An Adaptive-Scaling Tone Reservation Algorithm for PAR Reduction in OFDM Systems

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***Abstract*— Existing tone-reservation algorithms (such as the controlled clipper algorithm) for OFDM require a number of iterations to ensure the reduction of Peak-to-Average Power Ratio (PAR). Consequently, such algorithms entail high levels of computational complexity. In this paper, we propose a new adaptive-scaling tone reservation algorithm. It utilizes the filtered clipping noise as the PAR reduction signal, and adaptively scales it to reduce the PAR. Simulation results show that our proposed algorithm achieves a better PAR reduction and lower complexity than the controlled clipper algorithm.**

I. Introduction

Despite many advantages, Orthogonal Frequency Division Multiplexing (OFDM) systems suffer from the high Peak-to-Average Power Ratio (PAR) [1]. Large PAR requires a linear High Power Amplifier (HPA), which, however, is not efficiently used. If the linear range of HPA is not sufficient, large PAR leads to in-band distortion and out-of-band radiation [1]. Various PAR reduction techniques have been proposed, including clipping and filtering [2]–[4], tone reservation [5], multiple signal representation [6], [7], and coding [8].

In clipping and filtering, the signal is clipped and the out-of-band radiation is eliminated by filtering [2], [4], although the in-band clipping noise cannot be eliminated via filtering, leading to the increase of Bit-Error-Rate (BER). Coding techniques such as block codes, convolutional codes, Golay complementary sequences can also be used to reduce PAR [8]. However, these techniques result in low code rates when the number of subcarriers increases. Although multiple signal representation techniques such as Selected Mapping (SLM) and Partial Transmit Sequences (PTS) are suitable for a large number of subcarriers [6], [7], side information may be required at the receiver to decode data symbols. Incorrectly received side information causes burst errors.

The tone reservation technique mitigates these drawbacks and is particularly appropriate when there is a large number of subcarriers. It exploits a small number of unused subcarriers called reserved tones to reduce PAR, and the remaining subcarriers are used for data transmission, just like conventional OFDM. Since data symbols are not transmitted on reserved tones, the receiver simply omits the reserved tones, and only detects the symbols on data tones. Therefore, this technique does not require side information, and the BER is not worse than that of other PAR reduction techniques.

The controlled clipper algorithm [5] for the tone reservation technique obtains moderate PAR reduction. In this algorithm, an impulse-like signal subject to the tone-reservation constraints is used to suppress the high peaks of OFDM signals. The convergence rate slows down after several iterations. Therefore, a tradeoff has to be made to maintain reasonable computational complexity. Other algorithms, for example [3], [9], are also proposed for tone reservation. Nevertheless, they also require a number of iterations to ensure low PAR regrowth. When passband OFDM signal is considered, some of these algorithms (e.g. [9]) preprocess the complex signal to separate it to real numbers, which increases the complexity.

In this paper, we propose an adaptive-scaling algorithm for tone reservation. This algorithm utilizes the filtered clipping noise as the peak-canceling signal, and adaptively scales it to reduce the PAR. Simulation results show that the proposed algorithm has larger PAR reduction and lower complexity than the controlled clipper algorithm.

This paper is organized as follows: Section II characterizes the OFDM system and gives a brief review of the tone reservation technique. Section III analyzes PAR reduction using tone reservation. The adaptive-scaling algorithm is proposed in Section IV. In Section V, simulation results compare the proposed algorithm with the controlled clipper. Section VI makes the concluding remarks.

II. Tone Reservation Technique

A. Characterization of OFDM System

The time domain OFDM signal $x(t)$ may be written as

$$x(t) = \frac{1}{\sqrt{N}} \sum_{k=-\frac{N}{2}}^{\frac{N}{2}-1} X_k e^{j2\pi kt/T}, \quad 0 \le t \le T\,, \qquad (1)$$

where N is the number of subcarriers, T is the OFDM symbol period, and X_k's are data symbols. We call $\mathbf{X} = [X_{-\frac{N}{2}}, X_{-\frac{N}{2}+1}, ..., X_{\frac{N}{2}-1}]$ an OFDM block. In practice, JN samples of $x(t)$ are efficiently computed by an Inverse Discrete

