On Decoding, Mutual Information, and Antenna Selection Diversity for Quasi-Orthogonal STBC with Minimum Decoding Complexity

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Abstract—ABBA codes are an important class of quasi-orthogonal space-time block codes proposed by Tirkkonen et al. Recently, they have become more attractive for practical applications because Yuen et al. have shown that ABBA codes allow pair-wise real-symbol decoding (or equivalently, single-complex symbol decoding) complexity; it is the minimum decoding complexity (MDC) achievable by any non-OSTBC. Additionally, MDC-ABBA codes can achieve full diversity while their code rate is higher than that of OSTBC. In this paper, we present a new, general, simple, and closed-form method to decode MDC-ABBA codes. We explicitly derive the equivalent channel of MDC-ABBA codes and the maximum mutual information of MDC-ABBA. Furthermore, we prove that MDC-ABBA codes can achieve full diversity with transmit and/or receive antenna selection and full or limited feedback.

I. INTRODUCTION

ABBA codes [1], a class of quasi-orthogonal space-time block codes (QSTBC), have a higher code rate than orthogonal space-time block codes (OSTBC) [2]. Since ABBA codes allow low complexity pair-wise complex-symbol decoding and perform better than OSTBC [3], they have been widely studied for coherent and non-coherent transmissions, beamforming, and others. Recently, Yuen et al. (see [4] and references therein) have shown that ABBA codes also enable pair-wise real-symbol (PQRS) decoding; they call such codes minimum decoding complexity (MDC) codes. Thus, while the decoding complexity of ABBA-MDC codes is equal to single complex-symbol decoding complexity, their code rate higher than that of OSTBC. In the following, we reserve the term "ABBA" for the QSTBC proposed by Tirkkonen et al. [1] with pair-wise complex-symbol decoding [3] and the term "MDC-ABBA" for the ABBA codes with PQRS decoding [4].

Recently, several authors have investigated closed-loop methods using phase feedback for ABBA codes (see, e.g. [5]) so that single complex symbol decoding is possible. However, these methods may be unnecessary since the ABBA-QSTBC are already single-symbol decodable. A few decoders designed for ABBA codes with 4 or 6 transmit antennas [6], [7] are also presented.

Despite of many studies on ABBA codes, the equivalent channel of ABBA and also of MDC-ABBA codes is unknown in the most general case with arbitrary numbers of transmit (Tx) and receive (Rx) antennas. Additionally, suitable closed loop methods for MDC-ABBA codes are not studied so far.

In this paper, we propose a new, general, simple, and closed-form method to decode ABBA and MDC-ABBA codes. We show how the ABBA-space-time (ST) channel can be decoupled into parallel independent channels, each of which carries a pair of data symbols. Using this new representation of the equivalent channel, we derive the maximum mutual information (MMI) of ABBA/MDC-ABBA codes. Finally, we show that MDC-ABBA codes achieve full diversity with Tx/Rx antenna selection and with full or limited feedback [8].

II. DECODERS FOR ABBA AND MDC-ABBA CODES

A. System Model and Preliminaries

We consider a quasi-static Rayleigh flat fading multiple-antenna channel. The transmitter and receiver are equipped with M Tx and N Rx antennas. The receiver, but not the transmitter, completely knows the channel gains.

From matrix representation theory, the mapping of a block of K data symbols \((s_1, s_2, \ldots, s_K)\) into a \(T \times M\) code matrix of a STBC can be generally represented as follows [9]:

\[
\mathbf{X}_M = \sum_{k=1}^{K} (s_k \mathbf{A}_k + s_k^* \mathbf{B}_k)
\]

where \(\mathbf{A}_k\) and \(\mathbf{B}_k\), \((k = 1, 2, \cdots, K)\) are \(T \times M\) constant basis matrices, superscript \(\dagger\) denotes conjugate. The average energy of code matrices \(\mathbf{X} \in \mathbf{X}_M\) is constrained such that \(\mathbb{E}[\|\mathbf{X}\|^2] = T\). The code rate \(R_{X_M}\) of a STBC \(\mathbf{X}_M\), in symbols per channel use (pcu), is defined by \(R_{X_M} = K/T\).

