmit antennas in details. To provide more insight into the decoding of 4Gp-QSTBC, we will derive an equivalent code and the equivalent channel of F_8. Furthermore, using the equivalent channel of F_8, we can use a sphere decoder [14] to reduce the complexity of the ML search.

The equivalent code of F_8 is obtained by column permutations for the code matrix of F_8 in (9): the order of columns is changed to (1, 3, 5, 7, 2, 4, 6, 8). This order of permutations is also applied for the rows of F_8. We obtain a permutation-equivalent code of F_8 below:

$$G = \begin{bmatrix} D_1 & D_2 \\ -D_2^* & D_1^* \end{bmatrix} \quad (10)$$

where

$$D_1 = \begin{bmatrix} x_1 & x_2 & x_3 & x_4 \\ x_2 & x_1 & x_4 & x_3 \\ x_3 & x_4 & x_1 & x_2 \\ x_4 & x_3 & x_2 & x_1 \end{bmatrix}, \; D_2 = \begin{bmatrix} x_5 & x_6 & x_7 & x_8 \\ x_6 & x_5 & x_8 & x_7 \\ x_7 & x_8 & x_5 & x_6 \\ x_8 & x_7 & x_6 & x_5 \end{bmatrix}. \quad (11)$$

The sub-matrices D_1 and D_2 have a special form called *block-circulant matrix with circulant blocks* [20].

We next show how to decode the code G. For the sake of simplicity, we consider a single receive antenna. The generalization for multiple receive antennas is straightforward.

Assume that the transmit symbols are drawn from a constellation with unit average power. The transmit-receive signal model in (3) for the case of STBC G follows

$$Y = \sqrt{\rho/8}GH + Z. \quad (12)$$

Let

$$\underline{x} = [x_1 \;\; x_2 \;\; \ldots \;\; x_8]^T,$$

$$\underline{y} = [y_1 \;\; \ldots \;\; y_4 \;\; y_5^* \;\; \ldots \;\; y_8^*]^T,$$

$$\underline{z} = [z_1 \;\; \ldots \;\; z_4 \;\; z_5^* \;\; \ldots \;\; z_8^*]^T, \text{ and}$$

$$H_1 = \begin{bmatrix} h_1 & h_2 & h_3 & h_4 \\ h_2 & h_1 & h_4 & h_3 \\ h_3 & h_4 & h_1 & h_2 \\ h_4 & h_3 & h_2 & h_1 \end{bmatrix}, H_2 = \begin{bmatrix} h_5 & h_6 & h_7 & h_8 \\ h_6 & h_5 & h_8 & h_7 \\ h_7 & h_8 & h_5 & h_6 \\ h_8 & h_7 & h_6 & h_5 \end{bmatrix}. \quad (13)$$

We have an equivalent expression of (12) as

$$\underline{y} = \sqrt{\frac{\rho}{8}} \underbrace{\begin{bmatrix} H_1 & H_2 \\ H_2^* & -H_1^* \end{bmatrix}}_{\hat{H}} \underline{x} + \underline{z}. \quad (14)$$

Note that H_1 and H_2 are block-circulant matrices with circulant-blocks [20]. Thus they are commutative and so do H_1^* and H_2^*. We can multiply both sides of (14) with $\hat{H}^H$ to get

$$\underbrace{\hat{H}^H \underline{y}}_{\underline{w}} = \sqrt{\frac{\rho}{8}} \begin{bmatrix} H_1^* H_1 + H_2^* H_2 & 0 \\ 0 & H_1^* H_1 + H_2^* H_2 \end{bmatrix} \underline{x} + \underbrace{\hat{H}^H \underline{z}}_{\underline{n}}. \quad (15)$$

It is not hard to show that the noise elements of vector $\underline{n}$ are correlated with covariance matrix $\hat{H}^H \hat{H}$. Thus this noise vector can be whiten by multiplying both side of (15) with the matrix $(\hat{H}^H \hat{H})^{-1/2}$. Let $\widetilde{H} = H_1^* H_1 + H_2^* H_2$, (15) after the noise whitening step is equivalent to the following equations

$$\widetilde{H}^{-1/2} \underline{w}_1 = \sqrt{\rho/8} \widetilde{H}^{1/2} \underline{x}_1 + \underline{n}_1, \quad (16a)$$

$$\widetilde{H}^{-1/2} \underline{w}_2 = \sqrt{\rho/8} \widetilde{H}^{1/2} \underline{x}_2 + \underline{n}_2 \quad (16b)$$

where

$\underline{w}_1 = [w_1 \;\; w_2 \;\; w_3 \;\; w_4]^T$, $\underline{w}_2 = [w_5 \;\; w_6 \;\; w_7 \;\; w_8]^T$, $\underline{x}_1 = [x_1 \;\; x_2 \;\; x_3 \;\; x_4]^T$, $\underline{x}_2 = [x_5 \;\; x_6 \;\; x_7 \;\; x_8]^T$.

