Performance Analysis and Optimal Signal Designs for Minimum Decoding Complexity ABBA Codes

Dũng Ngọc Đào and Chintha Tellambura,
Department of Electrical and Computer Engineering, University of Alberta
Edmonton, Alberta T6G 2V4, Canada
Email: {dndung, chintha}@ece.ualberta.ca, Telephone: (1-780) 492 7228, Fax: (1-780) 492 1811

***Abstract*— ABBA codes, a class of quasi-orthogonal space-time block codes (STBC) proposed by Tirkkonen *et al.*, have been studied extensively for various applications. Yuen *et al.* have recently shown that a refined version of ABBA codes admit pair-wise real-symbol decoding, i.e. with minimum decoding complexity (MDC) achievable by non-orthogonal STBC. In this paper, we derive the exact symbol pair-wise error probability and the union bound on the symbol error rate (SER). The union bound is only 0.1 dB from the simulated SER at medium or high SNR < 10^{-2}. Thus, by minimizing the SER union bound, we can find the optimal signal designs for any constellation with an arbitrary geometrical shape. Furthermore, we propose a new method combining signal rotation and power allocation for inphase-quadrature power-unbalanced constellations such as rectangular QAM. Our new optimal signal designs perform better than the existing ones and offer lower encoding/decoding complexities.**

I. Introduction

ABBA codes [1], a class of quasi-orthogonal space-time block codes (QSTBC), have been proposed to increase the code rate of orthogonal space-time block codes (OSTBC) [2]. Since ABBA QSTBC have low complexity pair-wise complex-symbol decoding and performs better than OSTBC [3], they have been widely studied for coherent and non-coherent transmissions, beamforming, and others [4].

Recently, Yuen *et al.* [5], [6] have shown that the ABBA codes also enable pair-wise real-symbol (PWRS) decoding, which is the *minimum decoding complexity* (MDC) achievable by non-orthogonal space-time block codes (STBC); they call such codes MDC codes. Thus, while their code rate is higher than that of OSTBC, their decoding complexity is equal to single complex symbol decoding.

To design MDC-ABBA codes with full-diversity, conventional quadrature amplitude modulation (QAM) or phase-shift keying (PSK) signals need to be transformed [5]–[7]. The authors in [5]–[7] employ the coding gain metric [8] to derive the optimal signal transformation for QAM and 8PSK. Their analytical results are reported for QAM only. However, maximizing the coding gain is in fact equivalent to minimizing the worst-case codeword pair-wise error probability (CPEP); this provides no guarantee for minimizing symbol error rate (SER). In general, how to find the optimal signal designs for QAM, PSK, and other constellation with good minimum Euclidean distance such as lattice of equilateral triangular (TRI) (also called hexagonal (HEX)) or amplitude PSK (APSK) [9] in terms of minimal SER is still an open problem.

In this paper, we solve the problem of optimal signal designs in the minimal SER sense. The exact symbol pair-wise error probability (SPEP) and union bound on the SER are derived. For all the examined constellations, the SER union bound is only 0.1 dB from the simulated SER when SER $< 10^{-2}$. Thus, the union bound can be used to precisely predict the performance of MDC-ABBA codes and, moreover, to optimize the signal designs for any constellation. Furthermore, for the constellations with inphase-quadrature power-imbalances like rectangular QAM (QAM-R), we propose a new method combining signal rotation and power allocation. Our new signal designs for QAM-R perform better and have lower encoding/decoding complexities than that proposed in [7].

II. Equivalent Channel of MDC-ABBA Codes

A. System Model and Preliminaries

We consider data transmission over a quasi-static Rayleigh flat fading channel. The transmitter and receiver are equipped with M Tx and N Rx antennas. The receiver, but not the transmitter, completely knows the channel gains.

From matrix representation theory, the mapping of a block of K data symbols $(s_1, s_2, \cdots, s_K)$ into a $T \times M$ code matrix of a STBC can be generally represented [10] as $\mathcal{X}_M = \sum_{k=1}^{K} (s_k A_k + s_k^* B_k)$, where A_k and B_k, $(k = 1, 2, \cdots, K)$ are $T \times M$ constant basis matrices, superscript * denotes conjugate [1]. The average energy of code matrices $X \in \mathcal{X}_M$ is constrained such that $\mathbb{E}[\|X\|_{\mathsf{F}}^2] = T$.