We now review the main properties of OSTBC \(\mathbf{O}_M\) to be used later. The basis matrices of OSTBC satisfy [2]:

\[
\begin{align*}
\mathbf{A}_i^\dagger \mathbf{A}_i + \mathbf{B}_i^\dagger \mathbf{B}_i &= \mathbf{I}_M, & i = 1, 2, \cdots, K \\
\mathbf{A}_i^\dagger \mathbf{A}_j + \mathbf{B}_i^\dagger \mathbf{B}_j &= \mathbf{0}_M, & 1 \leq i < j \leq K \\
\mathbf{A}_i^\dagger \mathbf{B}_j + \mathbf{B}_i^\dagger \mathbf{A}_j &= \mathbf{0}_M, & i, j = 1, 2, \cdots, K.
\end{align*}
\]

From now on, superscripts \(\dagger\) and \(\ast\) denote matrix transpose and transpose conjugate. The \(n \times n\) identity and all-zero matrices are denoted by \(\mathbf{I}_n\) and \(\mathbf{0}_n\), respectively; \(\text{diag}(\mathbf{a})\) denotes a diagonal matrix with elements of vector \(\mathbf{a}\) on its main diagonal. \(\|X\|\) denotes Frobenius norm of matrix \(X\) and \(\otimes\) denotes Kronecker product [10]. \(\mathbb{E}[.]\) denotes average. A mean-\(\sigma^2\) and variance-\(\sigma^2\) circularly complex Gaussian random variable is written by \(\mathbf{CN}(m, \sigma^2)\). \(\Re(X)\) and \(\Im(X)\) stand for the real and imaginary parts of a matrix \(X\), respectively.
Let the data symbols are drawn from a constellation with unit average power. To guarantee the average power constraint, the OSTBC matrices are multiplied by a constant $\kappa = \sqrt{\text{E}_{\text{b}} \text{E}_{\text{m}}}$. For notational brevity, $\kappa$ is not shown.

We next examine the algebraic structure of ABBA QSTBC codes. Let $A_k$ and $B_k$ ($k = 1, 2, \cdots, K$) be the $t \times m$ basis matrices of an OSTBC $O_m$. Two blocks of data, each of $K$ symbols, are mapped into two code matrices $A$ and $B$ of $O_m$ as $A = \sum_{k=1}^{K} (s_k A_k + s_k^* B_k)$, $B = \sum_{k=1}^{K} (s_k^* A_k + s_k B_k)$. The ABBA code matrices for $M = 2m$ Tx antennas are constructed from $O_m$ as

$$Q_M = \begin{bmatrix} A & B \\ B & A \end{bmatrix} = \sum_{k=1}^{K} (C_k \otimes A_k + C_k^* \otimes B_k) \quad (3)$$

where $C_k = (s_k \Pi^0 + s_{k+K} \Pi^2)$. Note that $\Pi^1 = \Pi^{-1}$, $\Pi^2 = I_2$.

### B. Equivalent Channel of ABBA and MDC-ABBA Codes

For the sake of simplicity, we first consider one Rx antenna and generalize the results for $N \geq 1$ later. Let $h = [h_1, h_2, \cdots, h_M]^T$ denote the channel vector with $h_i \sim \mathcal{CN}(0, 1)$. Let $Q \in Q_M$ be a transmitted code vector, the Rx signal vector is $y = \sqrt{\rho \kappa} Q h + w$, where $w$ is noise vector with independently, identically distributed (i.i.d) entries $\sim \mathcal{CN}(0, 1)$; $\rho$ is the average Rx signal-to-noise ratio (SNR).

The conventional approach for decoding ABBA codes can start from expanding the metric $\|y - \sqrt{\rho \kappa} Q h\|^2$. In the following, we present a new method, in which the equivalent channel of ABBA codes is represented in an elegant form.