The noise vectors $\underline{n}_1 = \widetilde{H}^{-1/2}[n_1 \;\; n_2 \;\; n_3 \;\; n_4]^T$, $\underline{n}_2 = \widetilde{H}^{-1/2}[n_5 \;\; n_6 \;\; n_7 \;\; n_8]^T$ are uncorrelated and have elements with distribution $CN(0, 1)$.

At this point, the decoding of the 8 transmitted symbols in the code matrix G can be readily decoupled into 2 independent groups. However, since the code is a 4Gp-STBC, we can further decompose them into 4 groups in the following.

Denote the 2-by-2 discrete Fourier transform matrix by $F_2 = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$. The block-circulant matrices H_1 and H_2 can be diagonalized by a (real) unitary matrix $T = \frac{1}{2} F_2 \otimes F_2$ [20, Theorem 5.8.2, p. 185]. Note that $T^H = T$, therefore,

$$H_1 = T\Lambda_1 T,\ H_2 = T\Lambda_2 T \tag{17}$$

where Λ_1 and Λ_2 are diagonal matrices with eigenvalues of H_1 and H_2 in the main diagonal, respectively. Thus,

$$\widetilde{H} = H_1^* H_1 + H_2^* H_2 = T(\Lambda_1^H \Lambda_1 + \Lambda_2^H \Lambda_2)T \tag{18}$$

and hence $\widetilde{H}^{1/2} = T(\Lambda_1^H \Lambda_1 + \Lambda_2^H \Lambda_2)^{1/2} T$. Since $\widetilde{H}^{1/2}$ is a real matrix, (16) becomes

$$\widetilde{H}^{-1/2} \Re(\underline{w}_i) = \sqrt{\rho/8}\, \widetilde{H}^{1/2} \Re(\underline{x}_i) + \Re(\underline{n}_i), \tag{19a}$$

$$\widetilde{H}^{-1/2} \Im(\underline{w}_i) = \sqrt{\rho/8}\, \widetilde{H}^{1/2} \Im(\underline{x}_i) + \Im(\underline{n}_i), \tag{19b}$$

for i = 1, 2. Note that $\Re(\underline{x}_1) = [a_1 \quad a_2 \quad b_1 \quad b_2]^T := \underline{d}_1$, that is $\Re(\underline{x}_1)$ is only dependent on the two complex symbols s_1 and s_2 (see (4)). Similarly, $\Re(\underline{x}_2), \Im(\underline{x}_1)$ and $\Im(\underline{x}_2)$ depend on the pairs of symbols (s_3, s_4), (s_5, s_6), and (s_7, s_8), respectively.

From (19), the decoding of 8 transmitted symbols into four groups has been shown explicitly. One can apply a sphere decoder to decode the data vectors. The matrix $\widetilde{H}^{1/2}$ can be considered as the *equivalent channel* of the 4Gp-QSTBC G.

C. *Performance Analysis*

In (19), the four data vectors experience the same equivalent channel and the additive noise vectors have the same statistic; the PEPs of the four vectors are the same. We only need to consider the PEP of the vector $\underline{d}_1 = \Re(\underline{x}_1) = [a_1 \quad a_2 \quad b_1 \quad b_2]^T$. For notational simplicity, the sub-index 1 of d_1 is dropped.

Additionally, we can introduce redundancy on the signal space by using a 4-by-4 real rotation matrix R to the data vector $\underline{d}_1 = [a_1 \quad a_2 \quad b_1 \quad b_2]^T$ [21], hence $\underline{d}_1 = R[a_1 \quad a_2 \quad b_1 \quad b_2]^T$. To keep the transmit power unchanged, the rotation matrix is assumed orthogonal, i.e. $R^T R = I$.

From (18a), the PEP of the pair $(\underline{d}, \underline{d}')$ can be expressed by the Gaussian tail function as [22]

$$P(\underline{d}, \underline{d}' \mid \widetilde{H}) = Q\left(\sqrt{\frac{\rho}{8} \frac{|\widetilde{H}^{1/2} R \underline{\delta}|^2}{4N_0}}\right). \tag{20}$$

where N_0 = 1/2 is the variance of the elements of the white noise vector $\Re(\underline{n}_i)$ in (19a), $\underline{\delta} = \underline{d} - \underline{d}'$. From (18), one has

$$P(\underline{d}, \underline{d}' \mid \widetilde{H}) = Q\left(\sqrt{\frac{\rho \underline{\delta}^T R^T T^T (\Lambda_1^H \Lambda_1 + \Lambda_2^H \Lambda_2) T R \underline{\delta}}{16}}\right). \tag{21}$$