We now review the structure of ABBA codes. Let A_k and B_k $(k = 1, 2, \cdots, K)$ be the $t \times m$ basis matrices of an OSTBC $\mathcal{O}_m$. Let the data symbols are drawn from a constellation with unit average power. To guarantee the average power constraint, the OSTBC matrices are multiplied by a scalar $\kappa = \frac{1}{M \mathsf{R}_{\mathcal{O}_M}}$. For notational brevity, κ is not shown.

Two blocks of data, each of K symbols, are mapped into two code matrices $\mathcal{A}$ and $\mathcal{B}$ of $\mathcal{O}_m$ as $\mathcal{A} = \sum_{k=1}^{K} (s_k A_k + s_k^* B_k), \mathcal{B} = \sum_{k=1}^{K} \left(s_{k+K} A_k + s_{k+K}^* B_k\right)$.

[1]From now on, superscripts $^\mathsf{T}$ and † denote matrix transpose and transpose conjugate. The diagonal matrix with elements of vector $\boldsymbol{x}$ on the main diagonal is denoted by $\mathrm{diag}(\boldsymbol{x})$. $\|X\|_{\mathsf{F}}$ denotes Frobenius norm of matrix X and $\otimes$ denotes Kronecker product [11]. $\mathbb{E}[\cdot]$ denotes average. A mean-m and variance-σ^2 circularly complex Gaussian random variable is written by $\mathcal{CN}(m, \sigma^2)$. $\Re(X)$ and $\Im(X)$ denote the real and imaginary parts of a matrix X, respectively.

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The ABBA code matrices for $M = 2m$ Tx antennas are constructed from $\mathcal{O}_m$ as $\mathcal{Q}_M = \begin{bmatrix} \mathcal{A} & \mathcal{B} \\ \mathcal{B} & \mathcal{A} \end{bmatrix}$.

In the following, we reserve the term "ABBA" for the QSTBC proposed by Tirkkonen *et al.* [1] with pair-wise complex-symbol decoding [3] and the term "MDC-ABBA" for the ABBA codes with PWRS decoding [6].

B. Equivalent Channel of MDC-ABBA Codes

Let $Q \in \mathcal{Q}_M$ be a transmitted code matrix. Let $\mathcal{H}$ be the $M \times N$ channel matrix written as $\mathcal{H} = \begin{bmatrix} \boldsymbol{h}_1 & \boldsymbol{h}_2 & \cdots & \boldsymbol{h}_N \end{bmatrix}$, where $\boldsymbol{h}_n = \begin{bmatrix} h_{1n} & h_{2n} & \cdots & h_{Mn} \end{bmatrix}^\mathsf{T}$, $h_{i,j} \sim \mathcal{CN}(0,1)$. The Rx signal matrix is $\boldsymbol{Y} = \sqrt{\rho\kappa}\, Q\,\mathcal{H} + \boldsymbol{W}$, where $\boldsymbol{W}$ is noise matrix of size $T \times N$ with independently, identically distributed (i.i.d.) entries $\sim \mathcal{CN}(0,1)$; ρ is the average Rx signal-to-noise ratio (SNR). In [12], we have shown that the equivalent Tx/Rx equation of MDC-ABBA codes is given by

$$\underbrace{\hat{H}^{-1} \sum_{n=1}^{N} (E_{kn}^\dagger \boldsymbol{y}_n + F_{kn}^\mathsf{T} \boldsymbol{y}_n^*)}_{\hat{\boldsymbol{y}}_k} = \sqrt{\rho\kappa} \hat{H} \boldsymbol{s}_k + \underbrace{\hat{H}^{-1} \sum_{n=1}^{N} (E_{kn}^\dagger \boldsymbol{w}_n + F_{kn}^\mathsf{T} \boldsymbol{w}_n^*)}_{\hat{\boldsymbol{w}}_k} \quad (1)$$

where $\boldsymbol{y}_n$ and $\boldsymbol{w}_n$ are the received signal and additive noise vectors of the nth antenna,

$$E_{kn} = \begin{bmatrix} \boldsymbol{e}_{k1,n} & \boldsymbol{e}_{k2,n} \end{bmatrix}, \qquad \text{for } k = 1, 2, \ldots, K, \quad (2a)$$

$$\boldsymbol{e}_{ki,n} = \left(A_k \otimes \Pi^{i-1} \right) \boldsymbol{h}_n, \qquad \text{for } i = 1, 2, \quad (2b)$$

$$F_{kn} = \begin{bmatrix} \boldsymbol{f}_{k1,n} & \boldsymbol{f}_{k2,n} \end{bmatrix}, \quad (2c)$$

