On Decoding, Mutual Information, and Antenna Selection Diversity for Quasi-Orthogonal STBC with Minimum Decoding Complexity

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***Abstract*— ABBA codes are an important class of quasi-orthogonal space-time block codes proposed by Tirkkonen *et al.*. Recently, they have become more attractive for practical applications because Yuen *et al.* have shown that ABBA codes allow pair-wise real-symbol decoding (or equivalently, single-complex symbol decoding) complexity; it is the minimum decoding complexity (MDC) achievable by any non-OSTBC. Additionally, MDC-ABBA codes can achieve full diversity while their code rate is higher than that of OSTBC. In this paper, we present a *new, general, simple, and closed-form* method to decode MDC-ABBA codes. We explicitly derive the equivalent channel of MDC-ABBA codes and the maximum mutual information of MDC-ABBA. Furthermore, we prove that MDC-ABBA codes can achieve full diversity with transmit and/or receive antenna selection and full or limited feedback.**

I. Introduction

ABBA codes [1], a class of quasi-orthogonal space-time block codes (QSTBC), have a higher code rate than orthogonal space-time block codes (OSTBC) [2]. Since ABBA codes allow low complexity pair-wise complex-symbol decoding and perform better than OSTBC [3], they have been widely studied for coherent and non-coherent transmissions, beamforming, and others. Recently, Yuen *et al.* (see [4] and references therein) have shown that ABBA codes also enable pair-wise real-symbol (PWRS) decoding; they call such codes minimum decoding complexity (MDC) codes. Thus, while the decoding complexity of MDC-ABBA codes is equal to single complex-symbol decoding complexity, their code rate higher than that of OSTBC. In the following, we reserve the term "ABBA" for the QSTBC proposed by Tirkkonen *et al.* [1] with pair-wise complex-symbol decoding [3] and the term "MDC-ABBA" for the ABBA codes with PWRS decoding [4].

Recently, several authors have investigated closed-loop methods using phase feedback for ABBA codes (see, e.g. [5]) so that single complex symbol decoding is possible. However, these methods may be unnecessary since the ABBA-QSTBC are already single-symbol decodable. A few decoders designed for ABBA codes with 4 or 6 transmit antennas [6], [7] are also presented.

Despite of many studies on ABBA codes, the equivalent channel of ABBA and also of MDC-ABBA codes is unknown in the most general case with arbitrary numbers of transmit (Tx) and receive (Rx) antennas. Additionally, suitable closed loop methods for MDC-ABBA codes are not studied so far.

In this paper, we propose a new, general, simple, and closed-form method to decode ABBA and MDC-ABBA codes. We show how the ABBA space-time (ST) channel can be decoupled into parallel independent channels, each of which carries a pair of data symbols. Using this new representation of the equivalent channel, we derive the maximum mutual information (MMI) of ABBA/MDC-ABBA codes. Finally, we show that MDC-ABBA codes achieve full diversity with Tx/Rx antenna selection and with full or limited feedback [8].

II. Decoders for ABBA and MDC-ABBA Codes

A. System Model and Preliminaries

We consider a quasi-static Rayleigh flat fading multiple-antenna channel. The transmitter and receiver are equipped with M Tx and N Rx antennas. The receiver, but not the transmitter, completely knows the channel gains.

From matrix representation theory, the mapping of a block of K data symbols $(s_1, s_2, \cdots, s_K)$ into a $T \times M$ code matrix of a STBC can be generally represented as follows [9]:

$$\mathcal{X}_M = \sum_{k=1}^{K} (s_k A_k + s_k^* B_k) \quad (1)$$

where A_k and B_k, $(k = 1, 2, \cdots, K)$ are $T \times M$ constant basis matrices, superscript * denotes conjugate [1]. The average energy of code matrices $X \in \mathcal{X}_M$ is constrained such that $\mathbb{E}[\|X\|_\mathsf{F}^2] = T$. The code rate $\mathsf{R}_{\mathcal{X}_M}$ of a STBC $\mathcal{X}_M$, in symbols per channel use (pcu), is defined by $\mathsf{R}_{\mathcal{X}_M} = K/T$.

We now review the main properties of OSTBC $\mathcal{O}_M$ to be used later. The basis matrices of OSTBC satisfy [2]:

$$A_i^\dagger A_i + B_i^\dagger B_i = \boldsymbol{I}_M, \qquad i = 1, 2, \cdots, K \quad (2a)$$
$$A_i^\dagger A_j + B_j^\dagger B_i = \mathbf{0}_M, \qquad 1 \le i < j \le K \quad (2b)$$
$$A_i^\dagger B_j + A_j^\dagger B_i = \mathbf{0}_M, \qquad i, j = 1, 2, \cdots, K. \quad (2c)$$

[1]From now on, superscripts $^\mathsf{T}$ and † denote matrix transpose and transpose conjugate. The $n \times n$ identity and all-zero matrices are denoted by $\boldsymbol{I}_n$ and $\mathbf{0}_n$, respectively; $\mathrm{diag}(\boldsymbol{x})$ denotes a diagonal matrix with elements of vector $\boldsymbol{x}$ on its main diagonal. $\|X\|_\mathsf{F}$ denotes Frobenius norm of matrix X and $\otimes$ denotes Kronecker product [10]. $\mathbb{E}[\cdot]$ denotes average. A mean-m and variance-σ^2 circularly complex Gaussian random variable is written by $\mathcal{CN}(m, \sigma^2)$. $\Re(X)$ and $\Im(X)$ stand for the real and imaginary parts of a matrix X, respectively.