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Fourier Transform (IDFT)[1]

$$x_n = \frac{1}{\sqrt{N}} \sum_{k=-\frac{N}{2}}^{\frac{N}{2}-1} X_k e^{j2\pi \frac{nk}{JN}}, \quad n = 0, ..., JN-1, \tag{2}$$

where J is the oversampling factor. The PAR may be defined as

$$\xi = \frac{\max_{t\in[0,T)} |x(t)|^2}{P_{\text{av}}}, \tag{3}$$

where $P_{\text{av}} = 2\sigma^2 = E\{|x(t)|^2\} = E\{|X_k|^2\}$ is the average power. The PAR may also be computed using the discrete samples x_n similar to (3), and is approximately equal to ξ when $J \geq 4$ [10].

B. Tone Reservation Technique

Tone Reservation technique [5] reserves N_r tones for peak reduction and uses the other $(N - N_r)$ tones for data transmission. The tone reservation ratio $R = \frac{N_r}{N}$ is typically small. Assume that $\mathcal{R} = \{i_0, ..., i_{N_r-1}\}$ is the index set of these peak reduction tones (PRT), where $-\frac{N}{2} \leq i_0 < i_1 < \cdots < i_{N_r-1} \leq \frac{N}{2} - 1$. The index set $\mathcal{R}^c$ of data symbols is the complement of $\mathcal{R}$ in $\mathcal{N} = \{-\frac{N}{2}, ..., \frac{N}{2} - 1\}$. Let the peak-canceling signal vector $\mathbf{C} = [C_{-\frac{N}{2}}, ..., C_{\frac{N}{2}-1}]$. Then, $X_k C_k = 0$, and

$$X_k + C_k = \begin{cases} X_k & k \in \mathcal{R}^c, \\ C_k & k \in \mathcal{R}. \end{cases}$$

The time domain peak-reduced OFDM signal is

$$\hat{x}_n = \text{IDFT}\{\mathbf{X}_{\text{ext}} + \mathbf{C}_{\text{ext}}\} = x_n + c_n, \tag{4}$$

where $x_n = \text{IDFT}\{\mathbf{X}_{\text{ext}}\}$, and $c_n = \text{IDFT}\{\mathbf{C}_{\text{ext}}\}$. The PAR is then defined as

$$\xi = \frac{\max |x_n + c_n|^2}{E\{|x_n|^2\}}. \tag{5}$$

The relevant optimization problem can be stated as

$$\min_{\mathbf{C}} \xi. \tag{6}$$

Eq. (6) can be formulated as a Quadratically Constrained Quadratic Program. A complexity-reduced algorithm, called the controlled clipper [5], iteratively calculates

$$\bar{\mathbf{x}}^{(i+1)} = \bar{\mathbf{x}}^{(i)} - \mu \sum_{\bar{x}_n^{m(i)} > A} \alpha_n^{(i)} \mathbf{P}_n^{||_2} \tag{7}$$

where $\bar{\mathbf{x}}^{(i)}$ is the peak-reduced OFDM signal vector at the i-th iteration, $\bar{\mathbf{x}}^{(0)} = [x_0, x_1, ..., x_{JN-1}]$, $\alpha_n^{(i)}$ are clipping pulses at the i-th iteration, and $\mathbf{P}_n^{||_2}$ is the peak-canceling signal.

[1]In this paper, we use zero-insertion scheme to calculation x_n, i.e., the IDFT operation is applied to the extended vector $\mathbf{X}_{\text{ext}} = [X_0, ..., X_{\frac{N}{2}-1}, \underbrace{0, ..., 0}_{(J-1)N \text{ zeros}}, X_{-\frac{N}{2}}, X_{-\frac{N}{2}+1}, ..., X_{-1}]$.