From (3), we have

$$y = \sqrt{\rho \kappa} \sum_{k=1}^{K} \sum_{i=1}^{2} \left[ (\Pi_{i-1} \otimes A_k) h s_{k+(i-1)K} \right] + \left( \Pi_{i-1} \otimes B_k \right) h s_{k+(i-1)K} + w \quad (4)$$

Let $e_{ki} = (\Pi_{i-1} \otimes A_k) h$, $E_k = [e_{k1}, e_{k2}], f_{ki} = (\Pi_{i-1} \otimes B_k) h$, $F_k = [f_{k1}, f_{k2}]$, and $s_k = [s_k \ s_k^*]$, (4) can be rewritten as

$$y = \sqrt{\rho \kappa} \left[ \begin{array}{c} E_k \\ F_k \end{array} \right] \left[ \begin{array}{c} F_k^* \\ E_k^* \end{array} \right] + \left[ \begin{array}{c} s_k^T \\ s_k^{*T} \end{array} \right] + w \quad (5)$$

We now use a trick in [11] to decode OSTBC for our next derivation. The following equation is equivalent to (5):

$$\begin{bmatrix} y \\ y^* \end{bmatrix} = \sqrt{\rho \kappa} \begin{bmatrix} E_k \\ F_k \end{bmatrix} \begin{bmatrix} F_k^* \\ E_k^* \end{bmatrix} + \begin{bmatrix} s_k^T \\ s_k^{*T} \end{bmatrix} + \begin{bmatrix} w \\ w^* \end{bmatrix} \quad (6)$$

We can show that the columns of matrix $G$ are orthogonal.

**Proof:** We will show that the following equations hold:

$$E_k \begin{bmatrix} E_l \\ F_l \end{bmatrix} = E_k^* E_l + F_k^* F_l = 0_2 \quad \text{for } k \neq l \quad (7a)$$

$$E_k \begin{bmatrix} F_l \\ E_l \end{bmatrix} = E_k^* F_l + F_k^* E_l = 0_2 \quad \forall k, l \quad (7b)$$

We just provide the proof for (7a); (7b) can be shown similarly. Let $Z_k = (E_k^* E_l + F_k^* F_l)$, its element can be calculated as

$$[Z_k]_{ij} = e_{kj} e_{lj} + f_{kj}^* f_{lj} = h_1^H \left( (\Pi_{i-j}^0 \otimes (A_k A_l + B_k^* B_l)) \right) h_1$$

$$= \begin{cases} 0, & k \neq l; \\ h_1^H \left( (\Pi_{i-j}^1 \otimes I_m) \right) h_1, & k = l. \end{cases} \quad (8)$$

Thus, $Z_{kl} = 0_2$ if $k \neq l$.

Since for $k = l$, the matrices $Z_{kk} = Z \forall k$, where the entries of $Z$ are

$$z_{ij} = h_1^H \left( (\Pi_{i-j}^0 \otimes I_m) \right) h_1$$

in particular,

$$z_{11} = z_{22} = \sum_{m=1}^{m} (h_1 h_{1+m} + h_1 h_{1+m})$$

Therefore, $Z$ is a circulant real matrix and can be represented as

$$Z = \sum_{i=1}^{m} H_i^H H_i$$

where

$$H_i = \begin{bmatrix} h_{i1} & h_{i1+m} \\ h_{i2} & h_{i1} \end{bmatrix}$$

To separate the transmitted vector $s_k (k = 1, 2, \cdots, K)$ at the receiver, we multiply the two sides of (6) by $[E_k^T \ F_k^T]$ to get

$$E_k^T y + F_k^T y^* = \sqrt{\rho \kappa} Z s_k + (E_k^T w + F_k^T w^*)$$

Thus $[E_k^T \ F_k^T]$ plays the role of the spatial signature of the data vector $s_k$.