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Let the data symbols are drawn from a constellation with unit average power. To guarantee the average power constraint, the OSTBC matrices are multiplied by a constant $\kappa = \frac{1}{M\mathsf{R}_{\mathcal{O}_M}}$. For notational brevity, κ is not shown.

We next examine the algebraic structure of ABBA QSTBC codes. Let A_k and B_k $(k = 1, 2, \cdots, K)$ be the $t \times m$ basis matrices of an OSTBC $\mathcal{O}_m$. Two blocks of data, each of K symbols, are mapped into two code matrices $\mathcal{A}$ and $\mathcal{B}$ of $\mathcal{O}_m$ as $\mathcal{A} = \sum_{k=1}^{K}(s_k A_k + s_k^* B_k), \mathcal{B} = \sum_{k=1}^{K}\left(s_{k+K}A_k + s_{k+K}^* B_k\right)$. The ABBA code matrices for $M = 2m$ Tx antennas are constructed from $\mathcal{O}_m$ as

$$\mathcal{Q}_M = \begin{bmatrix} \mathcal{A} & \mathcal{B} \\ \mathcal{B} & \mathcal{A} \end{bmatrix} = \sum_{k=1}^{K}(\mathcal{C}_k \otimes A_k + \mathcal{C}_k^\dagger \otimes B_k) \tag{3}$$

where $\mathcal{C}_k = (s_k\Pi^0 + s_{k+K}\Pi)$, $\Pi = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$. Note that $\Pi = \Pi^{-1}, \Pi^2 = \boldsymbol{I}_2$.

B. Equivalent Channel of ABBA and MDC-ABBA Codes

For the sake of simplicity, we first consider one Rx antenna and generalize the results for $N > 1$ later.

Let $\boldsymbol{h} = [h_1 \; h_2 \; \cdots \; h_M]^\mathsf{T}$ denote the channel vector with $h_i \sim \mathcal{CN}(0, 1)$. Let $Q \in \mathcal{Q}_M$ be a transmitted code matrix, the Rx signal vector is $\boldsymbol{y} = \sqrt{\rho\kappa}\, Q\boldsymbol{h} + \boldsymbol{w}$, where $\boldsymbol{w}$ is noise vector with independently, identically distributed (i.i.d.) entries $\sim \mathcal{CN}(0, 1)$; ρ is the average Rx signal-to-noise ratio (SNR).

The conventional approach for decoding ABBA codes can start from expanding the metric $\|\boldsymbol{y} - \sqrt{\rho\kappa}Q\boldsymbol{h}\|_\mathsf{F}^2$. In the following, we present a new method, in which the equivalent channel of ABBA codes is represented in an elegant form.

From (3), we have

$$\boldsymbol{y} = \sqrt{\rho\kappa}\sum_{k=1}^{K}\sum_{i=1}^{2}\Big[\left(\Pi^{i-1} \otimes A_k\right)\boldsymbol{h}s_{k+(i-1)K} + \left(\Pi^{1-i} \otimes B_k\right)\boldsymbol{h}s_{k+(i-1)K}^*\Big] + \boldsymbol{w}\,. \tag{4}$$

Let $\boldsymbol{e}_{ki} = \left(\Pi^{i-1} \otimes A_k\right)\boldsymbol{h}, E_k = \begin{bmatrix}\boldsymbol{e}_{k1} & \boldsymbol{e}_{k2}\end{bmatrix}, \boldsymbol{f}_{ki} = \left(\Pi^{1-i} \otimes B_k\right)\boldsymbol{h}, F_k = \begin{bmatrix}\boldsymbol{f}_{k1} & \boldsymbol{f}_{k2}\end{bmatrix}$, and $\boldsymbol{s}_k = \begin{bmatrix}s_k & s_{k+K}\end{bmatrix}^\mathsf{T}$, (4) can be rewritten as

$$\boldsymbol{y} = \sqrt{\rho\kappa}\begin{bmatrix}E_1 & F_1 & E_2 & F_2 & \cdots & E_K & F_K\end{bmatrix} \times \begin{bmatrix}\boldsymbol{s}_1^\mathsf{T} & \boldsymbol{s}_1^\dagger & \boldsymbol{s}_2^\mathsf{T} & \boldsymbol{s}_2^\dagger & \cdots & \boldsymbol{s}_K^\mathsf{T} & \boldsymbol{s}_K^\dagger\end{bmatrix}^\mathsf{T} + \boldsymbol{w}\,. \tag{5}$$

We now use a trick in [11] to decode OSTBC for our next derivation. The following equation is equivalent to (5):

$$\begin{bmatrix}\boldsymbol{y} \\ \boldsymbol{y}^*\end{bmatrix} = \sqrt{\rho\kappa}\underbrace{\begin{bmatrix}E_1 & F_1 & \cdots & E_K & F_K \\ F_1^* & E_1^* & \cdots & F_K^* & E_K^*\end{bmatrix}}_{G} \times \begin{bmatrix}\boldsymbol{s}_1^\mathsf{T} & \boldsymbol{s}_1^\dagger & \cdots & \boldsymbol{s}_K^\mathsf{T} & \boldsymbol{s}_K^\dagger\end{bmatrix}^\mathsf{T} + \begin{bmatrix}\boldsymbol{w} \\ \boldsymbol{w}^*\end{bmatrix}. \tag{6}$$

We can show that the columns of matrix G are orthogonal.

