Non-Linear Precoding for OFDM Systems in Spatially-Correlated Frequency-Selective Fading MIMO Channels

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***Abstract*— This paper presents non-linear precoding design in closed-loop multiple-input multiple-output (MIMO) orthogonal frequency-division multiplexing (OFDM) over spatially-correlated, frequency-selective fading channels. Our analysis takes into consideration receiver channel mismatch due to imperfect channel estimates, and transmitter channel mismatch due to estimation errors, channel variations over feedback delay and feedback noise. We present a general spatially-correlated, frequency-selective fading MIMO channel model and derive the conditional means of the channel response. Exploiting the channel statistics, which are only available at the receiver, we design new non-linear zero-forcing (ZF) Tomlinson-Harashima precoding (THP) for uncoded MIMO OFDM. The channel statistics do not need to be sent back to the transmitter, which avoids the possible maximum-Doppler-shift transmitter mismatch. Our proposed precoders are robust against time variations, channel estimation errors and antenna correlations, and offer a significant system performance gain over conventional THP.**

***Index Terms*— THP, MIMO, OFDM, frequency-selective fading**

I. Introduction

Multiple-input multiple-output (MIMO) wireless links, created through multiple antenna arrays at both the transmitter and the receiver, offer high data rates by exploiting spatial dimension of transmission in addition to time and frequency dimensions. Orthogonal frequency-division multiplexing (OFDM) enables spectrally efficient MIMO transmission on frequency-selective fading channels. If the channel state information (CSI) is available at the transmitter, precoding can help to simplify the receiver through interference mitigation and to improve the data rates or bit error rate (BER) performance. Non-linear Tomlinson-Harashima precoding (THP), originally proposed for temporal pre-equalization to mitigate intersymbol interference (ISI) in dispersive channels [1]–[3], has recently been extended to flat-fading (multiuser) MIMO systems in order to combat the interference between different spatial transmission layers [4], [5], and reduce intercarrier interference (ICI) in MIMO OFDM systems [6], [7].

If perfect CSI (instantaneous or long-term statistics) is available at the transmitter, minimum mean-square error (MMSE) or zero-forcing (ZF) THP outperforms its linear counterparts [8], [9]. The CSI can be either estimated at the transmitter in time-division duplex (TDD) systems where the forward and reverse links use the same channel carrier frequency, or estimated at the receiver and fed back to the transmitter. Perfect CSI requires accurate channel estimation and time-invariant channels. However, estimation errors introduce channel mismatch at both the transmitter and the receiver, i.e., the channel information which is available at the transmitter and the receiver differs from the actual channel at the time of transmission. The time variations and feedback errors lead to unbalanced CSI between the transmitter and receiver, and make CSI transmitter mismatch more serious. As the CSI mismatch becomes severe, the performance of a TH precoder is significantly degraded [10], [11].

The problem of imperfect and different CSI at the transmitter and the receiver has been addressed in [8], [11]–[13]. [8] optimizes THP using a conditional probability density function for channel parameters given outdated and noisy training sequences. In [11], an adaptive THP structure is proposed to combat the interference or channel variations. An MMSE-TH precoder with estimation errors and a feedback delay has been studied in single-antenna systems in [12]. A general system model is investigated in [13], where linear precoding is designed for MIMO OFDM in spatially-correlated frequency-selective fading channels with estimation errors at the receiver. In both [12] and [13], the transmitter and the receiver are assumed to know a priori statistical channel knowledge, i.e., maximum Doppler shift has to be perfectly available at both the transmitter and the receiver, which, however, may be difficult to obtain. In the TDD mode, a Doppler-shift estimate may not be readily available at the transmitter; if a Doppler-shift estimate is fed back to the transmitter, further errors may occur due to imperfect feedback.

In this paper, we develop a TH precoder for MIMO OFDM in spatially-correlated frequency-selective fading channels, when channel is estimated at the receiver. The receiver imperfectly estimates the channel impulse response, resulting in a channel mismatch at the receiver, and sends the erroneous channel estimates back to the transmitter via a feedback channel, which further introduces delay and noise. Consequently, the channel mismatch at the transmitter is caused by estimation errors, feedback noise, channel time-variations and feedback delay. In the system model considered in [13], however, the mismatch effect is only assumed due to an additive noise

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and the solution for correlated antennas is not provided. We consider a more general OFDM system model with erroneous channel estimates, multiplicative time-varying effects, noisy feedback, and transmit and receive antenna correlations. We derive the conditional expectation of the channel matrix and design a new TH precoder using a statistical model for estimation errors and time variations. The statistical channel model, which requires the maximum Doppler shift estimate, is only available at the receiver. Our design avoids the possible maximum-Doppler-shift transmitter mismatch present in [12] and [13], and reduces feedback requirements. Our precoder also significantly reduces system degradation due to channel mismatch at both the transmitter and the receiver, and is robust on fast time-variant channels.

II. System Description

This section will introduce an N-subcarrier OFDM with M_T transmit antennas and M_R receive antennas in spatially-correlated frequency-selective fading MIMO channels. We then describe THP with perfect CSI at both the transmitter and the receiver.