We now generalize the result of (10) for the case of multiple receive antennas, $N \geq 1$. The subscript $n$ ($n = 1, 2, \cdots, N$) is added to the channel gain vector $h$. The channel matrix $H$ is therefore written as $H = [h_1 \ h_2 \ \cdots \ h_N]$, where $h_n = [h_{n1} \ h_{n2} \ \cdots \ h_{nM}]^T$.

It is not hard to show that the matrix $Z$ in (9) becomes

$$Z = \sum_{j=1}^{N} \sum_{i=1}^{m} H_{i,j}^H H_{j,i}$$

where

$$H_{i,j} = \begin{bmatrix} h_{i,j} & h_{i,j+m} \\ h_{i,j} & h_{i,j} \end{bmatrix}$$

Therefore, (10) is generalized for multiple Rx antennas as follows:

$$\sum_{n=1}^{N} (E_{kn}^T y_n + F_{kn}^T y_n^*) = \sqrt{\rho \kappa} Z s_k + \sum_{n=1}^{N} (E_{kn}^T w_n + F_{kn}^T w_n^*)$$

where $y_n$ is the received signal vector of the $n$th antenna,

$$E_{kn} = [e_{k1,n} \ e_{k2,n}]^T \quad \text{for } k = 1, 2, \cdots, K \quad (13a)$$

$$e_{k,n} = (A_k \otimes \Pi_{i-1}^0) h_n \quad \text{for } i = 1, 2 \quad (13b)$$

$$F_{kn} = [f_{k1,n} \ f_{k2,n}]^T \quad (13c)$$

$$f_{k,n} = (B_k \otimes \Pi_{i-1}^1) h_n \quad (13d)$$

and $w_k$ is noise vector with covariance matrix $V = \mathbb{E}[w_k w_k^T] = Z \neq I_M$. The color noise $w_k$ can be whitened.
by a whitening matrix $\hat{H}^{-1} = Z^{-\frac{1}{2}}$; (12) with whitened noise is given below

$$\hat{H}^{-1}\hat{y}_k = \sqrt{\rho c}\hat{H}s_k + \sqrt{\rho c}\hat{w}_k,$$  \hspace{1cm} (14)

Thus, (14) is the general equivalent Tx/Rx signal relation for ABBA codes.

C. General Decoder for ABBA Codes

To achieve full diversity, $K$ data symbols $s_{k+K}$ ($k = 1, 2, \ldots, K$) must be rotated by an angle $\alpha$ [3]. If $s_k \in \mathbb{S}$, where $\mathbb{S}$ is a unit average energy constellation, for example QAM, PSK, then $s_{k+K} \in e^{j\alpha} \mathbb{S}$. Including the complex symbol rotation, (14) becomes

$$\hat{H}^{-1}\hat{y}_k = \sqrt{\rho c}\hat{H}\text{diag}(1, e^{j\alpha})s_k + \hat{w}_k, \hspace{1cm} j^2 = -1.$$  \hspace{1cm} (15)

Let $R = \text{diag}(1, e^{j\alpha})$, $\bar{s} = [\bar{s}_1 \ \bar{s}_2]$, where $\bar{s}_1 \in \mathbb{S}, \bar{s}_2 \in e^{j\alpha} \mathbb{S}$ the maximum likelihood (ML) solution of (15) is

$$s_k = \arg\min_{\bar{s}_k} \|\hat{H}^{-1}\hat{y}_k - \sqrt{\rho c}\hat{H}R\bar{s}_k\|_F^2.$$  \hspace{1cm} (16)

One can use a sphere decoder [12] to solve (16). Additionally, the right hand side of (16) can be simplified so that

$$s_k = \arg\min_{\bar{s}_k} \left( \rho c R^\dagger Z R \bar{s}_k - 2\sqrt{\rho c}\Re(\hat{y}_k^\dagger R\bar{s}_k) \right).$$  \hspace{1cm} (17)