Proof: We will show that the following equations hold:

$$\begin{bmatrix}E_k \\ F_k^*\end{bmatrix}^\dagger\begin{bmatrix}E_l \\ F_l^*\end{bmatrix} = E_k^\dagger E_l + F_k^\mathsf{T}F_l^* = \boldsymbol{0}_2 \quad \text{for } k \neq l, \tag{7a}$$

$$\begin{bmatrix}E_k \\ F_k^*\end{bmatrix}^\dagger\begin{bmatrix}F_l \\ E_l^*\end{bmatrix} = E_k^\dagger F_l + F_k^\mathsf{T}E_l^* = \boldsymbol{0}_2 \quad \forall k, l. \tag{7b}$$

We just provide the proof for (7a); (7b) can be shown similarly. Let $Z_{kl} = (E_k^\dagger E_l + F_k^\mathsf{T}F_l^*)$, its element can be calculated as

$$[Z_{kl}]_{ij} = \boldsymbol{e}_{ki}^\dagger\boldsymbol{e}_{lj} + \boldsymbol{f}_{ki}^\mathsf{T}\boldsymbol{f}_{lj}^* = \boldsymbol{h}^\dagger[(\Pi^{j-i}) \otimes (A_k^\dagger A_l + B_k^\dagger B_l)]\boldsymbol{h} = \begin{cases} 0, & k \neq l; \\ \boldsymbol{h}^\dagger(\Pi^{j-i} \otimes \boldsymbol{I}_m)\boldsymbol{h}, & k = l. \end{cases} \tag{8}$$

Thus, $Z_{kl} = \boldsymbol{0}_2$ if $k \neq l$. □

Since for $k = l$, the matrices $Z_{kk} = Z\,\forall k$, where the entries of Z are $z_{ij} = \boldsymbol{h}^\dagger(\Pi^{j-i} \otimes \boldsymbol{I}_m)\boldsymbol{h}$. In particular, $z_{1,1} = z_{2,2} = \|\boldsymbol{h}\|_\mathsf{F}^2, z_{1,2} = z_{2,1} = \sum_{i=1}^{m}(h_i h_{i+m}^* + h_i^* h_{i+m})$. Therefore, Z *is a circulant real matrix* and can be represented as

$$Z = \sum_{i=1}^{m} H_i^\dagger H_i \tag{9}$$

where $H_i = \begin{bmatrix} h_i & h_{i+m} \\ h_{i+m} & h_i \end{bmatrix}$. To separate the transmitted vector $\boldsymbol{s}_k (k = 1, 2, \ldots K)$ at the receiver, we multiply the two sides of (6) with $\begin{bmatrix}E_k^\dagger & F_k^\mathsf{T}\end{bmatrix}$ to get

$$E_k^\dagger\boldsymbol{y} + F_k^\mathsf{T}\boldsymbol{y}^* = \sqrt{\rho\kappa}Z\boldsymbol{s}_k + (E_k^\dagger\boldsymbol{w} + F_k^\mathsf{T}\boldsymbol{w}^*)\,. \tag{10}$$

Thus $\begin{bmatrix}E_k^\dagger & F_k^\mathsf{T}\end{bmatrix}$ plays the role of the spatial signature of the data vector $\boldsymbol{s}_k$.

We now generalize the result of (10) for the case of multiple receive antennas, $N \geq 1$. The subscript n $(n = 1, 2, \ldots, N)$ is added to the channel gain vector $\boldsymbol{h}$. The channel matrix $\mathcal{H}$ is therefore written as $\mathcal{H} = \begin{bmatrix}\boldsymbol{h}_1 & \boldsymbol{h}_2 & \cdots & \boldsymbol{h}_N\end{bmatrix}$, where $\boldsymbol{h}_n = \begin{bmatrix}h_{1n} & h_{2n} & \cdots & h_{Mn}\end{bmatrix}^\mathsf{T}$.