A. System Model

We assume that the channel for each transmit-receive antenna pair has L resolvable paths. At the time i, the l-th path gain can be represented by an $M_R \times M_T$ complex-valued random matrix $\mathbf{h}(l,i)$ with entries $h_{u,v}(l,i)$, $u \in [1, M_T]$ and $v \in [1, M_R]$, which has a fixed complex component $\bar{\mathbf{h}}(l)$ and a zero-mean Gaussian random component $\widetilde{\mathbf{h}}(l,i)$ with time-invariant variance σ_l^2,

$$\mathbf{h}(l,i) = \bar{\mathbf{h}}(l) + \widetilde{\mathbf{h}}(l,i), \quad l = 0, \ldots, L-1, \tag{1}$$

where the specular component $\bar{\mathbf{h}}(l)$ is defined by (6) in [14]; the channel is Rayleigh fading if $\bar{\mathbf{h}}(l) = \mathbf{0}$, $\forall\, l$. The impact of transmit and receive antenna correlation is modeled as in [14]

$$\widetilde{\mathbf{h}}(l,i) = \sigma_l \mathbf{r}_R \widetilde{\mathbf{h}}_w(l,i) \mathbf{r}_T, \tag{2}$$

where $\widetilde{\mathbf{h}}_w(l,i)$ is an $M_R \times M_T$ matrix with independent and identically distributed (i.i.d.) zero-mean and unity-variance complex Gaussian random variables; $\mathbf{r}_T = \sqrt{\mathbf{R}_T}$ and $\mathbf{r}_R = \sqrt{\mathbf{R}_R}$. As in [15], the $M_T \times M_T$ $\mathbf{R}_T$ and $M_R \times M_R$ $\mathbf{R}_R$ are the transmit and receive antenna correlation matrices with entries

$$\begin{aligned} R_T(m,n) &= \mathcal{J}_0\left(2\pi|m-n|\zeta_T\right), \\ R_R(m,n) &= \mathcal{J}_0\left(2\pi|m-n|\zeta_R\right), \end{aligned} \tag{3}$$

where $\mathcal{J}_0$ is a zero-order Bessel function of the first kind. $\zeta_T = \Delta\frac{d_T}{\lambda}$ and $\zeta_R = \frac{d_R}{\lambda}$ is the normalized distance between the receive antennas; $\lambda = c/f_c$ is the wavelength of a narrow-band signal with center frequency f_c, Δ is the angle spread, and the transmit and receive antennas are spaced by d_T and d_R, respectively.

After reception and discarding the cyclic prefix, the channel matrix for the k-th subcarrier in a MIMO OFDM system can be given as in [14] and [16],

$$\begin{aligned} \mathbf{H}[k,i] &= \sum_{l=0}^{L-1} \mathbf{h}(l,i) e^{-\jmath\frac{2\pi}{N}kl} \\ &= \bar{\mathbf{h}}\left(\mathbf{F}[k] \otimes \mathbf{I}_{M_T}\right) + \mathbf{r}_R \widetilde{\mathbf{h}}_w(i) \mathbf{r}_T[k],. \end{aligned} \tag{4}$$

where $\bar{\mathbf{h}} = \left[\bar{\mathbf{h}}(0), \ldots, \bar{\mathbf{h}}(L-1)\right]$ and $\widetilde{\mathbf{h}}_w(i) = \left[\widetilde{\mathbf{h}}_w(0,i), \ldots, \widetilde{\mathbf{h}}_w(L-1,i)\right]$ are $M_R \times M_T L$ matrices. $\mathbf{F}[k] = \left[e^{-\jmath\frac{2\pi}{N}k0} \ldots e^{-\jmath\frac{2\pi}{N}k(L-1)}\right]^T$ is an L-dimensional vector. $\mathbf{r}_T[k] = \mathbf{r}_P^T \mathbf{F}[k] \otimes \mathbf{r}_T$ is an $M_T L \times M_T$ matrix with $\mathbf{r}_P = \operatorname{diag}\left[\sigma_0, \ldots, \sigma_{L-1}\right]$; $\mathbf{r}_P = \sqrt{\mathbf{R}_P}$ and $\mathbf{R}_T[k] = \mathbf{r}_T^H[k]\mathbf{r}_T[k]$.

Let $X_u[k,i]$ be the M-ary QAM symbol sent by the u-th transmit antenna on the subcarrier k of the i-th OFDM symbol. The received signal vector on the k-th subcarrier of the i-th OFDM symbol can be given by

$$\mathbf{Y}[k,i] = \mathbf{H}[k,i]\mathbf{X}[k,i] + \mathbf{W}[k,i], \tag{5}$$

where $\mathbf{Y}[k,i]$ is an M_R-dimensional vector. The $\mathbf{W}[k,i]$ is the noise vector where the entries $W_v[k,i] = \sum_{u=1}^{M_T} W_{u,v}[k,i]$ are additive white Gaussian noise (AWGN) samples with zero mean and variance σ_W^2, and $W_{u,v}[k,i]$, $\forall\, k,\, i$, are assumed i.i.d.. Stacking all the received signals, we get

$$\mathbf{Y}[i] = \mathbf{H}[i]\mathbf{X}[i] + \mathbf{W}[i], \tag{6}$$

where $\mathbf{Y}[i] = \left[\mathbf{Y}^T[0,i] \ldots \mathbf{Y}^T[N-1,i]\right]^T$ and $\mathbf{X}[i] = \left[\mathbf{X}^T[0,i] \ldots \mathbf{X}^T[N-1,i]\right]^T$; the $NM_R \times NM_T$ channel matrix $\mathbf{H}[i]$ is given by

$$\mathbf{H}[i] = \operatorname{diag}\left[\mathbf{H}[0,i] \ldots \mathbf{H}[N-1,i]\right]. \tag{7}$$

The model in [12] is a special case only when $N = 1$ and $M_T = M_R = 1$. In this paper, we consider the structure described by (5). MMSE or ZF THP can therefore be designed individually for each subcarrier.

B. Tomlinson-Harashima Precoding with Perfect CSI

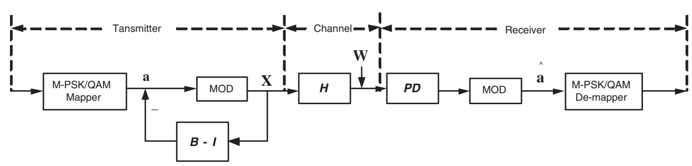

Fig. 1. Tomlinson-Harashima precoder in a MIMO OFDM link.