III. Analysis of PAR Reduction Using Tone Reservation

We consider clipping $x(t)$ using a soft limiter [11] and filtering the clipping noise to reserved tones. The clipped OFDM signal $\tilde{x}(t)$ is

$$\tilde{x}(t) = \begin{cases} Ae^{j\phi(t)}, & |x(t)| > A, \\ x(t), & |x(t)| \leq A, \end{cases} \tag{8}$$

where A is the predefined threshold and $\phi(t)$ is the phase of $x(t)$. The clipping noise is defined as

$$f(t) = x(t) - \tilde{x}(t). \tag{9}$$

Unless A is small, $f(t)$ is a series of pulses,

$$f(t) = \sum_i f_i(t - t_i),$$

where $f_i(t)$ is the i-th clipping pulse with pulse duration τ_i, and reaching its maximum at t_i. Using Taylor's series expansion, $f_i(t)$ can be approximated as [12]

$$f_i(t) = -\frac{1}{2} b_i t^2 + \frac{1}{8} b_i \tau_i^2, \quad -\frac{\tau_i}{2} \leq t < \frac{\tau_i}{2}, \tag{10}$$

where $b_i = -\ddot{x}(t_i - \frac{\tau_i}{2})$.

The frequency spectrum of $f_i(t)$ can be written as

$$F_i(\omega) = \frac{b_i \tau_i}{\omega^2} \left(\operatorname{sinc} \frac{\omega \tau_i}{2} - \cos \frac{\omega \tau_i}{2} \right), \tag{11}$$

where $\operatorname{sinc} x = \frac{\sin x}{x}$. $F_i(\omega)$ distributes over the whole frequency band from $\omega = -\infty$ to ∞. Figure 1 shows an example of $F_i(\omega)$. The solid curve represents $|F_i(\omega)|$. The dashed line illustrates the OFDM frequency band, where the cut-off frequency is

$$\omega_x = 2\pi f_x = 2\pi \frac{N}{2T}.$$

We observe that $F_i(\omega)$ consists of a large portion of out-of-band radiation. The in-band clipping noise is only a small portion of the main lobe of $F_i(\omega)$.

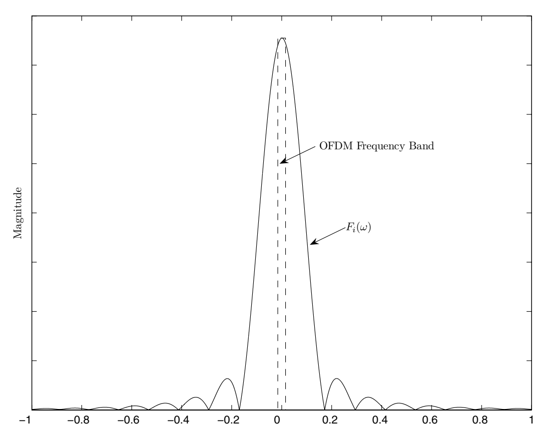

Fig. 1. Frequency spectrum of $f_i(t)$.

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Since τ_i is very small, we have $\frac{\omega_x \tau_i}{2} \ll 1$ when A/σ is large [4]. Thus the in-band part of $F_i(\omega)$ is approximated as

$$F_i(\omega) \approx \frac{b_i \tau_i^3}{12}, \quad |\omega| \le \omega_x \tag{12}$$

by using $\operatorname{sinc}\theta - \cos\theta \approx \frac{\theta^2}{3}$, when $\theta \ll 1$ ([13]). Now, the power spectrum of clipping noise can be found as

$$\begin{aligned} E\{|F_i(\omega)|^2\} \approx \frac{1}{144} \sum_i E\{b_i^2 \tau_i^6\} \\ + \frac{1}{144} \sum_i \sum_k E\{b_i b_k \tau_i^3 \tau_k^3\}, \quad |\omega| \le \omega_x. \end{aligned} \tag{13}$$

Since $E\{|F_i(\omega)|^2\}$ does not depend on the frequency ω, the power spectrum of clipping noise is white. This phenomenon has been observed in [14] by simulation.

To eliminate out-of-band radiation and in-band distortion in $f(t)$, we use an ideal multiple-passband filter with the frequency response as[2]

$$H_k = \begin{cases} 1, & k \in \mathcal{R}, \\ 0, & \text{otherwise.} \end{cases}$$