D. General Decoder of MDC-ABBA Codes

Since $\hat{H}$ is real, we can rewrite (12) by decoupling the real and imaginary parts of the two sides of (12) as

$$\begin{bmatrix} \hat{H}^{-1}\Re(\hat{y}_k) \\ \hat{H}^{-1}\Im(\hat{y}_k) \end{bmatrix} = \sqrt{\rho c} \begin{bmatrix} \hat{H} & 0_2 \\ 0_2 & H \end{bmatrix} \begin{bmatrix} \Re(s_k) \\ \Im(s_k) \end{bmatrix} + \begin{bmatrix} \Re(\hat{w}_k) \\ \Im(\hat{w}_k) \end{bmatrix}.$$  \hspace{1cm} (18)

In order to achieve full diversity, symbol transformations are required. There are two existing signal transformation methods proposed by (1) Yuen, Guan, and Tjhung [4] and (2) Wang, Wang, and Xia [13] for QAM constellations. Due to the space limit, we present the results with the signal transformation proposed by Yuen-Guan-Tjhung only.

Let the input symbols are $d_k = a_k + jb_k, d_{k+K} = a_{k+K} + jb_{k+K}$, where $d_k, d_{k+K} \in \mathbb{S}$. Let $s_k = p_k + jq_k, s_{k+K} = p_{k+K} + jq_{k+K}$ be the transmitted symbols. We can jointly transform the real input symbols $a_k, b_k, a_{k+K}$ and $b_{k+K}$ by a real transformation $R$ to generate transmitted symbols $p_k, q_k, p_{k+K},$ and $q_{k+K}$ as

$$\begin{bmatrix} p_k \\ p_{k+K} \\ q_k \\ q_{k+K} \end{bmatrix}^\dagger = R \begin{bmatrix} a_k \\ b_k \\ a_{k+K} \\ b_{k+K} \end{bmatrix}.$$  \hspace{1cm} (19)

It can be showed that the rotation in [4] is of the form

$$R_{YGT} = \begin{bmatrix} U & 0_2 \\ 0_2 & U \end{bmatrix},$$  \hspace{1cm} (20)

where

$$U = \begin{bmatrix} \cos(\alpha) \\ \sin(\alpha) \end{bmatrix},$$  \hspace{1cm} (21)

and optimal angle for QAM in terms of coding gain is $\alpha = \frac{1}{2} \arctan(\frac{1}{2}) = 13.2825^0$ [4].

Using (19) and (20), we rewrite (18) as

$$\begin{bmatrix} \hat{H}^{-1}\Re(\hat{y}_k) = \sqrt{\rho c}Hu \begin{bmatrix} a_k \\ b_k \end{bmatrix} + \Re(\hat{w}_k) \\ \hat{H}^{-1}\Im(\hat{y}_k) = \sqrt{\rho c}Hu \begin{bmatrix} a_{k+K} \\ b_{k+K} \end{bmatrix} + \Im(\hat{w}_k) \end{bmatrix},$$  \hspace{1cm} (22a)

(22b)

Let $c_k = \begin{bmatrix} a_k \\ b_k \end{bmatrix}^\dagger, c_{k+K} = \begin{bmatrix} a_{k+K} \\ b_{k+K} \end{bmatrix}^\dagger$, and $\hat{e} = \begin{bmatrix} \hat{a} \\ \hat{b} \end{bmatrix}^\dagger$ such that $\hat{d} = (\hat{a} + j\hat{b}) \in \mathbb{S}$. The ML solutions for (22a) and (22b) are

$$c_k = \arg\min_{\hat{e}} \left( \rho c e^\dagger UZU^\dagger \hat{e} - 2\sqrt{\rho c}e^\dagger U\Re(\hat{y}_k) \right),$$  \hspace{1cm} (23a)

$$c_{k+K} = \arg\min_{\hat{e}} \left( \rho c e^\dagger UZU^\dagger \hat{e} - 2\sqrt{\rho c}e^\dagger U\Im(\hat{y}_k) \right).$$  \hspace{1cm} (23b)

The above equations (23a) and (23b) are the general detection equations of MDC-ABBA codes.