This subsection briefly reviews the operation and matrix design of THP with perfect CSI at both the transmitter and the receiver. For simplicity, in this subsection we omit the time index of the matrices. We assume that $M_R \geq M_T$ for convenience of signal detection.

The structure of a TH precoder in MIMO OFDM links is depicted in Fig. 1. It consists of a feedforward matrix $\mathbf{D}$, a scaling matrix $\mathbf{P}$ and a modulo arithmetic device at the receiver and a modulo arithmetic feedback structure $\mathbf{B}$ at the transmitter. Given the channel matrix at both the transmitter and the receiver, we design the feedforward filter at the receiver and the feedback filter at the transmitter separately. A QR factorization of the channel matrix yields

$$\mathbf{H}[k] = \mathbf{D}^H[k]\mathbf{T}[k], \tag{8}$$

where the feedforward matrix $\mathbf{D}[k]$ is an $M_T \times M_R$ unitary matrix; $\mathbf{D}[k]\mathbf{D}^H[k] = \mathbf{I}_{M_T}$. The $\mathbf{T}[k] = \left[T_k(m,n)\right]$ is an $M_T \times M_T$ upper triangular matrix. An $M_T \times M_T$ diagonal

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scaling matrix at the receiver is given by $\mathbf{P}[k] = \left[1/T_k(n,n)\right]$. The feedback matrix at the transmitter $\mathbf{B}[k] = \mathbf{P}[k]\mathbf{T}[k]$ pre-equalizes the channel matrix and the feedforward filter according to the ZF criterion. The MMSE design and detailed discussion of THP can be found in [5]. The overall precoding matrices can be written as $\mathbf{B} = \text{diag}\left[\mathbf{B}[0]\ldots\mathbf{B}[N-1]\right]$, $\mathbf{D} = \text{diag}\left[\mathbf{D}[0]\ldots\mathbf{D}[N-1]\right]$, and $\mathbf{P} = \text{diag}\left[\mathbf{P}[0]\ldots\mathbf{P}[N-1]\right]$.

The modulo reduction $2\sqrt{M}$ at the transmitter is applied separately to the real and imaginary parts of the input for $\Re\left\{\text{MOD}_{2\sqrt{M}}(X)\right\} \in \left(-\sqrt{M},\sqrt{M}\right]$ and $\Im\left\{\text{MOD}_{2\sqrt{M}}(X)\right\} \in \left(-\sqrt{M},\sqrt{M}\right]$, such that a transmitted signal has constrained power. At the receiver, the transmitted data symbols are recovered using the same modulo operation as that at the transmitter. If the input sequence $a[k]$ is a sequence of i.i.d. samples, the output of the modulo device is also a sequence of i.i.d. random variables, and the real and imaginary parts are independent [5]. We therefore can assume $\text{E}\left[\mathbf{X}[k]\mathbf{X}^H[k]\right] = E_s\mathbf{I}_{M_T}$, $\forall\, k$.

Note that the THP design is based on the assumption that both the transmitter and receiver have high-quality channel information such that the feedback filter $\mathbf{B}$ can pre-equalize the cascade $\mathbf{PDH}$. In a practical system, the transmitter and the receiver have imperfect CSI due to time variations, feedback noise and estimation errors, which introduces residual interference. The non-zero interference limits the output signal-to-interference-plus-noise power ratio and severely degrades system's performance [10], [11].

III. Tomlinson-Harashima Precoder with Channel Mismatch

In this section, we propose a TH precoder with transmitter and receiver channel mismatch in a spatially-correlated frequency-selective fading MIMO channel. For simplicity, we only consider ZF THP. In our case, the receiver has erroneous channel estimates $\mathbf{H}_R[k,i]$ of the current actual, but unknown, channel $\mathbf{H}[k,i]$; the imperfect channel estimates are sent to the transmitter via a feedback channel which introduces noise and delay $\tau_{u,v}$. In principle, each transmit-receive antenna pair may have different time delay if the collocated antennas do not share the same oscillator. Consequently, the transmitter has the noise-corrupted estimate $\mathbf{H}_T[k,i-\tau]$ of the actual (unknown) but outdated channel matrix $\mathbf{H}[k,i-\tau]$, in which the $\{u,v\}$th channel is $\tau_{u,v}$ seconds older than that in the current channel matrix $\mathbf{H}[k,i]$. The actual channel matrices $\mathbf{H}[k,i]$ and $\mathbf{H}[k,i-\tau]$ are unknown at both the transmitter and the receiver. Therefore, due to estimation errors, the receiver possesses an imperfect channel matrix $\mathbf{H}_R[k,i]$, while the transmitter has an erroneous estimate $\mathbf{H}_T[k,i-\tau]$ of an outdated channel matrix due to estimation errors, channel time-variations and feedback noise.

A. Channel Statistics

In this subsection, we give a statistical channel model for the current actual channel $\mathbf{H}[k,i]$, outdated channel $\mathbf{H}[k,i-\tau]$ and their estimates $\mathbf{H}_R[k,i]$ and $\mathbf{H}_T[k,i-\tau]$. The $\text{vec}(\mathbf{A})$ is a vectorization of the matrix $\mathbf{A}$, and is denoted as $\vec{\mathbf{A}}$.