For consecutive reserved tones around the DC component, i.e., $\mathcal{R} = \{-\frac{N_r}{2}, -\frac{N_r}{2}+1, ..., \frac{N_r}{2}-1\}$, the i-th filtered clipping pulse $\hat{f}_i(t)$ has a simple closed-form as

$$\hat{f}_i(t) = \frac{b_i \tau_i^3 \omega_c}{12\pi} \operatorname{sinc} \omega_c (t - t_i), \tag{14}$$

where $\omega_c = 2\pi \frac{N_r}{2T} = R\omega_x$. Compared to $|f_i(t)|$, $|\hat{f}_i(t)|$ has a smaller magnitude but has a wider mainlobe, where the former implies peak regrowth and the latter implies that, if two clipping pulses are close to each other, reducing the magnitude of one pulse may unfortunately increases that of the other, depending on the phase difference of these two pulses. Moreover, since $\hat{f}(t)$ is the summation of a series of sinc functions, sidelobes of $\hat{f}(t)$ may introduce "new" peaks higher than A, which however do not exist in $f(t)$. Note that this conclusion holds for general cases where $\mathcal{R}$ is randomly chosen. Therefore, a good tone reservation algorithm must make a careful balance between reduced peaks and increased peaks as well as "newly generated" peaks.

IV. The New Algorithm and Its Complexity

A. Adaptive-scaling tone reservation algorithm

We now describe the adaptive-scaling tone reservation algorithm in oversampled discrete-time domain. The main idea of this algorithm is that we use $\hat{f}_n$ as a peak-canceling signal, and scale it by a factor β to minimize the out-of-range power P, i.e., the total power of those $|\hat{x}_n| > A$. The objective function is therefore

$$\min_\beta P \tag{15}$$

where

$$P = \sum_{|\hat{x}_n| > A} (|\hat{x}_n| - A)^2. \tag{16}$$

and $\hat{x}_n$ is the peak-reduced signal written as

$$\hat{x}_n = x_n - \beta \hat{f}_n = Ae^{j\phi_n} + f_n - \beta \hat{f}_n. \tag{17}$$

Eq. (16) can be rewritten as

$$P = \sum_{n \in \mathcal{S}_1} (|\hat{x}_n| - A)^2 - \sum_{\substack{n \in \mathcal{S}_1 \\ |\hat{x}_n| \le A}} (|\hat{x}_n| - A)^2 + \sum_{n \in \mathcal{S}_2} (|\hat{x}_n| - A)^2, \tag{18}$$

where $\mathcal{S}_1 = \{n : |f_n| > 0\}$ and $\mathcal{S}_2 = \{n : |f_n| = 0 \text{ and } |\hat{x}_n| > A\}$. Note that $\mathcal{S}_1$ is the index set of all clipping pulses. Since the power of any clipping pulse is a monotonic function of its amplitude [15], minimizing (18) is equivalent to minimizing

$$\hat{P} = \underbrace{\sum_{n \in \mathcal{S}_p} (|\hat{x}_n| - A)^2}_{P_1} - \underbrace{\sum_{\substack{n \in \mathcal{S}_p \\ |\hat{x}_n| \le A}} (|\hat{x}_n| - A)^2}_{P_2} + \underbrace{\sum_{n \in \mathcal{S}_p^+} (|\hat{x}_n| - A)^2}_{P_3}, \tag{19}$$

where $\mathcal{S}_p = \{n : n \in \mathcal{S}_1, |x_n| > |x_{n-1}| \text{ and } |x_n| \ge |x_{n+1}|\}$ is the index set of the peaks of f_n, and $\mathcal{S}_p^+ = \{n : n \in \mathcal{S}_2, |\hat{x}_n| > |\hat{x}_{n-1}| \text{ and } |\hat{x}_n| \ge |\hat{x}_{n+1}|\}$ is the index set of the peaks of "newly generated" pulses whose amplitudes are larger than A. That is, if $\hat{P}$ is minimized, P is also close-to-optimally minimized.

Eq. (19) implies that the optimal β, denoted as $\beta^{(\mathrm{opt})}$, must both minimize the peaks of x_n and prevent any large "newly generated" pulses, which in turn implies that β could not be large. Taking this fact into account, P_2 and P_3 are small and their difference can be omitted. Therefore, we have

$$\begin{aligned} \hat{P} \approx P_1 &= \sum_{n \in \mathcal{S}_p} |\hat{x}_n - Ae^{j\hat{\phi}_n}|^2 \\ &= \sum_{n \in \mathcal{S}_p} |f_n - \beta \hat{f}_n + A(e^{j\phi_n} - e^{j\hat{\phi}_n})|^2, \end{aligned} \tag{20}$$

where $\hat{\phi}_n$ is the phase of $\hat{x}_n$.