It is not hard to show that the matrix Z in (9) becomes

$$Z = \sum_{j=1}^{N}\sum_{i=1}^{m} H_{i,j}^\dagger H_{i,j} \tag{11}$$

where $H_{i,j} = \begin{bmatrix} h_{i,j} & h_{i+m,j} \\ h_{i+m,j} & h_{i,j} \end{bmatrix}$. Therefore, (10) is generalized for multiple Rx antennas as follows:

$$\underbrace{\sum_{n=1}^{N}(E_{kn}^\dagger\boldsymbol{y}_n + F_{kn}^\mathsf{T}\boldsymbol{y}_n^*)}_{\hat{\boldsymbol{y}}_k} = \sqrt{\rho\kappa}Z\boldsymbol{s}_k + \underbrace{\sum_{n=1}^{N}(E_{kn}^\dagger\boldsymbol{w}_n + F_{kn}^\mathsf{T}\boldsymbol{w}_n^*)}_{\bar{\boldsymbol{w}}_k} \tag{12}$$

where $\boldsymbol{y}_n$ is the received signal vector of the nth antenna,

$$E_{kn} = \begin{bmatrix}\boldsymbol{e}_{k1,n} & \boldsymbol{e}_{k2,n}\end{bmatrix}, \quad \text{for } k = 1, 2, \ldots, K, \tag{13a}$$

$$\boldsymbol{e}_{ki,n} = \left(A_k \otimes \Pi^{i-1}\right)\boldsymbol{h}_n, \quad \text{for } i = 1, 2, \tag{13b}$$

$$F_{kn} = \begin{bmatrix}\boldsymbol{f}_{k1,n} & \boldsymbol{f}_{k2,n}\end{bmatrix}, \tag{13c}$$

$$\boldsymbol{f}_{ki,n} = \left(B_k \otimes \Pi^{1-i}\right)\boldsymbol{h}_n\,, \tag{13d}$$

and $\bar{\boldsymbol{w}}_k$ is noise vector with covariance matrix $V = \mathbb{E}[\bar{\boldsymbol{w}}_k\bar{\boldsymbol{w}}_k^\dagger] = Z \neq \boldsymbol{I}_M$. The color noise $\bar{\boldsymbol{w}}_k$ can be whitened by a whitening matrix $\hat{H}^{-1} = Z^{-\frac{1}{2}}$; (12) with whitened noise is given below

$$\hat{H}^{-1}\hat{\boldsymbol{y}}_k = \sqrt{\rho\kappa}\hat{H}\boldsymbol{s}_k + \underbrace{\hat{H}^{-1}\bar{\boldsymbol{w}}_k}_{\hat{\boldsymbol{w}}_k}. \quad (14)$$

Thus, (14) *is the general equivalent Tx/Rx signal relation for ABBA codes.*

C. General Decoder for ABBA Codes

To achieve full diversity, K data symbols s_{k+K} ($k = 1, 2, \ldots, K$) must be rotated by an angle α [3]. If $s_k \in \mathcal{S}$, where $\mathcal{S}$ is a unit average energy constellation, for example QAM, PSK, then $s_{k+K} \in e^{\mathrm{j}\,\alpha}\mathcal{S}$. Including the complex symbol rotation, (14) becomes

$$\hat{H}^{-1}\hat{\boldsymbol{y}}_k = \sqrt{\rho\kappa}\hat{H}\,\mathrm{diag}(1, e^{\mathrm{j}\,\alpha})\boldsymbol{s}_k + \hat{\boldsymbol{w}}_k, \qquad \mathrm{j}^2 = -1. \quad (15)$$

Let $R = \mathrm{diag}(1, e^{\mathrm{j}\,\alpha})$, $\bar{\boldsymbol{s}} = [\bar{s}_1 \;\; \bar{s}_2]$, where $\bar{s}_1 \in \mathcal{S}, \bar{s}_2 \in e^{\mathrm{j}\,\alpha}\mathcal{S}$ the maximum likelihood (ML) solution of (15) is

$$\boldsymbol{s}_k = \arg\min_{\bar{\boldsymbol{s}}} \|\hat{H}^{-1}\hat{\boldsymbol{y}}_k - \sqrt{\rho\kappa}\hat{H}R\bar{\boldsymbol{s}}\|_{\mathsf{F}}^2. \quad (16)$$

One can use a sphere decoder [12] to solve (16). Additionally, the right hand side of (16) can be simplified so that

$$\boldsymbol{s}_k = \arg\min_{\bar{\boldsymbol{s}}_k} \left(\rho\kappa\bar{\boldsymbol{s}}_k^{\mathsf{T}}R^{\dagger}ZR\bar{\boldsymbol{s}}_k - 2\sqrt{\rho\kappa}\Re(\hat{\boldsymbol{y}}_k^{\dagger}R\bar{\boldsymbol{s}}_k)\right). \quad (17)$$

D. General Decoder of MDC-ABBA Codes

Since $\hat{H}$ is real, we can rewrite (12) by decoupling the real and imaginary parts of the two sides of (12) as

$$\begin{bmatrix}\hat{H}^{-1}\Re(\hat{\boldsymbol{y}}_k)\\ \hat{H}^{-1}\Im(\hat{\boldsymbol{y}}_k)\end{bmatrix} = \sqrt{\rho\kappa}\begin{bmatrix}\hat{H} & \boldsymbol{0}_2\\ \boldsymbol{0}_2 & \hat{H}\end{bmatrix}\begin{bmatrix}\Re(\boldsymbol{s}_k)\\ \Im(\boldsymbol{s}_k)\end{bmatrix} + \begin{bmatrix}\Re(\hat{\boldsymbol{w}}_k)\\ \Im(\hat{\boldsymbol{w}}_k)\end{bmatrix}. \quad (18)$$

In order to achieve full-diversity, signal transformations are required. There are two existing signal transformation methods proposed by (1) Yuen, Guan, and Tjhung [4] and (2) Wang, Wang, and Xia [13] for QAM constellations. Due to the space limit, we present the results with the signal transformation by Yuen-Guan-Tjhung only.