Without loss of generality, we build up the time-variant channel model as follows:

- The tap vector for the $\{u,v\}$th antenna pair $\mathbf{h}_{u,v}(i) = \left[h_{u,v}(0,i),\ldots,h_{u,v}(L-1,i)\right]^T$ has the same statistical distribution with $\mathbf{h}_{u',v'}(i)$, $\forall\ i,\ u\neq u',\ v\neq v'$. The taps are time-varying according to Clarke's 2-D isotropic scattering model with maximum Doppler shift $f_{D_{u,v}}$ [17]. Since $\mathbf{h}_{u,v}(i-\tau_{u,v})$ is a delayed version of $\mathbf{h}_{u,v}(i)$, they are jointly Gaussian with an autocovariance matrix

$$\text{E}\left[\left(\mathbf{h}_{u,v}(i)-\bar{\mathbf{h}}_{u,v}\right)\left(\mathbf{h}_{u,v}(i-\tau_{u,v})-\bar{\mathbf{h}}_{u,v}\right)^H\right] = J_{u,v}\mathbf{R}_P, \tag{9}$$

where $J_{u,v} = \mathcal{J}_0(2\pi\epsilon_{u,v})$, and $\epsilon_{u,v} = f_{D_{u,v}}\tau_{u,v}$; $f_{D_{u,v}}$ is the maximum Doppler shift of the $\{u,v\}$th channel. Here, we consider the most general case of $\epsilon_{u',v'}\neq\epsilon_{u,v}$, $\forall\ u'\neq u,\ v'\neq v$. The time-varying channel can hence be written as

$$\mathbf{h}_{u,v}(i) = \bar{\mathbf{h}}_{u,v} + J_{u,v}\widetilde{\mathbf{h}}_{u,v}(i-\tau_{u,v}) + \mathbf{g}_{u,v}, \tag{10}$$

where $\mathbf{g}_{u,v}$ has entries $g_{u,v}\sim\mathcal{CN}(0,\Omega_{g_{u,v}})$, and $\mathbf{\Omega}_{g_{u,v}} = \left[\Omega_{g_{u,v}}\right] = (1-J_{u,v}^2)\mathbf{R}_P$; $\mathbf{g}_{u,v}$ is independent of $\mathbf{h}_{u,v}(i)$, $\forall\ i,u,v$.

- The channel matrix $\mathbf{H}[k,i]$ satisfies

$$\begin{aligned}\mathbf{C}_{\vec{\mathbf{H}}\vec{\mathbf{H}}}[k] &= \text{E}\left[\text{vec}\left(\mathbf{H}[k,i]-\bar{\mathbf{H}}[k,i]\right)\text{vec}\left(\mathbf{H}[k,i]-\bar{\mathbf{H}}[k,i]\right)^H\right]\\ &= \text{E}\left[\vec{\widetilde{\mathbf{H}}}[k,i]\vec{\widetilde{\mathbf{H}}}^H[k,i]\right] = \mathbf{R}_T^T[k]\otimes\mathbf{R}_R.\end{aligned} \tag{11}$$

The channel estimates at the receiver are maximum likelihood (ML) estimates and can be expressed as

$$\mathbf{H}_R[k,i] = \mathbf{H}[k,i] + \mathbf{e}[k], \tag{12}$$

where $\mathbf{e}[k]$ is the estimation error with entries $e_{u,v}[k]\sim\mathcal{CN}(0,\Omega_{e_{u,v}})$, $\forall\, i,k$. The covariance of $\vec{\mathbf{H}}_R[k,i]$ hence is

$$\mathbf{C}_{\vec{\mathbf{H}}_\mathbf{R}\vec{\mathbf{H}}_\mathbf{R}}[k] = \text{E}\left[\vec{\widetilde{\mathbf{H}}}_R[k,i]\vec{\widetilde{\mathbf{H}}}_R^H[k,i]\right] = \mathbf{R}_T^T[k]\otimes\mathbf{R}_R + \text{diag}\left(\vec{\mathbf{\Omega}}_e\right), \tag{13}$$

where $\mathbf{\Omega}_e = \left[\Omega_{e_{u,v}}\right]$ is the $M_R\times M_T$ estimation-error matrix; $\vec{\mathbf{\Omega}}_e = \text{vec}(\mathbf{\Omega}_e)$. And the covariance of the $\vec{\mathbf{H}}[k,i]$ and $\vec{\mathbf{H}}_R[k,i]$ is

$$\mathbf{C}_{\vec{\mathbf{H}}\vec{\mathbf{H}}_\mathbf{R}}[k] = \text{E}\left[\vec{\widetilde{\mathbf{H}}}[k,i]\vec{\widetilde{\mathbf{H}}}_R^H[k,i]\right] = \mathbf{R}_T^T[k]\otimes\mathbf{R}_R. \tag{14}$$

- The transmitter channel matrix $\mathbf{H}_T[k,i-\tau]$ is a noise-corrupted estimate of the actual but unknown $\mathbf{H}[k,i-\tau]$, which can be modeled by

$$\mathbf{H}_T[k,i-\tau] = \mathbf{H}[k,i-\tau] + \mathbf{e}[k] + \mathbf{q}[k], \tag{15}$$

where $\mathbf{q}[k]$ is feedback noise with entries $q_{u,v}[k]\sim\mathcal{CN}(0,\Omega_{q_{u,v}})$, $\forall\, i,k$. The $\mathbf{e}[k]$ and $\mathbf{q}[k]$ are independent of all other stochastic processes. Combined with (9), we obtain the covariance

$$\begin{aligned}\mathbf{C}_\tau[k] &= \text{E}\left[\vec{\widetilde{\mathbf{H}}}[k,i]\vec{\widetilde{\mathbf{H}}}^H[k,i-\tau]\right]\\ &= \mathbf{F}^T[k]\mathbf{R}_P\mathbf{F}^*[k]\left(\mathbf{r}_T^T\otimes\mathbf{r}_R\right)\text{diag}\left(\vec{\mathbf{J}}\right)\left(\mathbf{r}_T^*\otimes\mathbf{r}_R\right),\end{aligned} \tag{16}$$

where $\mathbf{J} = \left[J_{u,v}\right]$ is the $M_R\times M_T$ time-varying coefficient matrix. Obviously, the time variations have

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a multiplicative effect. A proof of (16) is given in the Appendix. Similarly, we can obtain $\mathbf{C}_{\vec{\mathbf{H}}_\mathbf{T}\vec{\mathbf{H}}_\mathbf{R}}[k] = \mathbf{C}_\tau[k] + \text{diag}\left(\vec{\boldsymbol{\Omega}}_e\right)$ and $\mathbf{C}_{\vec{\mathbf{H}}_\mathbf{T}\vec{\mathbf{H}}_\mathbf{T}}[k] = \mathbf{R}_T^T[k] \otimes \mathbf{R}_R + \text{diag}\left(\vec{\boldsymbol{\Omega}}_e\right) + \text{diag}\left(\vec{\boldsymbol{\Omega}}_q\right)$.