$$\boldsymbol{f}_{ki,n} = \left(B_k \otimes \Pi^{1-i} \right) \boldsymbol{h}_n, \quad (2d)$$

$$\hat{H}^2 = Z = \sum_{j=1}^{N} \sum_{i=1}^{m} H_{i,j}^\dagger H_{i,j}, \quad (3)$$

and $H_{i,j} = \begin{bmatrix} h_{i,j} & h_{i+m,j} \\ h_{i+m,j} & h_{i,j} \end{bmatrix}$. The noise vector $\hat{\boldsymbol{w}}_k$ is color with covariance matrix $V = \mathbb{E}[\hat{\boldsymbol{w}}_k \hat{\boldsymbol{w}}_k^\dagger] = \hat{H}^2 \neq \boldsymbol{I}_M$. Therefore this color noise is whitened by the matrix $\hat{H}^{-1}$.

We can show that Z is a real and circulant matrix. Then (1) can be rewritten by decoupling the real and imaginary parts of the two sides of (1) as

$$\underbrace{\begin{bmatrix} \Re(\hat{\boldsymbol{y}}_k) \\ \Im(\hat{\boldsymbol{y}}_k) \end{bmatrix}}_{\bar{\boldsymbol{y}}_k} = \sqrt{\rho\kappa} \underbrace{\begin{bmatrix} Z & \boldsymbol{0}_2 \\ \boldsymbol{0}_2 & Z \end{bmatrix}}_{\widetilde{H}} \begin{bmatrix} \Re(\boldsymbol{s}_k) \\ \Im(\boldsymbol{s}_k) \end{bmatrix} + \underbrace{\begin{bmatrix} \Re(\hat{\boldsymbol{w}}_k) \\ \Im(\hat{\boldsymbol{w}}_k) \end{bmatrix}}_{\bar{\boldsymbol{w}}_k}. \quad (4)$$

Thus, the real and imaginary parts of the transmitted vector $\boldsymbol{s}_k$ can be separately detected. Including the noise whitening matrix $\hat{H}^{-1}$, the general equivalent Tx/Rx signal relation of MDC-ABBA codes are:

$$\hat{H}^{-1} \Re(\hat{\boldsymbol{y}}_k) = \sqrt{\rho\kappa} \hat{H} \Re(\boldsymbol{s}_k) + \hat{H}^{-1} \Re(\hat{\boldsymbol{w}}_k), \quad (5a)$$

$$\hat{H}^{-1} \Im(\hat{\boldsymbol{y}}_k) = \sqrt{\rho\kappa} \hat{H} \Im(\boldsymbol{s}_k) + \hat{H}^{-1} \Im(\hat{\boldsymbol{w}}_k). \quad (5b)$$

In (5), $\hat{H}$ is the *equivalent channel* of MDC-ABBA codes. We have some important properties of $\hat{H}$ as follows.

Lemma 1: The equivalent channel matrix $\hat{H}$ and its inversion $\hat{H}^{-1}$ are real and circulant.

The detection of vectors $\Re(\boldsymbol{s}_k)$ and $\Im(\boldsymbol{s}_k)$ in (5) involves only 2 real symbols. Therefore, the maximum likelihood (ML) detection of MDC-ABBA codes is very simple.

In order to achieve full-diversity, signal transformations are required before sending the data symbols to the channels. We will next derive the optimal signal transformation for MDC-ABBA codes. Nevertheless, it is necessary to analyze the encoding and decoding of existing signal transformations proposed by (1) Yuen, Guan, and Tjhung (YGT) [6] and (2) Wang, Wang, and Xia (WWX) [7]. Note that the coding gain metric [8] is used to optimize signal transformation in [6], [7], which may not be optimal in terms of minimal SER.