Since $\beta^{(\mathrm{opt})}$ is not large and $|\hat{f}_n|_{\max} \ll |f_n|_{\max}$ [15][3], we can see that $|x_n| = |Ae^{j\phi_n} + f_n| \gg |\beta \hat{f}_n|$, i.e., $\beta \hat{f}_n$ could not significantly change the phase of x_n. Therefore, $\hat{\phi}_n \approx \phi_n$ and

$$\hat{P} \approx \sum_{n \in \mathcal{S}_p} |f_n - \beta \hat{f}_n|^2. \tag{21}$$

The optimal solution is

$$\beta^{(\mathrm{opt})} = \frac{\mathfrak{Re}[\sum_{n \in \mathcal{S}_p} f_n \hat{f}_n^*]}{\sum_{n \in \mathcal{S}_p} |\hat{f}_n|^2}, \tag{22}$$

where $\mathfrak{Re}[x]$ is the real part of x, and $(\cdot)^*$ represents complex conjugate.

[2]Although $\mathcal{R}$ is a discrete set, here we slightly misuse $\mathcal{R}$ to represent the reserved tones in the continuous frequency domain for simplicity. In this case, each item $i \in \mathcal{R}$ represents a frequency band with width $\frac{2}{N}\omega_x$ and the central frequency $\frac{2i}{N}\omega_x$.

[3]This is not the case if a small clipping pulse is close to a large clipping pulse. However, this case can be treated as a part of P_2 and has little effect on $\beta^{(\mathrm{opt})}$.

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The adaptive-scaling tone reservation algorithm can now be summarized as follows:

Algorithm 1 (Adaptive-Scaling Algorithm):

Initialization:

1) Set up A and $\mathcal{R}$, and choose a maximum iteration number L.

Runtime:

For each length $(N - N_r)$ OFDM symbol,

1) Calculate x_n using (2).
2) If $\max_{0 \le n \le JN-1} |x_n| > A$, go to step 3; otherwise, transmit x_n and terminate the algorithm.
3) Calculate f_n using (9).
4) Calculate $\hat{f}_n$ as follows.
    a) Convert f_n to the frequency domain to obtain F_k. A DFT operation is required.
    b) Set $F_k = 0$ for all $k \notin \mathcal{R}$.
    c) Convert F_k to the time domain to obtain $\hat{f}_n$. An IDFT operation is required.
5) Find the peaks of f_n to get $\mathcal{S}_p$.
6) Calculate $\beta^{(\mathrm{opt})}$ using (22).
7) Calculate the peak-reduced OFDM signal as $\hat{x}_n = x_n - \beta^{(\mathrm{opt})}\hat{f}_n$. If $\max |\hat{x}_n| > A$ and the iteration number is less than L, let $x_n = \hat{x}_n$ and go to Step 3. Otherwise, transmit $\hat{x}_n$ and terminate the algorithm.

B. Analysis of Computational Complexity

We only consider the runtime complexity. Moreover, the rumtime Steps 1 and 2 are not counted because all OFDM systems must execute Step 1, and all PAR reduction techniques require at least one iteration of these two steps.

Denote the number of nonzero samples in f_n as N_f. The complexity of Step 3 is $\mathcal{O}(N_f)$. However, N_f is a function of N. To see this, we calculate the mean of N_f as

$$\bar{N}_f = N_p \bar{\tau} f_s,$$

where $\bar{\tau}$ is the average clipping pulse duration, $f_s = \frac{JN}{T}$ is the sampling frequency, and N_p is the average number of pulses in an OFDM signal duration, which can be calculated as [15]

$$N_p = N\sqrt{\frac{\pi}{6}}\frac{A}{\sigma}e^{-A^2/(2\sigma^2)}. \tag{23}$$

Note that N_p is also the average size of $\mathcal{S}_p$. Thus,

$$\bar{N}_f = JNe^{-A^2/(2\sigma^2)},$$

and the complexity of Step 3 is $\mathcal{O}(N)$. However, its constant of proportionality is small. For example, when $N = 512$, $A = 6\,\mathrm{dB}$, and $\sigma = \frac{1}{\sqrt{2}}$, $N_f \approx 38.2$ and $N_p \approx 19.5$.