B. New THP Design

At the transmission time i, we design the non-linear TH precoder based on the receiver channel matrix $\mathbf{H}_R[k,i]$, which is the imperfect estimate of the current actual and unknown channel $\mathbf{H}[k,i]$, and the transmitter channel matrix $\mathbf{H}_T[k,i-\tau]$, which is a noise-corrupted estimate of $\mathbf{H}[k,i-\tau]$. The $\mathbf{H}[k,i]$ and $\mathbf{H}[k,i-\tau]$ are unknown at both the transmitter and the receiver, and $\mathbf{H}_R[k,i] \neq \mathbf{H}[k,i] \neq \mathbf{H}[k,i-\tau] \neq \mathbf{H}_T[k,i-\tau]$. The channel statistics knowledge, including the Doppler shift and error variances, is only available at the receiver.

Exploiting the above statistical channel model, the receiver can obtain the statistics of $\vec{\mathbf{H}}[k,i]$ given $\vec{\mathbf{H}}_R[k,i]$ with expectation and variance [18]

$$\begin{aligned}\mathbf{H}_{\vec{\mathbf{H}}|\vec{\mathbf{H}}_R}[k,i] &= \overline{\vec{\mathbf{H}}}[k] + \mathbf{C}_{\vec{\mathbf{H}}\vec{\mathbf{H}}_R}[k]\mathbf{C}^{-1}_{\vec{\mathbf{H}}_R\vec{\mathbf{H}}_R}[k]\tilde{\vec{\mathbf{H}}}_R[k,i] \\ \mathbf{C}_{\vec{\mathbf{H}}|\vec{\mathbf{H}}_R}[k] &= \mathbf{C}_{\vec{\mathbf{H}}\vec{\mathbf{H}}}[k] - \mathbf{C}_{\vec{\mathbf{H}}\vec{\mathbf{H}}_R}[k]\mathbf{C}^{-1}_{\vec{\mathbf{H}}_R\vec{\mathbf{H}}_R}[k]\mathbf{C}_{\vec{\mathbf{H}}_R\vec{\mathbf{H}}}[k].\end{aligned} \tag{17}$$

In our case, substituting (11), (13), and (14), the conditional expectation of $\vec{\mathbf{H}}[k,i]$ becomes

$$\begin{aligned}&\mathbf{H}_{\vec{\mathbf{H}}|\vec{\mathbf{H}}_R}[k,i] = \overline{\vec{\mathbf{H}}}[k,i] \\ &+ \left(\mathbf{R}_T^T[k] \otimes \mathbf{R}_R\right)\left[\mathbf{R}_T^T[k] \otimes \mathbf{R}_R + \text{diag}\left(\vec{\boldsymbol{\Omega}}_e\right)\right]^{-1}\tilde{\vec{\mathbf{H}}}_R[k,i].\end{aligned} \tag{18}$$

The conditional covariance which gives the degree of the channel uncertainty can be given by

$$\begin{aligned}&\mathbf{C}_{\vec{\mathbf{H}}|\vec{\mathbf{H}}_R}[k] = \\ &\left(\mathbf{R}^T[k] \otimes \mathbf{R}_R\right)\left[\mathbf{R}_T^T[k] \otimes \mathbf{R}_R + \text{diag}\left(\vec{\boldsymbol{\Omega}}_e\right)\right]^{-1}\text{diag}\left(\vec{\boldsymbol{\Omega}}_e\right).\end{aligned} \tag{19}$$

Similarly, we can obtain the conditional expectation of $\vec{\mathbf{H}}_T[k,i-\tau]$ given the receiver channel matrix $\mathbf{H}_R[k,i]$

$$\begin{aligned}&\mathbf{H}_{\vec{\mathbf{H}}_T|\vec{\mathbf{H}}_R}[k,i-\tau] = \overline{\vec{\mathbf{H}}}[k] \\ &+ \left[\mathbf{C}_\tau + \text{diag}\left(\vec{\boldsymbol{\Omega}}_e\right)\right]\left[\mathbf{R}_T^T[k] \otimes \mathbf{R}_R + \text{diag}\left(\vec{\boldsymbol{\Omega}}_e\right)\right]^{-1}\tilde{\vec{\mathbf{H}}}_R[k,i].\end{aligned} \tag{20}$$

The M_RM_T-dimensional vectors $\mathbf{H}_{\vec{\mathbf{H}}|\vec{\mathbf{H}}_R}[k,i]$ in (18) and $\mathbf{H}_{\vec{\mathbf{H}}_T|\vec{\mathbf{H}}_R}[k,i-\tau]$ in (20) can be reshaped to the $M_R \times M_T$ conditional channel matrices $\mathbf{H}_{\mathbf{H}|\mathbf{H}_R}[k,i]$ and $\mathbf{H}_{\mathbf{H}_T|\mathbf{H}_R}[k,i-\tau]$.

