A Simple Construction of 16-QAM Codewords with Low PMEPR for OFDM Signals

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***Abstract*— Golay sequences have been introduced to construct 16-QAM (quaternary amplitude modulation) code for the orthogonal frequency division multiplexing (OFDM), reducing the peak-to-mean envelope power ratio (PMEPR). As an alternative way to construct Golay sequences, the construction of 16-QAM using Rudin-Shapiro polynomials (RSP) was also reported recently in several literatures. In this paper, we develop a simple effcient construction of 16-QAM using generalized RSP, which provides high code rate and large Hamming distance while tightly controlling the PMEPR.**

I. Introduction

Orthogonal frequency division multiplexing (OFDM) eliminates the need for complex equalizers in wide-band fading channels, while efficient hardware implementations can be realized using the fast Fourier transform (FFT). However, a major drawback of OFDM signals is the high peak-to-mean envelope power ratio (PMEPR) of the uncoded OFDM signal. Some popular PMEPR reduction techniques include signal distortion techniques [1], [2], coding [3], [4], [5], [6], multiple signal representation [7], [8], [9], [10], modified signal constellation [11], pilot tone methods [12] and others.

An idea introduced in [13] and developed in [14] is to use the Golay sequences [15] to encode OFDM signals with a PMEPR of at most 2. These sequences have been employed as pilot sequences by European Telecommunications Standards Institute (ETSI) Broad Radio Acess Networks (BRAN). Recently Davis and Jedwab [3] made an attractive theoretical advance on this work and observed that the 2^h-ary Golay sequences of length 2^m can be obtained from certain second order cosets of the classical first order Reed-Muller code. As a consequence of this intrinsic observation, Davis and Jedwab [4] were able to obtain $(m!/2)2^{h(m+1)}$ codewords (DJ-code) for the phase shift keying (PSK) OFDM signals of 2^m carriers with good error-correcting capabilities, efficient encoding and decoding, and a PMEPR of at most 2. Futher investigation on DJ-code has been done in [6], which employs Golay set [16] to increase the code rate by relaxing the PMEPR.

Since quadrature amplitude modulation (QAM) sequences are widely used in OFDM, RöBing and Tarokh[17] has studied the Golay sequences for 16-QAM OFDM signals. By decomposing a 16-QAM symbol uniquely into a pair of quaternary PSK (QPSK) symbols, they construct $[(m!/2)4^{(m+1)}]^2$ 16-QAM OFDM signals of length 2^m starting from Golay QPSK sequences and provide bound 3.6 on their PMEPR. More detailed construction of 16-QAM OFDM signals using Golay sequneces has been recently reported in [18].

As a simple way to construct Golay sequences, the Rudin-Shapiro polynomials (RSP) [19] have been studied in [20], [21], [22]. They obtain $2^{h(m+1)}$ codewords for 2^h-ary OFDM signals of 2^m carriers. Although the code rate of RSP based construction is lower than that of DJ-code, this simple construction matches with some efficient decoding scheme [23]. Further study to increase the code rate has been reported in [20]. Recently, RSP has also been used to construct 16-QAM code in [24], which obtains $m4^{m+3}-256(m-1)$ codewords of length 2^m. In this paper, we develop a simple way to design the 16-QAM OFDM signals using the generalized RSP [20]. We can obtain $4^{3(2k)+2}$ codewords of length 2^{2k} and provide bound 7.2 for PMEPR. Let $m=2k$, we actually construct 4^{3m+2} codewords of length 2^m, which is more than those presented in [18], [24] for moduratly large carriers.

II. Prelimilaries

Let j be the imaginary unit, i.e., $j^2=-1$. For an M-ary phase modulation OFDM, let $\xi^{\mathbb{Z}_M}=\{\xi^k : k\in\mathbb{Z}_M\}$, where $\xi=\exp(2\pi j/M)$ and $\mathbb{Z}_M=\{0,\cdots,M-1\}$.

A. OFDM signals, instantaneous power and PMEPR

For a codeword $c=(c_0,\ldots,c_{n-1})$ with $c_\ell\in\xi^{\mathbb{Z}_M}$, the n subcarrier complex baseband OFDM signal can be mathematically simplified as

$$s_c(z):=\sum_{\ell=0}^{n-1}c_\ell z^\ell, \tag{1}$$

where $z=e^{j2\pi t}$. The instantaneous power of the complex envelope $s_c(z)$ is defined by

$$P_c(z):=|s_c(z)|^2. \tag{2}$$

The peak-to-mean envelope power ratio (PMEPR) of the codeword c is defined as

$$\mathrm{PMEPR}(c):=\frac{1}{n}\sup_{|z|=1}P_c(z). \tag{3}$$

B. Aperiodic auto-correlation and Golay sequences

For a sequence $a \in \mathbb{C}^n$, the aperiodic auto-correlation function $R_a(\cdot)$ is defined by

$$R_a(\ell) = \begin{cases} \sum_{k=0}^{n-\ell-1} a_{k+\ell}\bar{a}_k, & \ell = 0, 1, \cdots, n-1, \\ 0, & \text{otherwise}, \end{cases}$$

where $\bar{a}_k$ is the complex conjugate of a_k.

Golay sequences were originally introduced to deal with the optical problem of multislit spectrometry. Golay also predicted that it will have possible application in communication engineering [15], which was recently fulfilled by the works done in [3], [4], [5], [6]. The original Golay sequences are defined only for the binary sequences [15]. However, it can be easily extended to the M-ary sequences as recently done in [4], [5], [6].