Let the input symbols are $d_k = a_k + \mathrm{j}\,b_k, d_{k+K} = a_{k+K} + \mathrm{j}\,b_{k+K}$, where $d_k, d_{k+K} \in \mathcal{S}$. Let $s_k = p_k + \mathrm{j}\,q_k, s_{k+K} = p_{k+K} + \mathrm{j}\,q_{k+K}$ be the transmitted symbols. We can jointly transform the real input symbols a_k, b_k, a_{k+K} and b_{k+K} by a real transformation $\mathcal{R}$ to generate transmitted symbols p_k, q_k, p_{k+K}, and q_{k+K} as

$$\begin{bmatrix}p_k & p_{k+K} & q_k & q_{k+K}\end{bmatrix}^{\mathsf{T}} = \mathcal{R}\begin{bmatrix}a_k & b_k & a_{k+K} & b_{k+K}\end{bmatrix}^{\mathsf{T}}. \quad (19)$$

It can be showed that the rotation in [4] is of the form

$$\mathcal{R}_{YGT} = \begin{bmatrix}U & \boldsymbol{0}_2\\ \boldsymbol{0}_2 & U\end{bmatrix} \quad (20)$$

where

$$U = \begin{bmatrix}\cos(\alpha) & \sin(\alpha)\\ \sin(\alpha) & -\cos(\alpha)\end{bmatrix}, \quad (21)$$

and optimal angle for QAM in terms of coding gain is $\alpha = \frac{1}{2}\arctan(\frac{1}{2}) = 13.2825^0$ [4].

Using (19) and (20), we rewrite (18) as

$$\hat{H}^{-1}\Re(\hat{\boldsymbol{y}}_k) = \sqrt{\rho\kappa}\hat{H}U\begin{bmatrix}a_k & b_k\end{bmatrix}^{\mathsf{T}} + \Re(\hat{\boldsymbol{w}}_k), \quad (22a)$$

$$\hat{H}^{-1}\Im(\hat{\boldsymbol{y}}_k) = \sqrt{\rho\kappa}\hat{H}U\begin{bmatrix}a_{k+K} & b_{k+K}\end{bmatrix}^{\mathsf{T}} + \Im(\hat{\boldsymbol{w}}_k). \quad (22b)$$

Let $\boldsymbol{c}_k = \begin{bmatrix}a_k & b_k\end{bmatrix}^{\mathsf{T}}$, $\boldsymbol{c}_{k+K} = \begin{bmatrix}a_{k+K} & b_{k+K}\end{bmatrix}^{\mathsf{T}}$, and $\bar{\boldsymbol{c}} = \begin{bmatrix}\bar{a} & \bar{b}\end{bmatrix}^{\mathsf{T}}$ such that $\bar{d} = (\bar{a} + \mathrm{j}\,\bar{b}) \in \mathcal{S}$. The ML solutions for (22a) and (22b) are

$$\boldsymbol{c}_k = \arg\min_{\bar{\boldsymbol{c}}} \left(\rho\kappa\bar{\boldsymbol{c}}^{\mathsf{T}}UZU\bar{\boldsymbol{c}} - 2\sqrt{\rho\kappa}\bar{\boldsymbol{c}}^{\mathsf{T}}U\Re(\hat{\boldsymbol{y}}_k)\right), \quad (23a)$$

$$\boldsymbol{c}_{k+K} = \arg\min_{\bar{\boldsymbol{c}}} \left(\rho\kappa\bar{\boldsymbol{c}}^{\mathsf{T}}UZU\bar{\boldsymbol{c}} - 2\sqrt{\rho\kappa}\bar{\boldsymbol{c}}^{\mathsf{T}}U\Im(\hat{\boldsymbol{y}}_k)\right). \quad (23b)$$

The above equations (23a) *and* (23b) *are the general detection equations of MDC-ABBA codes.*

From (14) and (22), $\hat{H}$ is the *equivalent channel* of ABBA and MDC-ABBA codes. The important properties of the equivalent channel $\hat{H}$ are given as follows.

Lemma 1: The equivalent channel matrix $\hat{H}$ and its inversion $\hat{H}^{-1}$ are real and circulant.

III. Maximum Mutual Information

The MMI of ABBA (and also MDC-ABBA) codes can be calculated using the equivalent channel $\hat{H}$ [9].

$$\begin{aligned}\mathsf{C}_{\mathcal{Q}_{2m}} &= \frac{K}{T}\,\mathbb{E}\left\{\log_2\det\left(\boldsymbol{I}_2 + \rho\kappa\hat{H}^{\dagger}\hat{H}\right)\right\}\\ &= \frac{K}{T}\,\mathbb{E}\left[\log_2\det\left(\boldsymbol{I}_2 + \rho\kappa\sum_{j=1}^{N}\sum_{i=1}^{m}H_{i,j}^{\dagger}H_{i,j}\right)\right].\end{aligned} \quad (24)$$

The coefficient $\frac{K}{T}$ appears because that the MMI of ABBA codes is a sum of MMI of K orthogonal blocks of data averaged over T channel uses.