III. Analyzing the Existing Signal Designs

Let the input symbols are $d_k = a_k + \mathsf{j}\, b_k, d_{k+K} = a_{k+K} + \mathsf{j}\, b_{k+K}, (k = 1, 2, \ldots, K)$; they are drawn from a unit average energy constellation $\mathcal{S}$, for example QAM, PSK. Let $s_k = p_k + \mathsf{j}\, q_k, s_{k+K} = p_{k+K} + \mathsf{j}\, q_{k+K}$ be the transmitted symbols. We can jointly transform the real input symbols a_k, b_k, a_{k+K} and b_{k+K} by a real transformation $\mathcal{R}$ to generate transmitted symbols p_k, q_k, p_{k+K}, and q_{k+K} as

$$\begin{aligned} \begin{bmatrix} \Re(\boldsymbol{s}_k)^\mathsf{T} & \Im(\boldsymbol{s}_k)^\mathsf{T} \end{bmatrix}^\mathsf{T} &= \begin{bmatrix} p_k & p_{k+K} & q_k & q_{k+K} \end{bmatrix}^\mathsf{T} \\ &= \mathcal{R} \underbrace{\begin{bmatrix} a_k & b_k & a_{k+K} & b_{k+K} \end{bmatrix}^\mathsf{T}}_{\hat{\boldsymbol{c}}_k} \end{aligned} \quad (6)$$

1. *Signal rotation proposed by Yuen et al.* [5], [6]:
In [6], the transmitted symbols are generated as follows:

$$\Re(\boldsymbol{s}_k) = \begin{bmatrix} p_k & p_{k+K} \end{bmatrix}^\mathsf{T} = R \begin{bmatrix} a_k & b_k \end{bmatrix}^\mathsf{T}, \quad (7a)$$

$$\Im(\boldsymbol{s}_k) = \begin{bmatrix} q_k & q_{k+K} \end{bmatrix}^\mathsf{T} = R \begin{bmatrix} a_{k+K} & b_{k+K} \end{bmatrix}^\mathsf{T} \quad (7b)$$

where R is a unitary matrix, $R = \begin{bmatrix} \cos(\alpha) & \sin(\alpha) \\ \sin(\alpha) & -\cos(\alpha) \end{bmatrix}$. The optimal angle for QAM is $\alpha = \frac{1}{2} \arctan(\frac{1}{2}) = 13.2825°$.

2. *Signal transformation proposed by Wang et al.* [7]: Wang *et al.* present more general signal transformations and show that there are three permutation-equivalent cases can be derived to achieve PWRS decoding. We consider only the first case with following signal transformation.

$$\begin{bmatrix} p_k & q_k & p_{k+K} & q_{k+K} \end{bmatrix}^\mathsf{T} = \mathcal{R}_W \hat{\boldsymbol{c}}_k \quad (8)$$

where $\mathcal{R}_W = \begin{bmatrix} U_1 & U_2 \\ U_1 R_1 & U_2 R_2 \end{bmatrix}$, and U_1, U_2, R_1, R_2 are 2×2 real matrices, $R_1^2 = \boldsymbol{I}_2, R_2^2 = \boldsymbol{I}_2$.

However, the symbol mapping in [7] is slightly different from (6): the p_{k+K} and q_k are permuted compared with the arrangement in (6) such that

$$\begin{aligned} \begin{bmatrix} \Re(\boldsymbol{s}_k)^\mathsf{T} & \Im(\boldsymbol{s}_k)^\mathsf{T} \end{bmatrix}^\mathsf{T} &= \begin{bmatrix} p_k & p_{k+K} & q_k & q_{k+K} \end{bmatrix}^\mathsf{T} \\ &= \pi \begin{bmatrix} p_k & q_k & p_{k+K} & q_{k+K} \end{bmatrix}^\mathsf{T} = \underbrace{\pi \mathcal{R}_W}_{\widehat{\mathcal{R}}_W} \hat{\boldsymbol{c}}_k \end{aligned} \quad (9)$$

where

$$\pi = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}. \qquad (10)$$

Substituting $\widehat{\mathcal{R}}_W$ into (4), we have

$$\bar{\boldsymbol{y}}_k = \sqrt{\rho\kappa}\widetilde{H}\widehat{\mathcal{R}}_W\hat{\boldsymbol{c}}_k + \bar{\boldsymbol{w}}_k. \qquad (11)$$

The matrix $\widetilde{H}\widehat{\mathcal{R}}_W$ in (11) is not block-diagonal; thus PWRS decoding seems to be impossible. However, by multiplying to sides of (11) with $\widehat{\mathcal{R}}_W^{\mathsf{T}}$, we again obtain another block diagonal matrix $\widehat{\mathcal{R}}_W^{\mathsf{T}}\widetilde{H}\widehat{\mathcal{R}}_W$. Further details on the decoding of MDC-ABBA codes with signal transformation of Wang-Wang-Xia are omitted due to the space limit.

We make the following observations on the signal transformations by Yuen *et al.* and Wang *et al.*.