From (14) and (22), $H$ is the equivalent channel of ABBA and MDC-ABBA codes. The important properties of the equivalent channel $\hat{H}$ are given as follows.

Lemma 1: The equivalent channel matrix $\hat{H}$ and its inversion $\hat{H}^{-1}$ are real and circulant.

III. MAXIMUM MUTUAL INFORMATION

The MMI of ABBA (and also MDC-ABBA) codes can be calculated using the equivalent channel $\hat{H}$ [9].

$$C_{Q_{2m}} = \frac{K}{T} \mathbb{E} \left\{ \log_2 \det \left( I_2 + \rho c \hat{H}^\dagger \hat{H} \right) \right\}$$

$$= \frac{K}{T} \mathbb{E} \left\{ \log_2 \det \left( I_2 + \rho c \sum_{j=1}^{N} \sum_{i=1}^{m} H_{i,j}^\dagger H_{i,j} \right) \right\}. \hspace{1cm} (24)$$

The coefficient $\frac{K}{T}$ appears because that the MMI of ABBA codes is a sum of MMI of $K$ orthogonal blocks of data averaged over $T$ channel uses.

We can use a unitary discrete Fourier transform matrix $F_2$ to diagonalize the circulant matrices $H_{i,j}$ without changing the distribution of $C_Q$. Let $\lambda_{i,j,p}$ $(p = 1, 2)$ be the eigenvalues of $H_{i,j}$. Since the vectors of eigenvalues are the Fourier transform of the channel vector $[h_1, h_{1+m}]^\dagger$. Thus, $\lambda_{i,j,p}$ are independent and $\lambda_{i,j,p} \sim C \mathcal{N}(0, 1)$. By denoting $\Lambda_{i,j} = \text{diag}(\lambda_{i,j,1}, \lambda_{i,j,2}, \ldots)$, (24) becomes

$$C_Q = R_{\mathcal{O}_m} \mathbb{E} \left\{ \log_2 \det \left( I_2 + \frac{\rho}{mR_{\mathcal{O}_m}} \sum_{j=1}^{N} \sum_{i=1}^{m} |\lambda_{i,j,1}|^2 \right) \right\}. \hspace{1cm} (25)$$

In (25), $p = 1$ or $2$ does not change the distribution of $C_Q$; therefore, we can set $p = 1$ without loss of generality. Furthermore, let $\hat{H} \in \mathbb{C}^{m \times N}$ with entries $\lambda_{i,j,1}$, we have $\sum_{j=1}^{N} |\lambda_{i,j,1}|^2 = \|H\|_F^2$. We arrive at the new expression of $C_Q$ below.

$$C_{Q_{2m}} = R_{\mathcal{O}_m} \mathbb{E} \left\{ \log_2 \det \left[ I_2 + \frac{\rho}{mR_{\mathcal{O}_m}} \|H\|_F^2 \right] \right\} = C_{O_m} \hspace{1cm} (26)$$

where $C_{O_m}$ is the MMI of the underlying OSTBC $\mathcal{O}_m$ [9], [14], which is used to construct ABBA codes. Therefore,
Substituting $Z$ into (28), one has 

$$x = \sum_{j=1}^{N} \sum_{i=1}^{M/2} \left[ (F_2 R \Delta)^{\dagger} \text{diag}(|\lambda_{i,j,1}|^2, |\lambda_{i,j,2}|^2) (F_2 R \Delta) \right].$$

Let $[\beta_1, \beta_2]$ = $F_2 R \Delta$, note that $\beta_1$ and $\beta_2$ are real. Thus,

$$x = \sum_{j=1}^{N} \sum_{i=1}^{M/2} \left| \beta_1^2 |\lambda_{i,j,1}|^2 + \beta_2^2 |\lambda_{i,j,2}|^2 \right| .$$

(30)