We can use a unitary discrete Fourier transform matrix F_2 to diagonalize the circulant matrices $H_{i,j}$ without changing the distribution of $\mathsf{C}_{\mathcal{Q}}$. Let $\lambda_{i,j,p}$ $(p = 1, 2)$ be the eigenvalues of $H_{i,j}$. Since the vectors of eigenvalues are the Fourier transform of the channel vector $[h_i, h_{i+m}]^{\mathsf{T}}$. Thus, $\lambda_{i,j,p}$ are independent and $\lambda_{i,j,p} \sim \mathcal{CN}(0, 1)$. By denoting $\boldsymbol{\Lambda}_{i,j} = \mathrm{diag}\,(\lambda_{i,j,1}, \lambda_{i,j,2})$, (24) becomes

$$\mathsf{C}_{\mathcal{Q}} = \mathsf{R}_{\mathcal{O},m}\,\mathbb{E}\left[\log_2\det\left(1 + \frac{\rho}{m\mathsf{R}_{\mathcal{O},m}}\sum_{j=1}^{N}\sum_{i=1}^{m}|\lambda_{i,j,p}|^2\right)\right]. \quad (25)$$

In (25), $p = 1$ or 2 does not change the distribution of $\mathsf{C}_{\mathcal{Q}}$; therefore, we can set $p = 1$ without loss of generality. Furthermore, let $\bar{H} \in \mathbb{C}^{m \times N}$ with entries $\lambda_{i,j,1}$, we have $\sum_{j=1}^{N}\sum_{i=1}^{|\lambda_{i,j,1}|^2} = \|\bar{H}\|_{\mathsf{F}}^2$. We arrive at the new expression of $\mathsf{C}_{\mathcal{Q}}$ below.

$$\mathsf{C}_{\mathcal{Q}_{2m}} = \mathsf{R}_{\mathcal{O}_m}\,\mathbb{E}\left\{\log_2\det\left[1 + \frac{\rho}{m\mathsf{R}_{\mathcal{O}_m}}\|\bar{H}\|_{\mathsf{F}}^2\right]\right\} = \mathsf{C}_{\mathcal{O}_m} \quad (26)$$

where $\mathsf{C}_{\mathcal{O},m}$ is the MMI of the underlying OSTBC $\mathcal{O}_m$ [9], [14], which is used to construct ABBA codes. Therefore,

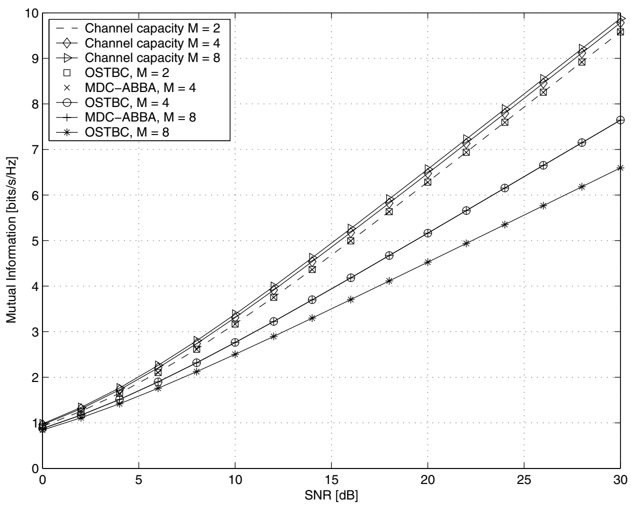

Fig. 1. Channel capacity and maximum mutual information of ABBA/MDC-ABBA codes and OSTBC over multiple-input single-output channels.

1) The MMI of ABBA/MDC-ABBA codes for $M = 2m$ Tx antennas equals to that of OSTBC for m Tx antennas; i.e., by doubling number of Tx antennas and replacing OSBTC by ABBA/MDC-ABBA codes, one can get higher diversity benefit but not the capacity benefit.
2) Compared with OSTBC, MDC-ABBA codes can attain larger portion of channel capacity.

The MMI of ABBA/MDC-ABBA codes and OSTBC (maximal rates), and channel capacity illustrated in Fig. 1 (for $M = 2, 4, 8$ and $N = 1$) agree with the above analysis.

IV. MDC-ABBA Codes with Antenna Selection

Since $\Re(\hat{\boldsymbol{w}}_k)$ and $\Im(\hat{\boldsymbol{w}}_k)$ in (22) are real Gaussian vectors with i.i.d. entries (zero-mean and variance $N_0 = 1/2$), the data vectors $\begin{bmatrix} a_k & b_k \end{bmatrix}^\mathsf{T}$ and $\begin{bmatrix} a_{k+K} & b_{k+K} \end{bmatrix}^\mathsf{T}$ $k = 1, 2, \ldots, K$) experience the same channels. Thus, they are subject to the same error probability. Furthermore, the pair-wise error probability (PEP) of each vector is also the symbol PEP (SPEP). The subscript k of symbols can be omitted for brevity.