Let $\underline{\beta} = TR\underline{\delta}$. Remember that Λ_i is a diagonal matrix with eigenvalues of H_i on the main diagonal. Let $\lambda_{i,k}\ (i = 1,2; k = 1,2,3,4)$ be the eigenvalues of H_i. Then $\Lambda_i = diag(\lambda_{i,1}, \lambda_{i,2}, \lambda_{i,3}, \lambda_{i,4})$. Furthermore,

$$[\lambda_{i,1}, \lambda_{i,2}, \lambda_{i,3}, \lambda_{i,4}]^T = (F_2 \otimes F)_2 [h_1 \quad h_2 \quad h_3 \quad h_4]^T.$$

Since h_i have distribution CN(0, 1), $\lambda_{i,k}$ have distribution CN(0, 4). Now the PEP in (21) can be written as

$$P(\underline{d}, \underline{d}' \mid \widetilde{H}) = Q\left(\sqrt{\frac{\rho \sum_{i=1}^{2} \sum_{k=1}^{4} \beta_k^2 |\lambda_{i,k}|^2}{16}}\right). \tag{22}$$

Applying the Craig's formula for the conditional PEP in [23], one has

$$P(\underline{d}, \underline{d}' \mid \widetilde{H}) = \frac{1}{\pi} \int_0^{\pi/2} \exp\left(\frac{-\rho \sum_{i=1}^{2} \sum_{k=1}^{4} \beta_k^2 |\lambda_{i,k}|^2}{32 \sin^2 \alpha}\right) d\alpha \cdot \tag{23}$$

We can apply a method based on the moment generating function [24], [25] to obtain the unconditional PEP in the following:

$$P(\underline{d}, \underline{d}') = \frac{1}{\pi} \int_0^{\pi/2} \left[\prod_{i=1}^{4} \left(1 + \frac{\rho \beta_i^2}{8 \sin^2 \alpha}\right)\right]^{-2} d\alpha \cdot \tag{24}$$

Since there are four vectors to be decoded in each code matrix, the codeword PEP is therefore bounded by 4 times the PEP given in (24). Assume that there are σ possible vectors $\underline{d}$, the union bound on the frame error rate (FER) is

$$P_u = 4 \times \frac{2}{\sigma} \sum_{i=1}^{\sigma} \sum_{k=i+1}^{\sigma} P(\underline{d}^i, \underline{d}^k) \cdot \tag{25}$$

We now examine the tightness of the union bound (25) compared with the simulated FER. Recall that the signal rotation R plays an important role on the decoding performance of 4Gp-QSTBC. In [13], the symbols s_1, s_3, s_5, s_7 are rotated by and angle γ_1, and the other symbols are rotated by an angle γ_2. This type of complex signal rotations is equivalent to the real signal rotation, denoted by R_{YGT}, below.

$$R_{YGT} = \begin{bmatrix} \cos\gamma_1 & \sin\gamma_1 & 0 & 0 \\ \sin\gamma_1 & -\cos\gamma_1 & 0 & 0 \\ 0 & 0 & \cos\gamma_2 & \sin\gamma_2 \\ 0 & 0 & \sin\gamma_2 & -\cos\gamma_2 \end{bmatrix}. \tag{26}$$

For this class of rotation matrix and 4QAM, the values $\gamma_1 = 7^0$ and $\gamma_1 = 23^0$ maximize the coding gain [2]. In Fig. 1, the FER of 4Gp-STBC G with the best found rotation of the form in (26) is plotted for 4QAM and 16QAM. The union bound becomes tight at FER <10^{-2}.

The tight union bound at medium and high SNR suggests that this bound can be used to optimize the signal rotation R. In the most general case, the 4-by-4 orthogonal matrix R has no less than 4 independent entries. Therefore, an exhaustive search becomes impractical.

To overcome this problem, we can optimize R based on the asymptotic bound at high SNR. If $\beta_i \neq 0 \ \forall i = 1,\ldots,4$, then $1 + \rho\beta_i^2 / 8\sin^2\alpha \approx \rho\beta_i^2 / 8\sin^2\alpha$ at high SNR, the approximation of the exact PEP in (24) is

$$P(\underline{d},\underline{d}') \approx \frac{2^7 \rho^{-8} 16!}{8!8!} \prod_{i=1}^{4} |\beta|^{-4} . \tag{27}$$

The asymptotic bound in (27) unveils an important property of the 4Gp-QSTBC at high SNR: The PEP is heavily dependent on the product distance $\prod_{i=1}^{4} |\beta_i|$ (see, e.g. [21]). Recall that $\underline{\beta} = TR\underline{\delta}$; we can consider the product matrix TR is a combined rotation matrix for the data vector $\underline{d}$.