Let $\bar{\beta}_1 = \min(|\beta_1|, |\beta_2|)$, $\bar{\beta}_2 = \max(|\beta_1|, |\beta_2|)$, we have

$$x \geq \sum_{j=1}^{N} \sum_{i=1}^{M/2} \left[ \bar{\beta}_2^2 (|\lambda_{i,j,1}|^2 + |\lambda_{i,j,2}|^2) \right] \leq \sum_{j=1}^{N} \sum_{i=1}^{M/2} \left[ \bar{\beta}_1^2 (|\lambda_{i,j,1}|^2 + |\lambda_{i,j,2}|^2) \right].$$

Since $|\lambda_{i,j,1}|^2 + |\lambda_{i,j,2}|^2 = |h_{i,j}|^2 + |h_{i,M/2,j}|^2$, we get

$$\bar{\beta}_1^2 \|\mathcal{H}\|^2 \leq x \leq \bar{\beta}_2^2 \|\mathcal{H}\|^2 .$$

(31)

Actually, $\hat{H}$ is dependent on $\mathcal{H}$, we thus rewrite the upper and lower bounds of conditional SPEP as

$$Q \left( \sqrt{\frac{\rho_1 \bar{\beta}_1^2 \|\mathcal{H}\|^2}{2}} \right) \leq P(d \rightarrow \hat{d} | \mathcal{H}) \leq Q \left( \sqrt{\frac{\rho_1 \bar{\beta}_2^2 \|\mathcal{H}\|^2}{2}} \right) .$$

(32)

If both $\beta_1$ and $\beta_2$ are nonzero for all distinct pairs of symbols, the lower and upper bounds of SPEP of MDC-OSTBC in (32) are simply a SPEP of some OSTBC transmitted over the same channel $\mathcal{H}$ with different SNR scales. Therefore, as long as $\beta_1$ and $\beta_2$ are nonzero, the SPEP of MDC-ABBA codes is bounded by two full-diversity SPEP curves, hence MDC-ABBA codes must achieve full diversity. We can show that the condition $\beta_1$ and $\beta_2$ are nonzero for all distinct pairs of symbols is also the necessary and sufficient condition for full diversity MDC-ABBA codes using the rank-determinant criteria with codeword PEP [4], [15]. The details of proof is omitted due to the space limit.

With transmit antenna selection (TAS), only $M$ out of $M_t$ available Tx antennas are used. The effective channel of MDC-ABBA codes with TAS is $\mathcal{H}$, which consists of $M$ columns with largest norm of $\mathcal{H}$ matrix. In this case, the matrix $\mathcal{H}$ in (32) is replaced by $\mathcal{H}$. It is similar to the case of OSTBC with TAS [8]. Since OSTBC achieve full diversity with TAS, MDC-ABBA codes also achieve full diversity with TAS; more important, full diversity can be obtained with limited feedback [8]. The similar explanation can be given with receive antenna selection (RAS). Therefore, with TAS and RAS, MDC-ABBA codes always achieve full diversity with full or limited feedback.

V. SIMULATION RESULTS

We present the simulation results using the new decoders for ABBA and MDC-ABBA codes to compare their performances. The diversity of MDC-ABBA codes with antenna selection is also verified. All signal constellations use Gray-bit mapping.

The performances of ABBA and MDC-ABBA codes for an open loop 4 Tx/1 Rx antenna system are compared in Fig. 2. While the performance of MDC-ABBA codes with 4- and 16QAM closely approach to that of ABBA codes, the former outperforms the latter with 8QAM square (8QAM-S).

1) The MMI of ABBA/MDC-ABBA codes for $M = 2m$ Tx antennas equals to that of OSTBC for $m$ Tx antennas; i.e., by doubling number of Tx antennas and replacing OSTBC by ABBA/MDC-ABBA codes, one can get higher diversity benefit but not the capacity benefit.

2) Compared with OSTBC, MDC-ABBA codes can attain larger portion of channel capacity.