Consider two arbitrary symbols $d = a + \mathrm{j}\, b$ and $\hat{d} = \hat{a} + \mathrm{j}\, \hat{b}$. Denote $\delta_1 = a - \hat{a}, \delta_2 = b - \hat{b}, \Delta = [\delta_1\ \delta_2]^\mathsf{T}$, the conditional SPEP of d and $\hat{d}$ can be expressed using the Gaussian Q-function as

$$P(d \to \hat{d}|\hat{H}) = Q\left(\sqrt{\frac{\rho\kappa\|\hat{H}R\Delta\|^2}{4N_0}}\right). \qquad (27)$$

We will derive a convenient form of the argument of the Gaussian Q-function above in the following.

$$x \triangleq \|\hat{H}R\Delta\|^2 = (R\Delta)^\dagger \hat{H}^\dagger \hat{H}(R\Delta) = (R\Delta)^\dagger Z(R\Delta). \quad (28)$$

Using the DFT matrix F_2 to diagonalize Z, we have

$$Z = \sum_{j=1}^{N}\sum_{i=1}^{M/2} H_{i,j}^\dagger H_{i,j} = \sum_{j=1}^{N}\sum_{i=1}^{M/2} F_2 \operatorname{diag}(|\lambda_{i,j,1}|^2, |\lambda_{i,j,2}|^2)F_2. \qquad (29)$$

Substituting Z into (28), one has $x = \sum_{j=1}^{N}\sum_{i=1}^{M/2}\left[(F_2R\Delta)^\dagger \operatorname{diag}(|\lambda_{i,j,1}|^2, |\lambda_{i,j,2}|^2)(F_2R\Delta)\right]$. Let $[\beta_1\ \beta_2]^\mathsf{T} = F_2R\Delta$, note that β_1 and β_2 are real. Thus,

$$x = \sum_{j=1}^{N}\sum_{i=1}^{M/2}\left[\beta_1^2|\lambda_{i,j,1}|^2 + \beta_2^2|\lambda_{i,j,2}|^2\right]. \qquad (30)$$

Let $\bar{\beta}_1 = \min(|\beta_1|, |\beta_2|), \bar{\beta}_2 = \max(|\beta_1|, |\beta_2|)$, we have $x \geq \sum_{j=1}^{N}\sum_{i=1}^{M/2}\left[\bar{\beta}_1^2\left(|\lambda_{i,j,1}|^2 + |\lambda_{i,j,2}|^2\right)\right], x \leq \sum_{j=1}^{N}\sum_{i=1}^{M/2}\left[\bar{\beta}_2^2\left(|\lambda_{i,j,1}|^2 + |\lambda_{i,j,2}|^2\right)\right]$.

Since $[\lambda_{i,j,1}\quad \lambda_{i,j,2}]^\mathsf{T} = F_2[h_{i,j}\quad h_{i+M/2,j}]^\mathsf{T}$, we get $|\lambda_{i,j,1}|^2 + |\lambda_{i,j,2}|^2 = |h_{i,j}|^2 + |h_{i+M/2,j}|^2$. Therefore

$$\bar{\beta}_1^2\|\mathcal{H}\|^2 \leq x \leq \bar{\beta}_2^2\|\mathcal{H}\|^2. \qquad (31)$$

Actually, $\hat{H}$ is dependent on $\mathcal{H}$, we thus rewrite the upper and lower bounds of conditional SPEP as

$$Q\left(\sqrt{\frac{\rho\kappa\bar{\beta}_2^2\|\mathcal{H}\|_\mathsf{F}^2}{2}}\right) \leq P(d \to \hat{d}|\mathcal{H}) \leq Q\left(\sqrt{\frac{\rho\kappa\bar{\beta}_1^2\|\mathcal{H}\|_\mathsf{F}^2}{2}}\right). \qquad (32)$$

If both $\bar{\beta}_1$ and $\bar{\beta}_2$ are nonzero for all distinct pairs of symbols, the lower and upper bounds of SPEP of MDC-QSTBC in (32) are simply a SPEP of some OSTBC transmitted over the same channel $\mathcal{H}$ with different SNR scales. Therefore, as long as $\bar{\beta}_1$ and $\bar{\beta}_2$ are nonzero, the SPEP of MDC-ABBA codes is bounded by two full-diversity SPEP curves, hence MDC-ABBA codes must achieve full diversity. We can show that the condition $\bar{\beta}_1$ and $\bar{\beta}_2$ are nonzero for all distinct pairs of symbols is also the necessary and sufficient condition for full diversity MDC-ABBA codes using the rank-determinant criteria with codeword PEP [4], [15]. The details of proof is omitted due to the space limit.

With transmit antenna selection (TAS), only M out of M_t available Tx antennas are used. The effective channel of MDC-ABBA codes with TAS is $\bar{\mathcal{H}}$, which consists of M columns with largest norm of $\mathcal{H}$ matrix. In this case, the matrix $\mathcal{H}$ in (32) is replaced by $\bar{\mathcal{H}}$. It is similar to the case of OSTBC with TAS [8]. Since OSTBC achieve full diversity with TAS, MDC-ABBA codes also achieve full diversity with TAS; more important, full diversity can be obtained with limited feedback [8]. The similar explanation can be given with receive antenna selection (RAS). Therefore, with TAS and RAS, MDC-ABBA codes always achieve full diversity with full or limited feedback.