The exponent of SNR in (27) is -8. This indicates that the maximum diversity order of 4Gp-QSTBC is 8 and it is achievable if the product distance is non-zero for all possible data vectors. Furthermore, at high SNR, the asymptotic bound becomes very tight to the union bound and therefore, very tight to the FER. Therefore, the larger the product distance, the lower FER can be obtained. This observation is very similar to the diversity-coding gain concept due to Tarokh *et al.* [2]. Thus, we can optimize the rotation by R so that the minimum product distance

$$d_{p,\min} = \min_{\forall \underline{d} \neq \underline{d}'} \prod_{i=1}^{4} |\beta_i| \tag{28}$$

is nonzero and maximized.

Note that the searches for the best rotation matrix R based on the union bound (25) and the worst-case PEP (28) can be run independently. In addition, one can use the coding gain metric [2] to search for the matrix R [13]. The rotation matrix minimizing the union bound of FER should yield the lowest FER compared with the best rotation found by optimizing the worst-case PEP and coding gain. However, we are still searching for a method to optimize the union bound.

If the complex signals are drawn from QAM, the (real) elements of $\underline{d}$ are in the set $(\pm 1, \pm 3, \pm 5, \ldots)$. The best known rotations for QAM in terms of maximizing the minimum product distance are provided in [26],[27]. Denoting the signal rotation presented in [26],[27] by R_{BOV}, the signal rotation for our 4Gp-QSTBC is given by

$$R = TR_{BOV} \tag{29}$$

The FER of 4Gp-QSTBC with 16QAM using signal rotation in (29) and the best rotation in (26) (in terms of coding gain) are compared in Fig. 2. Clearly, the rotation in (29) performs better at high SNR.

IV. Conclusion

We have presented a new method to decode and optimize the performance of 4Gp-STBC. The new decoding method enables the application of sphere decoders to reduce the decoding complexity. Additionally, the equivalent channel of 4Gp-STBC is presented. This channel is used to derived the exact PEP and furthermore, to optimize the performance of 4Gp-STBC. The newly proposed signal rotation yields better performance than the existing signal rotation method.

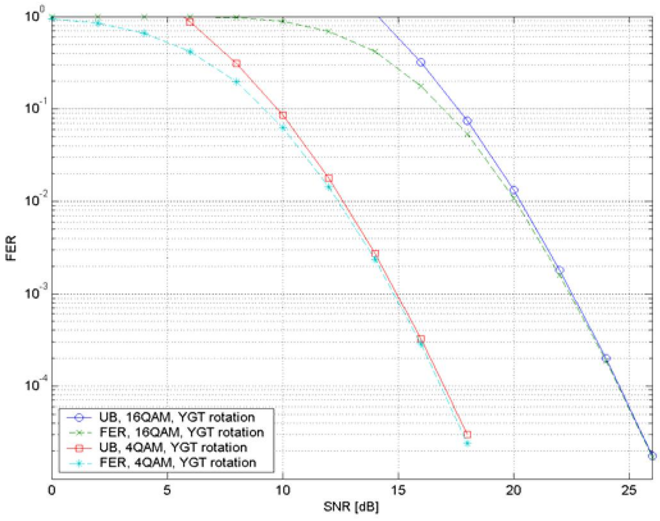

Figure 1. Union bound on the FER and simulated FER of 4Gp-STBC with 8 transmit/1 receive antennas.

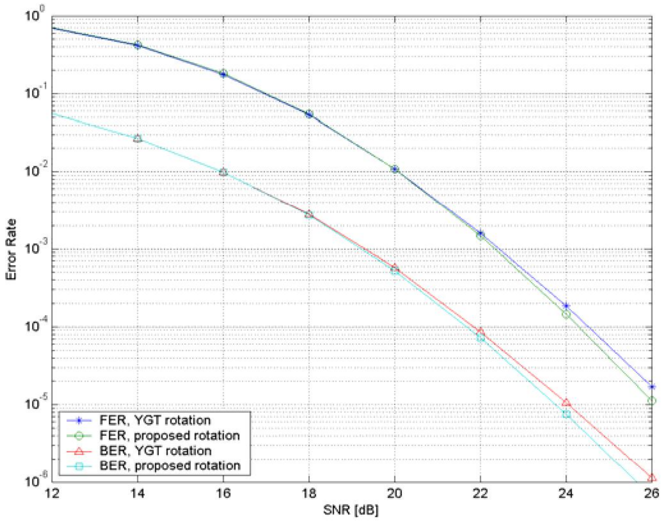

Figure 2. Comparison of the performance of 4Gp-STBC with the proposed signal rotation and the one proposed by Yuen, Guan, and Tjhung, 16QAM, 8 transmit/1 receive antennas.

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