The conditional channel matrices $\mathbf{H}_{\mathbf{H}|\mathbf{H}_R}[k,i]$ and $\mathbf{H}_{\mathbf{H}_T|\mathbf{H}_R}[k,i-\tau]$ can be seen as equivalent channels exploiting the channel statistics and uncertainty structure to mitigate the impact of CSI mismatch. They are calculated at the receiver given $\mathbf{H}_R[k,i]$ and are unknown at the transmitter. At the transmitter, the feedback matrix $\mathbf{B}_T[k,i-\tau] = \mathbf{P}_T[k,i-\tau]\mathbf{T}_T[k,i-\tau]$ is constructed with the transmitter channel matrix $\mathbf{H}_T[k,i-\tau]$, where $\mathbf{T}_T[k]$ is an upper triangular matrix obtained by the QR factorization of $\mathbf{H}_T[k,i-\tau]$. Since through (20) the receiver knows the matrix $\mathbf{H}_{\mathbf{H}_T|\mathbf{H}_R}[k,i-\tau]$, which is the equivalent channel of the transmitter channel matrix $\mathbf{H}_T[k,i-\tau]$, we therefore can estimate $\mathbf{B}_T[k,i-\tau]$ at the receiver,

$$\mathbf{B}_{TR}[k,i-\tau] = \mathbf{P}_{TR}[k,i-\tau]\mathbf{T}_{TR}[k,i-\tau], \tag{21}$$

where $\mathbf{T}_{TR}[k,i-\tau]$ is an upper triangular matrix obtained by the QR factorization of $\mathbf{H}_{\mathbf{H}_T|\mathbf{H}_R}[k,i-\tau]$. Under the ZF criterion, the feedforward matrix can be designed as

$$\begin{aligned}\mathbf{D}_R[k,i] &= \mathbf{T}_{TR}[k,i-\tau]\mathbf{H}^\dagger_{\mathbf{H}|\mathbf{H}_R}[k,i] = \mathbf{T}_{TR}[k,i-\tau]\times \\ &\mathrm{E}^{-1}\left(\mathbf{H}^H[k,i]\mathbf{H}[k,i] \mid \mathbf{H}_R[k,i]\right)\mathbf{H}^H_{\mathbf{H}|\mathbf{H}_R}[k,i],\end{aligned} \tag{22}$$

where $\mathrm{E}\left(\mathbf{H}^H[k,i]\mathbf{H}[k,i] \mid \mathbf{H}_R[k,i]\right)$ is an $M_T \times M_T$ matrix with entries $C(m,n) = \sum_{v=1}^{M_R} C'((n-1)M_R+v,(m-1)M_R+v)$, where the $C'(j,p)$ is the $\{j,p\}$th entry in $\mathbf{C}_{\vec{\mathbf{H}}\vec{\mathbf{H}}^H|\vec{\mathbf{H}}_R}[k]$,

$$\begin{aligned}\mathbf{C}_{\vec{\mathbf{H}}\vec{\mathbf{H}}^H|\vec{\mathbf{H}}_R}[k] &= \mathrm{E}\left(\vec{\mathbf{H}}[k,i]\vec{\mathbf{H}}^H[k,i] \mid \mathbf{H}_R[k,i]\right) \\ &= \mathbf{C}_{\vec{\mathbf{H}}|\vec{\mathbf{H}}_R}[k] + \mathbf{H}_{\vec{\mathbf{H}}|\vec{\mathbf{H}}_R}[k]\mathbf{H}^H_{\vec{\mathbf{H}}|\vec{\mathbf{H}}_R}[k].\end{aligned} \tag{23}$$

After modulo reduction at the receiver, the end-to-end cascade channel is

$$\begin{aligned}&\mathbf{P}_{TR}[k,i-\tau]\mathbf{D}_R[k,i]\mathbf{H}[k,i]\mathbf{B}_T^{-1}[k,i-\tau] = \\ &\mathbf{P}_{TR}[k,i-\tau]\mathbf{T}_{TR}[k,i-\tau]\mathbf{H}^\dagger_{\mathbf{H}|\mathbf{H}_R}[k,i]\mathbf{H}[k,i] \\ &\times \mathbf{T}_T^{-1}[k,i-\tau]\mathbf{P}_T^{-1}[k,i-\tau],\end{aligned} \tag{24}$$

where the matrix $\mathbf{H}_{\mathbf{H}|\mathbf{H}_R}[k,i]$ is an equivalent channel matrix of the actual $\mathbf{H}[k,i]$, $\mathbf{P}_{TR}[k,i-\tau]$ and $\mathbf{T}_{TR}[k,i-\tau]$ are the predicted matrices of $\mathbf{P}_T[k,i-\tau]$ and $\mathbf{T}_T[k,i-\tau]$ at the receiver, respectively. As the quality of CSI approaches perfect, (24) approaches $\mathbf{I}_{M_T}$.

In our THP design, the receiver calculates the equivalent channels based on the channel statistics, estimates the transmitter feedback matrix, and designs the feedforward matrix according to the equivalent channel matrices. The channel statistics are not needed at the transmitter. This avoids the impact of feedback errors on Doppler shift and the resulting further degradation.

IV. Simulation Results

We show how the new TH precoder mitigates the effect of a spatially-correlated frequency-selective fading environment. A 64-subcarrier QPSK OFDM system is considered. We consider time variations in terms of the normalized Doppler shift $\epsilon_{u,v}$ in the interval of $[-0.15, 0.15]$, where values of $\epsilon_{u,v}$ are assumed to be uniformly distributed. The maximum possible number of distinct normalized Doppler shift values is $M_T \times M_R$. In this interval, the time-variation coefficient $J_{u,v} \in [1, 0.79)$. The variance of estimation errors can be controlled around $\Omega_e = \frac{\sigma_W^2}{10}$. The angle of arrival spread is $12°$, i.e., $\Delta \approx 0.2$. Since the feedback channel capacity is usually much smaller than that of the downlink traffic channel, we assume the noise variance of the feedback link to be $\Omega_q = \frac{\sigma_W^2}{100}$.