The overall complexity is mainly determined by the DFT/IDFT pair in Step 4. The complexities of directly calculating DFT/IDFT are $\mathcal{O}(JNN_f)$ and $\mathcal{O}(JNN_p)$, respectively, or are $\mathcal{O}(N^2)$ with small constants of proportionality when J and A are fixed. On the other hand, if FFT and IFFT are used, the complexity is $\mathcal{O}(N \log_2 N)$. Since the constants of proportionality are small, the adaptive-scaling algorithm can obtain near-optimal PAR reduction with low computational complexity.

V. Simulation Results

In this section, we compare the adaptive-scaling (AS) and the controlled clipper (CC) algorithms in terms of the PAR complementary cumulative distribution function (CCDF) $F(\xi_0) = \Pr[\xi > \xi_0]$, which can be interpreted as the probability that the PAR exceeds a clipping level ξ_0 – we refer this as the clip probability. We use $N = 512$, $J = 4$, and 10^6 uniformly distributed unitary QPSK OFDM blocks (i.e., $|X_k| = 1$) are simulated.

Fig. 2 compares the two algorithms on a randomly selected $\mathcal{R}$, where $A = 6\,\mathrm{dB}$ and $R = 5\%$. The iteration number of AS and CC is denoted as L. For a 10^{-4} clip probability, AS with one iteration reduces the PAR to $9.1\,\mathrm{dB}$, which is $0.5\,\mathrm{dB}$ better than CC with four iterations. With 16 iterations, AS obtains $4.5\,\mathrm{dB}$ PAR reduction a 10^{-4} clip probability, and is $0.2\,\mathrm{dB}$ better than CC with 40 iterations.

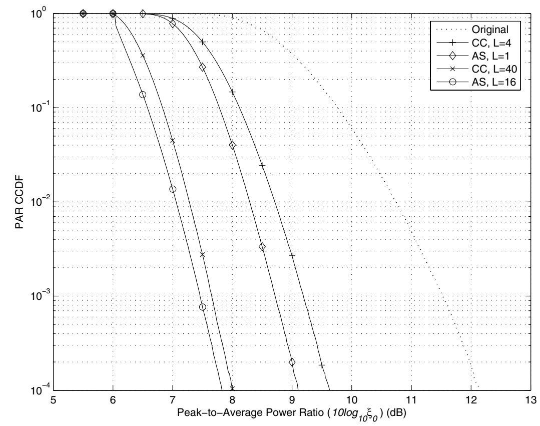

Fig. 2. PAR CCDF comparison of AS and CC, where $R = 5\%$, $A = 6\,\mathrm{dB}$, and $\mathcal{R}$ is randomly selected.

Fig. 3 compares AS and CC for $A = 4\,\mathrm{dB}$ and $R = 20\%$. For a 10^{-4} clip probability, CC with four iterations obtains $3.8\,\mathrm{dB}$ PAR reduction, which is $1.2\,\mathrm{dB}$ less than AS with one iteration. While CC with 40 iterations obtains $6.3\,\mathrm{dB}$ reduction, AS with 16 iterations obtains $7.4\,\mathrm{dB}$ reduction.

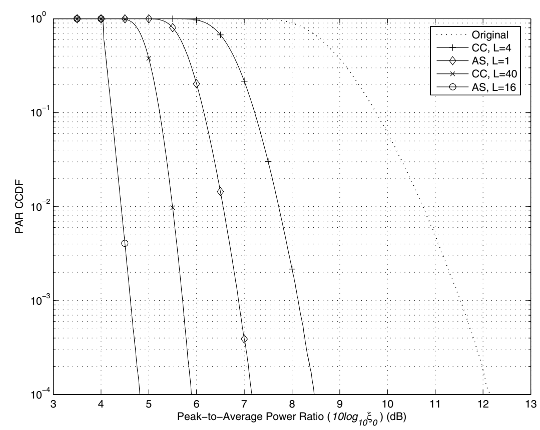

Fig. 3. PAR CCDF comparison of AS and CC, where $R = 20\%$, $A = 4\,\mathrm{dB}$, and $\mathcal{R}$ is randomly selected.

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The average computation time of these algorithms is listed in Table I, which is obtained on a Pentium IV 2.8G computer using Matlab 7 Service Pack 2. When $R = 5\%$, the average computation time of AS with one iteration is only 30% of that of CC with four iterations, and that of AS with 16 iterations is 65% of that of CC with 40 iterations.