- Encoding complexity: The 4×4 transformation $\mathcal{R}_W$ of Wang *et al.* has higher encoding complexity compared with the 2×2 rotation R of Yuen *et al.*.
- Decoding complexity: Multiplying both sides of (11) with $\widehat{\mathcal{R}}_W^{\mathsf{T}}$ to decode MDC-ABBA codes with WWX transformation slightly increases the decoding complexity.
- Performance: For square QAM (QAM-S), the transformation in [7, Theorem 2] provides no SNR gain compared with the rotation proposed by Yuen *et al.* [5], [6]. The transformation in [7, Theorem 3] performs better with rectangular QAM (QAM-R) at the cost of higher encoding/decoding complexities.

IV. Optimal Signal Designs

We will only consider the signal rotation due to Yuen *et al.* for deriving the exact SPEP because their rotation is mathematically convenient. More important, we will show that by combining power allocation and signal rotation for inphase-quadrature power-unbalanced constellations like QAM-R, we can achieve better performance than using the transformation in [7, Theorem 3], however, with less complexity.

A. Exact Symbol Pair-Wise Error Probability

From (6) and (7), we can rewrite (5) as

$$\hat{H}^{-1}\Re(\hat{\boldsymbol{y}}_k) = \sqrt{\rho\kappa}\hat{H}R\begin{bmatrix}a_k & b_k\end{bmatrix}^{\mathsf{T}} + \hat{H}^{-1}\Re(\hat{\boldsymbol{w}}_k), \qquad (12a)$$

$$\hat{H}^{-1}\Im(\hat{\boldsymbol{y}}_k) = \sqrt{\rho\kappa}\hat{H}R\begin{bmatrix}a_{k+K} & b_{k+K}\end{bmatrix}^{\mathsf{T}} + \hat{H}^{-1}\Im(\hat{\boldsymbol{w}}_k). \qquad (12b)$$

Since $\hat{H}^{-1}\Re(\hat{\boldsymbol{w}}_k)$ and $\hat{H}^{-1}\Im(\hat{\boldsymbol{w}}_k)$ are real random Gaussian vectors with i.i.d. entries (zero-mean and variance $N_0 = 1/2$), the information vectors $\begin{bmatrix}a_k & b_k\end{bmatrix}^{\mathsf{T}}$ and $\begin{bmatrix}a_{k+K} & b_{k+K}\end{bmatrix}^{\mathsf{T}}$ $(k = 1, 2, \ldots, K)$ experience the same channels; they are subject to the same error probability. We thus can consider the error probability of one of the two vectors only; the subscript of symbols can be omitted for short. Furthermore, the pair-wise error probability of each vector is also the SPEP.

Consider two distinct symbols $d = a + \mathrm{j}\,b$ and $\hat{d} = \hat{a} + \mathrm{j}\,\hat{b}$. Let $\delta_1 = a - \hat{a}, \delta_2 = b - \hat{b}, \Delta = [\delta_1\ \delta_2]^{\mathsf{T}}$, the conditional SPEP of d and $\hat{d}$ can be expressed by the Gaussian Q-function as

$$P(d \to \hat{d}|\hat{H}) = Q\left(\sqrt{\frac{\rho\kappa|\hat{H}R\Delta|^2}{4N_0}}\right). \qquad (13)$$

From Section II-B, we have shown that $\hat{H}$ is a 2×2 real circulant matrix. Hence $\hat{H}^{\dagger}\hat{H} = \hat{H}\hat{H} = \hat{H}^2 = Z$, where Z is given in (3). We can use eigenvalue decomposition for $H_{i,j}$ so that $H_{i,j} = F_2^{\dagger}\Lambda_{i,j}F_2$, where $\Lambda_{i,j} = \mathrm{diag}(\lambda_{i,j,1}, \lambda_{i,j,2})$ and $[\lambda_{i,j,1}\ \lambda_{i,j,2}]^{\mathsf{T}} = F_2[h_{i,j}\ h_{i+M/2,j}]^{\mathsf{T}}$. Since $h_{i,j}$ and $h_{i+M/2,j}$ are i.i.d. $\sim \mathcal{CN}(0,1)$, so do the $\lambda_{i,j,1}$ and $\lambda_{i,j,2}$. Thus,

$$Z = \sum_{j=1}^{N}\sum_{i=1}^{M/2} F_2\,\mathrm{diag}(|\lambda_{i,j,1}|^2, |\lambda_{i,j,2}|^2)F_2. \qquad (14)$$