Fig. 2 and 3 give the BER as a function of signal-to-noise power ratio (SNR) for different values of the normalized

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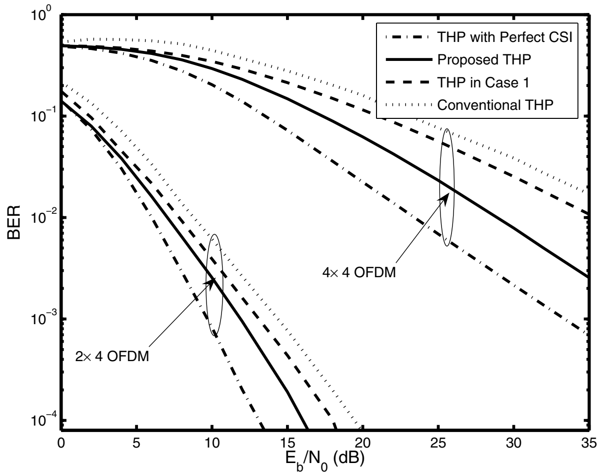

Fig. 2. BER with proposed THP as a function of the SNR for 2×4 and 4×4 64-subcarrier QPSK OFDM systems in transmit-antenna-correlated Rayleigh fading channels. $\zeta_T = 0.25$.

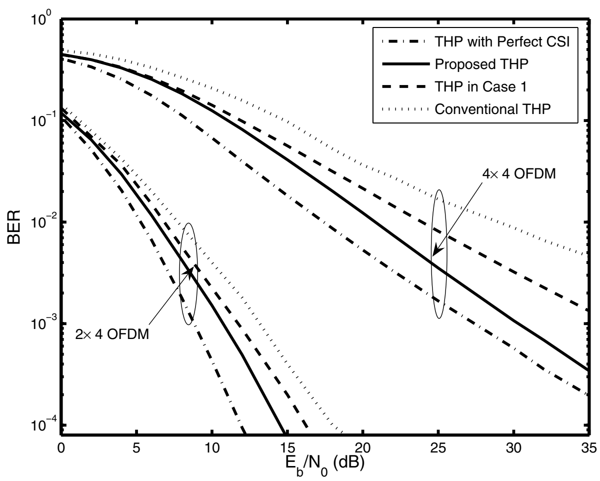

Fig. 3. BER with proposed THP as a function of the SNR for 2×4 and 4×4 64-subcarrier QPSK OFDM systems in transmit-antenna-correlated Rayleigh fading channels. $\zeta_T = 0.4$.

Doppler shift in 2×4 and 4×4 OFDM systems. A Rayleigh fading channel with only transmit antenna correlations is considered. The vehicular B channel specified by ITU-R M. 1225 [19], is used where the channel taps are zero-mean complex Gaussian random processes with variances of $-4.9\,$dB, $-2.4\,$dB, $-15.2\,$dB, $-12.4\,$dB, $-27.6\,$dB, and $-18.4\,$dB relative to the total power gain. [12] and [13] assume the maximum Doppler shift information at the transmitter, which may have possible Doppler-shift mismatch. The performance of ZF-THP with the maximum Doppler shift known at the transmitter is shown as a reference (Case 1). The BERs of conventional THP with channel mismatch and THP with perfect CSI at both the transmitter and the receiver are also shown for comparison. Obviously, THP in Case 1 is sensitive to the imperfect knowledge of Doppler shift at the transmitter. Our TH precoder thus significantly outperforms THP in Case 1, if the feedback link is noisy. The proposed THP improves the system performance significantly compared with conventional THP. Furthermore, as the transmit antenna spacing increases (the spatial correlation is weaker), the BER improvements increase.

In Fig. 4, a Ricean fading channel with only transmit antenna correlations is considered. The Ricean channel specified by ITU-R M. 1225 [19], is used where the channel taps are complex Gaussian random processes with variances of $-0.44\,$dB, $-14.42\,$dB, $-15.74\,$dB, $-16.52\,$dB, and $-19.51\,$dB relative to the total power gain. The first tap is Ricean fading with the Rice factor of $3\,$dB. Similarly, our THP outperforms THP in Case 1, where the transmitter needs to know Doppler-shift information. Since the transmitter Doppler-shift mismatch introduces uncertainty at the transmitter, THP in Case 1 has only marginal BER gain over conventional THP.

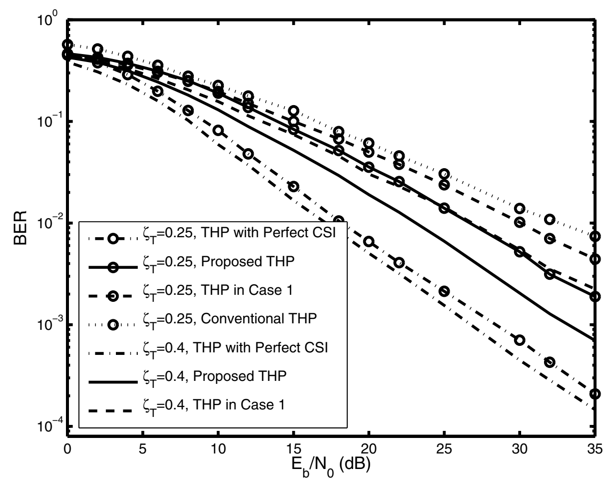

Fig. 4. BER with proposed THP as a function of the SNR for 2×4 and 4×4 64-subcarrier QPSK OFDM systems in transmit-antenna-correlated Ricean fading channels. $\zeta_T = 0.25$ and $\zeta_T = 0.4$.