V. Simulation Results

We present the simulation results using the new decoders for ABBA and MDC-ABBA codes to compare their performances. The diversity of MDC-ABBA codes with antenna selection is also verified. All signal constellations use Gray-bit mapping.

The performances of ABBA and MDC-ABBA codes for an open loop 4 Tx/1 Rx antenna system are compared in Fig. 2. While the performance of MDC-ABBA codes with 4- and 16QAM closely approach to that of ABBA codes, the former outperforms the latter with 8QAM square (8QAM-S)

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with signal points $(\pm 1, \pm \mathrm{j}, \pm 1 \pm \mathrm{j})$ (unnormalized power). Therefore the Gray-bit mapping may be not the optimal bit mapping for ABBA codes. Performance of OSTBC rate 3/4 symbol pcu [2] with 16QAM (3 bits pcu) is also plotted in Fig. 2. The MDC-ABBA code with 8QAM-S gains 0.5 dB over OSTBC with the same spectral efficiency of 3 bits pcu.

Performances of an MDC-ABBA code designed for 3 Tx antennas with TAS are presented in Fig. 3. The number of available antennas $M_t = 4$ and 1 Rx antenna. Compared with the open loop case, the MDC-ABBA code with TAS and 16QAM gains about 1.2 dB. Especially, the performance of $\binom{4}{3}$ TAS is slightly better than that of an ideal imaginative rate-one OSTBC using the same 16QAM. It is worthwhile to remember that the performance of an ideal hypothetical rate-one OSTBC is also the performance limit of ABBA-QSTBC with phase feedback scheme in [5]. The performance of the MDC-ABBA code is also compared with that of OSTBC for the same spectral efficiency of 3 bits pcu and TAS. In this case, MDC-ABBA codes gains 0.8 dB.

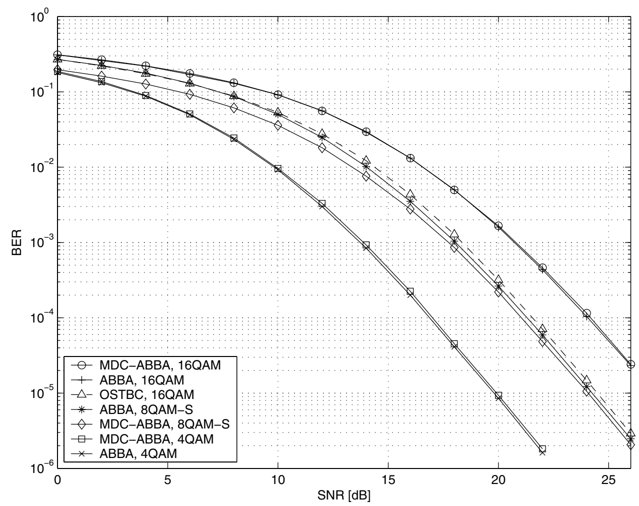

Fig. 2. Performances of MDC-ABBA codes for 4 Tx antennas compared with ABBA codes and OSTBC.

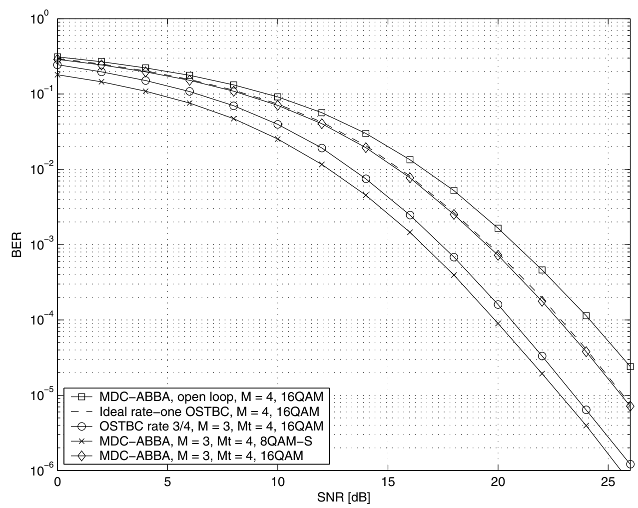

Fig. 3. Performances of MDC-ABBA codes and OSTBC designed for $M = 3$ with transmit antennas selection, $M_t = 4$, and 1 Rx antenna.

VI. Conclusion

We have presented a new general and closed form method to decode ABBA and MDC-ABBA codes. The general equivalent channel of these codes has been shown explicitly and it is used to derive the maximum mutual information of the codes. MDC-ABBA codes with Tx or Rx antenna selections and with full or limited feedback is also proved to achieve full diversity. Not only our analysis shows that the MDC-ABBA codes can achieve a higher portion of channel capacity than OSTBC, but also our simulations show that the former performs better than the latter. Therefore, MDC-ABBA codes might be a good candidate to replace OSTBC for open loop wireless channels with more than 2 transmit antennas.

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