Let $x \triangleq |\hat{H}R\Delta|^2 = (R\Delta)^{\dagger}\hat{H}^{\dagger}\hat{H}(R\Delta)$, one has

$$\begin{aligned} x &= \sum_{j=1}^{N}\sum_{i=1}^{M/2}\left[(F_2R\Delta)^{\dagger}\,\mathrm{diag}(|\lambda_{i,j,1}|^2, |\lambda_{i,j,2}|^2)(F_2R\Delta)\right] \\ &= \sum_{j=1}^{N}\sum_{i=1}^{M/2}\left[\beta_1^2|\lambda_{i,j,1}|^2 + \beta_2^2|\lambda_{i,j,2}|^2\right] \end{aligned} \qquad (15)$$

where $[\beta_1\ \beta_2]^{\mathsf{T}} = F_2R\Delta$, and β_1 and β_2 are real.

We now apply the Craig's formular [13] to derive the conditional SPEP in (13).

$$\begin{aligned} P(d \to \hat{d}|\hat{H}) &= Q\left(\sqrt{\frac{\rho\kappa x}{2}}\right) = \frac{1}{\pi}\int_0^{\pi/2}\exp\left(\frac{-\rho\kappa x}{4\sin^2\theta}\right)d\theta \\ &= \frac{1}{\pi}\int_0^{\pi/2}\prod_{j=1}^{N}\prod_{i=1}^{M/2}\exp\left(-\frac{\rho\kappa\left(\beta_1^2|\lambda_{i,j,1}|^2 + \beta_2^2|\lambda_{i,j,2}|^2\right)}{4\sin^2\theta}\right)d\theta. \end{aligned} \qquad (16)$$

Since $\lambda_{i,j,1}$ and $\lambda_{i,j,2}$ are i.i.d $\sim \mathcal{CN}(0,1)$, we can apply a method based on the moment generation function (MGF) [14], [15] to obtain the unconditional SPEP in the following:

$$P(d \to \hat{d}) = \frac{1}{\pi}\left[\left(1 + \frac{\rho\kappa\beta_1^2}{4}\right)\left(1 + \frac{\rho\kappa\beta_2^2}{4}\right)\right]^{-MN/2}. \qquad (17)$$

B. Optimal Signal Rotations Based on tight SER Union Bound

Assume that d_i and d_j, $i, j = 1, \ldots, L$, are signals drawn from a constellation $\mathcal{S}$ of size L. From the SPEP expression (17), we can find the union bound on the SER of constellation $\mathcal{S}$ with MDC-ABBA codes as

$$P_u(\mathcal{S}) = \frac{2}{L}\sum_{i=1}^{L-1}\sum_{j=i+1}^{L}P(d_i \to d_j) \qquad (18)$$

The SER union bound of square QAM (QAM-S) with signal rotation R due to Yuen *et al.* and $\alpha = 13.2825°$ are plotted in Fig. 1. The geometrical shape of 8QAM-S is sketched in Fig. 2 and the Gray-bit mapping is used in all simulations.

The difference between the simulated SER and the union bound is only about 0.1 dB when SER $< 10^{-2}$. Therefore,

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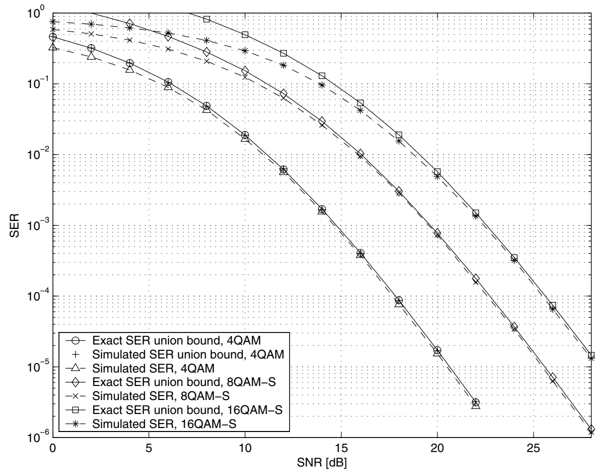

Fig. 1. Union bound compared with simulated SER of QAM signals, 4 Tx/1 Rx antennas.