In Fig. 5, both transmit and receive antenna correlations are considered; $\zeta_T = 0.25$ and $\zeta_R = 0.4$. The proposed THP has substantial BER gain over conventional THP. Our THP in spatially-correlated Ricean channels perform better than in Rayleigh channels. However, the BER gap between THP with perfect CSI and our THP in Ricean channels is larger than that in Rayleigh channels, i.e., Ricean fading channels have more detrimental impact on imperfect-CSI THP than Rayleigh channels.

V. Conclusion

We have proposed a new TH precoder for OFDM in spatially-correlated frequency-selective fading MIMO channels. The MIMO receiver estimates the channel and conveys an imperfect estimate of channel response to the transmitter through a feedback link introducing delay and noise. Consequently, the receiver mismatch is caused by imperfect estimates, and the transmitter mismatch is introduced by time variations, estimation errors and feedback noise. We have exploited statistical knowledge of the channel at the receiver for the new THP design. Our THP avoids the possible Doppler

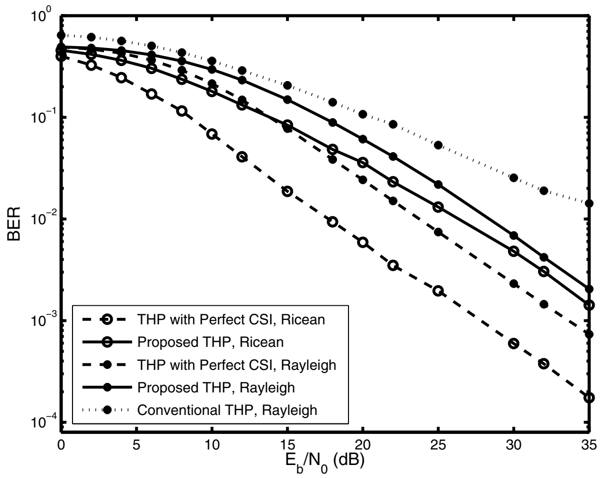

Fig. 5. BER with proposed THP as a function of the SNR for 4×4 64-subcarrier QPSK OFDM systems in spatially-correlated Rayleigh and Ricean fading channels. $\zeta_T = 0.25$ and $\zeta_R = 0.4$.

shift transmitter mismatch due to imperfect feedback links and considerably improves the system performance.

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VI. Appendix

Proof of (16). Note that $\mathrm{vec}(\mathbf{ABC}) = (\mathbf{C}^T \otimes \mathbf{A})\mathrm{vec}(\mathbf{B})$.

$$\begin{aligned}\mathbf{C}_\tau[k] &= \mathrm{E}\left[\overrightarrow{\widetilde{\mathbf{H}}}[k,i]\overrightarrow{\widetilde{\mathbf{H}}}^H[k,i-\tau]\right] \\ &= \left(\mathbf{r}_T^T[k] \otimes \mathbf{r}_R\right) \mathrm{E}\left[\overrightarrow{\widetilde{\mathbf{h}}_w}(i)\overrightarrow{\widetilde{\mathbf{h}}_w}^H(i-\tau)\right]\left(\mathbf{r}_T^*[k] \otimes \mathbf{r}_R\right),\end{aligned} \tag{25}$$

where the vector can be given by

$$\overrightarrow{\widetilde{\mathbf{h}}_w}(i) = \left[\tilde{h}_{w_{1,1}}(0,i) \ \dots \ \tilde{h}_{w_{M_T,M_R}}(0,i) \dots \tilde{h}_{w_{M_T,M_R}}(L-1,i)\right]^T. \tag{26}$$

Since $\widetilde{h}_{w_{u,v}}(l,i)$ are i.i.d. zero-mean and unit-variance complex Gaussian random variables, the $LM_RM_T \times LM_RM_T$ auto-correlation matrix of (26) is

$$\begin{aligned}&\mathrm{E}\left[\overrightarrow{\widetilde{\mathbf{h}}_w}(i)\overrightarrow{\widetilde{\mathbf{h}}_w}^H(i-\tau)\right] \\ &= \mathrm{diag}\left(\underbrace{J_{1,1} \ \dots \ J_{M_T,M_R}}_{l=0} \ \cdots \ \underbrace{J_{1,1} \ \dots \ J_{M_T,M_R}}_{l=L-1}\right) \\ &= \mathbf{I}_L \otimes \mathrm{diag}\left(J_{1,1} \ \dots \ J_{1,M_R} \ \dots \ J_{M_T,M_R}\right) = \mathbf{I}_L \otimes \mathrm{diag}\left(\overrightarrow{\mathbf{J}}\right).\end{aligned} \tag{27}$$

Substituting (27) into (25), we have

$$\begin{aligned}\mathbf{C}_\tau[k] &= \left(\mathbf{r}_T^T[k] \otimes \mathbf{r}_R\right)\left(\mathbf{I}_L \otimes \mathrm{diag}\left(\overrightarrow{\mathbf{J}}\right)\right)\left(\mathbf{r}_T^*[k] \otimes \mathbf{r}_R\right) \\ &= \left(\left(\mathbf{F}^T[k]\mathbf{r}_P\right) \otimes \mathbf{r}_T^T \otimes \mathbf{r}_R\right)\left(\mathbf{I}_L \otimes \mathrm{diag}\left(\overrightarrow{\mathbf{J}}\right)\right) \\ &\times \left(\left(\mathbf{r}_P^H\mathbf{F}^*[k]\right) \otimes \mathbf{r}_T^* \otimes \mathbf{r}_R\right) \\ &= \mathbf{F}^T[k]\mathbf{R}_P\mathbf{F}^*[k]\left(\mathbf{r}_T^T \otimes \mathbf{r}_R\right)\mathrm{diag}\left(\overrightarrow{\mathbf{J}}\right)\left(\mathbf{r}_T^* \otimes \mathbf{r}_R\right).\end{aligned} \tag{28}$$

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