The MMI of ABBA/MDC-ABBA codes and OSTBC (maximal rates), and channel capacity illustrated in Fig. 1 (for $M = 2, 4, 8$ and $N = 1$) agree with the above analysis.

IV. MDC-ABBA CODES WITH ANTENNA SELECTION

Since $\Re(\hat{w}_k)$ and $\Im(\hat{w}_k)$ in (22) are real Gaussian vectors with i.i.d. entries (zero-mean and variance $N_0 = 1/2$), the data vectors $[a_k \ b_k]^T$ and $[a_{k+K} \ b_{k+K}]^T \ k = 1, 2, \ldots, K$ experience the same channels. Thus, they are subject to the same error probability. Furthermore, the pair-wise error probability (PEP) of each vector is also the symbol PEP (SPEP). The subscript $k$ of symbols can be omitted for brevity.

Consider two arbitrary symbols $\hat{d} = a + j b$ and $\hat{d} = \hat{a} + j \hat{b}$. Denote $\delta_1 = a - \hat{a}$, $\delta_2 = b - \hat{b}$, $\Delta = [\delta_1 \ \delta_2]^T$, the conditional SPEP of $\hat{d}$ and $\hat{d}$ can be expressed using the Gaussian $Q$-function as

$$P(d \rightarrow \hat{d} | \hat{H}) = Q \left( \sqrt{\frac{\rho \|\hat{H} R \Delta\|^2}{4N_0}} \right) .$$

(27)

We will derive a convenient form of the argument of the Gaussian $Q$-function above in the following.

$$x \triangleq \|\hat{H} R \Delta\|^2 = (R \Delta)^{\dagger} \hat{H}^\dagger (R \Delta) = (R \Delta)^{\dagger} (R \Delta).$$

(28)

Using the DFT matrix $F_2$ to diagonalize $Z$, we have

$$Z = \sum_{j=1}^{N} \sum_{i=1}^{M/2} H_{i,j}^{\dagger} H_{i,j} = \sum_{j=1}^{N} \sum_{i=1}^{M/2} F_2 \text{diag}(|\lambda_{i,j,1}|^2, |\lambda_{i,j,2}|^2) F_2 .$$

(29)
with signal points \((\pm 1, \pm j, \pm 1, \pm j)\) (unnormalized power). Therefore the Gray-bit mapping may be not the optimal bit mapping for ABBA codes. Performance of OSTBC rate 3/4 symbol pcu [2] with 16QAM (3 bits pcu) is also plotted in Fig. 2. The MDC-ABBA code with 8QAM-S gains 0.5 dB over OSTBC with the same spectral efficiency of 3 bits pcu.

Performances of an MDC-ABBA code designed for 3 Tx antennas with TAS are presented in Fig. 3. The number of available antennas \(M_t = 4\) and 1 Rx antenna. Compared with the open loop case, the MDC-ABBA code with TAS and 16QAM gains about 1.2 dB. Especially, the performance of \((\frac{1}{2})\) TAS is slightly better than that of an ideal imaginative rate-one OSTBC using the same 16QAM. It is worthwhile to remember that the performance of an ideal hypothetical rate-one OSTBC is also the performance limit of ABBA-QSTBC with phase feedback scheme in [5]. The performance of the MDC-ABBA code is also compared with that of OSTBC for the same spectral efficiency of 3 bits pcu and TAS. In this case, MDC-ABBA codes gains 0.8 dB.

VI. CONCLUSION

We have presented a new general and closed form method to decode ABBA and MDC-ABBA codes. The general equivalent channel of these codes has been shown explicitly and it is used to derive the maximum mutual information of the codes. MDC-ABBA codes with Tx or Rx antenna selections and with full or limited feedback is also proved to achieve full diversity. Not only our analysis shows that the MDC-ABBA codes can achieve a higher portion of channel capacity than OSTBC, but also our simulations show that the former performs better than the latter. Therefore, MDC-ABBA codes might be a good candidate to replace OSTBC for open loop wireless channels with more than 2 transmit antennas.

REFERENCES