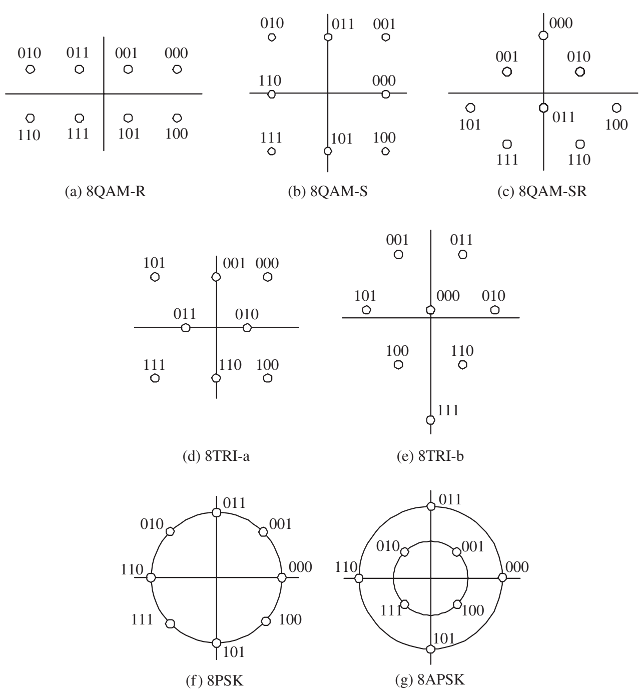

Fig. 2. Geometrical shapes of 8-ary constellations.

the union bound can be used to predict the SER performance of MDC-ABBA codes accurately, as well as to optimize the signal rotation R.

For popular constellations, the optimal rotation angles that that minimize the union bound are summarized in Table I. We choose the SNR such that the SER of corresponding optimal rotation angle is about 10^{-8}. At such low SER, these optimal rotation angles also yield full-diversity MDC codes.

The union bounds for several 4-, 8- and 16-ary constellations are illustrated in Fig. 3. Compared with QAM, TRI performs quite well when they are used for fading channels [9], and for OSTBC and ABBA codes [3]. In these scenarios, the signals having higher minimum Euclidean distance tend to

TABLE I

OPTIMAL ROTATION ANGLES OF POPULAR CONSTELLATIONS

Constellation	Optimal α	Constellation	Optimal α
4QAM	$14.382°$	8QAM-S	$12.268°$
4TRI	$31.155°$	8QAM-R	$13.166°$
8PSK	$5.915°$, $39.085°$	8QAM-SR	$31.964°$
8APSK	$33.472°$	16PSK	$24.883°$, $42.617°$
8TRI-a	$30.284°$	16TRI	$0°$
8TRI-b	$0°$	16QAM-S	$13.195°$

TABLE II

OPTIMAL POWER ALLOCATION AND SIGNAL ROTATION FOR QAM-R

Constellation	μ_1	μ_2	Optimal α
8QAM-R	0.9055	1.3784	$0°$
32QAM-R	0.8972	1.3487	$1.954°$

perform better. However, this conclusion is not always valid for MDC-ABBA codes. For example, 8TRI-b has the best minimum Euclidean distance among 8-ary constellations, but its performance is worse than 8QAM constellations.

We also compare the frame error rate (FER) of MDC-ABBA codes with the new optimal signal rotation and existing transformations for square-rotated 8QAM (8QAM-SR) in Fig. 4. Our new optimal signal rotation gains remarkable SNR at high SNR compared with the signal rotation in [6] and performs slightly better than the signal transformation in [7], however, with lower encoding/decoding complexities.

It is worthwhile to recognize that in Fig. 4, while ABBA codes (with pair-wise complex-symbol decoding) have smaller FER compared with MDC-ABBA codes, the BER of the former is inferior to that of the latter. It means that for 8QAM-SR the Gray-bit mapping may not be optimal for ABBA codes.

The new optimal rotation angles for QAM (square or rectangular) constellations are very close to the proposed angle $\alpha = 13.2825$ by minimizing CPEP [6]; hence the SNR gains are negligible compared to the results of [6] and [7, Theorem 2]. We will next present a new approach, which is applicable to find the optimal rotation angle for QAM-R so that the MDC-ABBA codes perform better but have lower encoding/decoding complexity than that proposed in [7, Theorem 3].

C. Optimal Signal Rotations with Power Allocations

For QAM-R, for example 8QAM-R in Fig. 2, the average powers of the real and imaginary parts of the signal points are different. We may allocate different powers to the real and imaginary parts of QAM-R signals to get better overall SER.