TABLE I
AVERAGE COMPUTATION TIME IN MILLISECOND OF AS AND CC

	$R = 5\%$, $A = 6\,\text{dB}$	$R = 20\%$, $A = 4\,\text{dB}$
CC, $L = 4$	5.9080	6.0753
CC, $L = 40$	50.962	52.734
AS, $L = 1$	1.7961	1.9189
AS, $L = 16$	33.006	35.604

Figs. 4 and 5 compare the two algorithms on a consecutive PRT set $\mathcal{R} = \{0, 1, ..., N_r - 1\}$, where $A = 6\,\text{dB}$ and $R = 5\%$ in Fig. 4, and $A = 4\,\text{dB}$ and $R = 20\%$ in Fig. 5. While AS continues to maintain its performance advantage over CC, both algorithms exhibit significant performance degradation compared to Figs. 2 and 3. The reason may be that the consecutive PRT set has much wider mainlobe and much larger sidelobes compared to a random PRT set. For $R = 5\%$, compared to the original OFDM system, CC with four iterations reduces the PAR by about $0.6\,\text{dB}$ at a 10^{-1} clip probability, but increases the PAR by about $0.4\,\text{dB}$ at a 10^{-4} clip probability. CC with 40 iterations performs $0.4\,\text{dB}$ better than CC with four iterations. On the other hand, AS obtains about $1.2\,\text{dB}$ PAR reduction with one iteration and about $1.9\,\text{dB}$ PAR reduction with 16 iterations at a 10^{-4} clip probability, respectively.

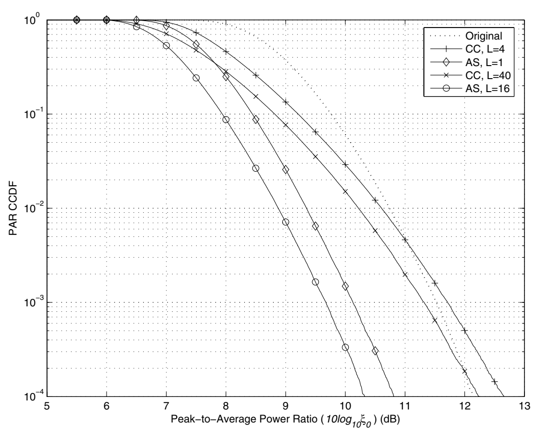

Fig. 4. PAR CCDF comparison of AS and CC, where $R = 5\%$, $A = 6\,\text{dB}$, and $\mathcal{R}$ is consecutive.

By increasing R to 20%, CC obtains about $0.6\,\text{dB}$ and $0.8\,\text{dB}$ reduction for four iterations and 40 iterations, respectively, at a 10^{-4} clip probability, while AS gives about $2\,\text{dB}$ and $2.6\,\text{dB}$ reduction for one iteration and 16 iterations, respectively.

VI. CONCLUSIONS

In this paper, we have proposed an adaptive-scaling algorithm for tone reservation. This algorithm utilizes the filtered

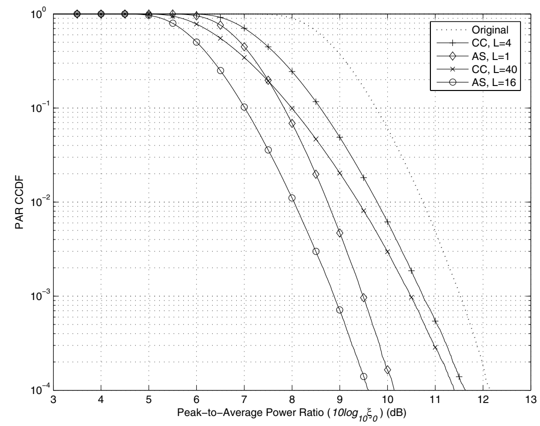

Fig. 5. PAR CCDF comparison of AS and CC, where $R = 20\%$, $A = 4\,\text{dB}$, and $\mathcal{R}$ is consecutive.

clipping noise as the PAR reduction signal, and adaptively scales it to reduce the PAR. Simulation results show that the PAR and the complexity of the proposed algorithm are lower than that of the controlled clipper algorithm.

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