In particular, the real and imaginary of QAM-R signals are scaled by constants called power loading coefficients μ_1 and μ_2 , respectively, before they are rotated. The average energy of the constellation is kept unchanged. For example, the 8QAM-R with signal points $\{(\pm 3 \pm \mathrm{j}, \pm 1 \pm \mathrm{j})/\sqrt{48}\}$ has constraint equation for power loading coefficients μ_1 and μ_2 as $5\mu_1^2 + \mu_2^2 = 6$. The best found power loading coefficients and rotation angle for 8- and 32QAM-R are given in Table II.

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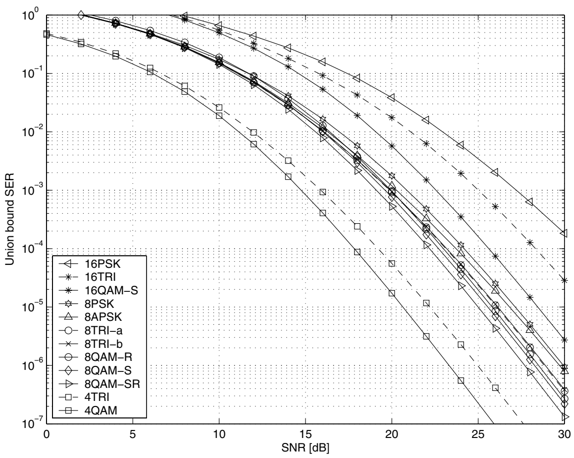

Fig. 3. SER union bound of 4-, 8-, 16-ary constellations, 4 Tx/1 Rx antennas.

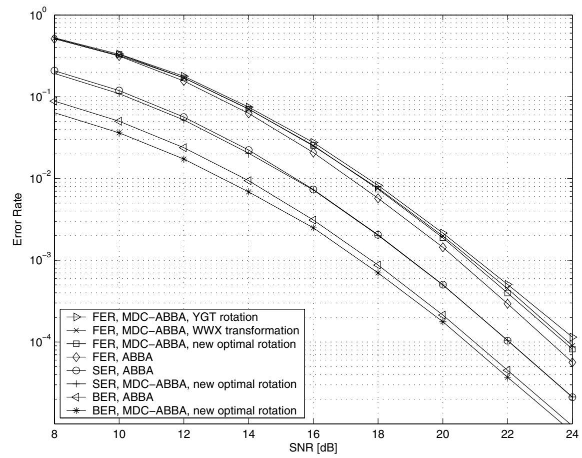

Fig. 4. Performance comparison of ABBA codes and MDC-ABBA codes using 8QAM-SR, 4 Tx/1 Rx antennas.

The FER of MDC-ABBA codes with our new power loading scheme for QAM-R is compared with the existing signal transformations in Fig. 5. Our proposed scheme gains remarkably compared with the signal rotation method due to Yuen-Guan-Tjhung and also performs slightly better than the signal transformation method due to Wang-Wang-Xia with lower encoding/decoding complexities.

V. Conclusion

We have presented a method to calculate the exact symbol-wise error probability and the exact union bound on SER. Since the union bound is tight at medium and high SNR, it can conveniently be used to analyze the performance of MDC-ABBA codes and, furthermore, to optimize signal rotations for minimizing SER of any constellation with arbitrary geometrical shapes. We have also proposed a new method combining the optimal power allocation and signal rotation to find the best signal transformation for inphase-quadrature power-imbalanced constellations such as rectangular QAM. Our new signal designs perform better than the existing ones and also have lower encoding/decoding complexities.

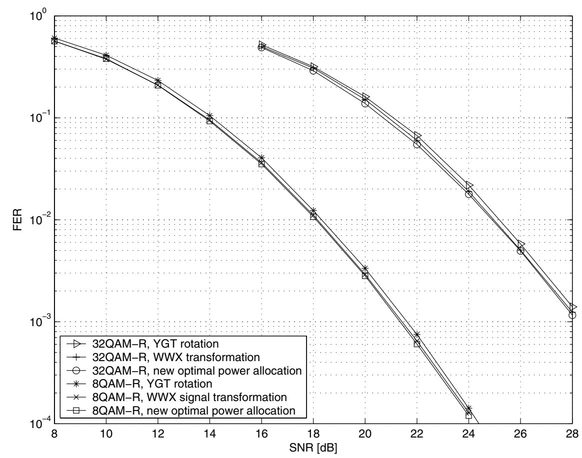

Fig. 5. Performances of MDC-ABBA codes with new optimal power allocation and existing signal designs for QAM-R, 4 Tx/1 Rx antennas.

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