A pair of sequences a and b of length n are said to form a Golay pair if

$$P_a(z) + P_b(z) = 2n. \quad (4)$$

The sequences a and b are called Golay sequences.

Obviously, $\mathrm{PMEPR}(a) \leq 2$ if a is a Golay sequence, which tightly controls the PMEPR of the underlying OFDM signal $s_a(z)$ by 2. This is a big advantage of using Golay sequences to reduce PMEPR for OFDM signals. Using the aperiodic auto-correlation function, the instantaneous power $P_a(z)$ of the sequence a can be represented as

$$P_a(z) = R_a(0) + \sum_{\ell=1}^{n-1} \left[R_a(\ell)z^{\ell} + \bar{R}_a(\ell)z^{-\ell} \right].$$

Therefore, equation (4) is equivalent to

$$R_a(\ell) + R_b(\ell) = 2n\delta(\ell),$$

where $\delta(\ell)$ is Dirac sequence, which takes the value 1 at 0, and takes the value 0 elsewhere.

C. Rudin-Shapiro polynomials

Besides those introduced in the introduction, the early application of Rudin-Shapiro polynomials (RSP) [19] to constructing encoding and decoding schemes for OFDM can be found in [25].

For a $k \geq 0$, an RSP pair $(A(z), B(z))$ is recursively defined as

$$\begin{cases} A_{k+1}(z) = A_k(z) + \xi_k z^{2^k} B_k(z), \\ B_{k+1}(z) = A_k(z) - \xi_k z^{2^k} B_k(z), \end{cases} \quad (5)$$

where $A_0(z) = B_0(z) = 1$ and ξ_k is any element in $\xi^{\mathbb{Z}_M}$.

Formula (5) recursively produces the polynomials $A_k(z)$ and $B_k(z)$ of degree $2^k - 1$ for any $k > 0$. In general, for $n = 2^m$, let the sequences a and b be, respectively, the coefficients of the polynomials $A_m(z)$ and $B_m(z)$. The 2^m-subcarrier OFDM signals are $s_a(z) = A_m(z)$ and $s_b(z) = B_m(z)$. For example, for $m = 3$, we have $n = 8$ and the codewords

$$\begin{cases} a = (1 \ \ \xi_0 \ \ \xi_1 \ \ -\xi_1\xi_0 \ \ \xi_2 \ \ \xi_2\xi_0 \ \ -\xi_2\xi_1 \ \ \xi_2\xi_1\xi_0), \\ b = (1 \ \ \xi_0 \ \ \xi_1 \ \ -\xi_1\xi_0 \ \ \xi_2 \ \ -\xi_2\xi_0 \ \ \xi_2\xi_1 \ \ -\xi_2\xi_1\xi_0). \end{cases}$$

From (5), it is clear that

$$P_a(z) + P_b(z) = |s_a(z)|^2 + |s_b(z)|^2 = |A_m(z)|^2 + |B_m(z)|^2$$

Noting $|A_m(z)|^2 + |B_m(z)|^2 = 2\left[|A_{m-1}(z)|^2 + |B_{m-1}(z)|^2\right]$ and repeating the process, we have

$$P_a(z) + P_b(z) = 2^m \left[|A_0(z)|^2 + |B_0(z)|^2\right] = 2n.$$

This shows that a and b form a Golay pair. Therefore, RSP pair constitute a subset of Golay pair. Hence the PMEPR of an RSP is at most 2.

By the recursive formula (5), one can construct M^m Golay sequences. Since for any $\eta \in \xi^{\mathbb{Z}_M}$, ηa is also a Golay sequence if a is a Golay sequence, one can construct M^{m+1} Golay sequences by RSP.

III. New construction of 16-QAM codewords

Our construction of 16-QAM is based on the generalized RSP and the decomposition of 16-QAM symbols. In the following, we will firstly introduce the generalized RSP, then construct the 16-QAM codewords using generalized RSP.

A. Generalized RSP

We now introduce the generalized RSP, for which we write the formula (5) in the matrix form. Let

$$\mathbf{A}_k^2(z) = \begin{pmatrix} A_k(z) \\ B_k(z) \end{pmatrix}, \ \mathbf{B}_k^2(z) = \begin{pmatrix} A_k(z) \\ z^{2^k} B_k(z) \end{pmatrix},$$

and

$$\mathbf{T}_k^2 = \begin{pmatrix} 1 & \xi_k \\ 1 & -\xi_k \end{pmatrix}.$$

Then one can rewrite formula (5) in the matrix form

$$\mathbf{A}_{k+1}^2(z) = \mathbf{T}_k^2 \mathbf{B}_k^2(z).$$

This immediately suggests an extension of RSP. Let $\theta = \exp(j2\pi/N)$. Extend $\mathbf{A}_k^2(z)$, $\mathbf{B}_k^2(z)$ and $\mathbf{T}_k^2$ respectively to $\mathbf{A}_k^N(z)$, $\mathbf{B}_k^N(z)$ and $\mathbf{T}_k^N$ as

$$\mathbf{A}_k^N(z) = \begin{pmatrix} A_{k+1}^0(z) \\ A_{k+1}^1(z) \\ \vdots \\ A_{k+1}^{N-1}(z) \end{pmatrix}, \ \mathbf{B}_k^N(z) = \begin{pmatrix} A_k^0(z) \\ z^{N^k} A_k^1(z) \\ \vdots \\ z^{(N-1)N^k} A_k^{N-1}(z) \end{pmatrix},$$

$$\mathbf{T}_k^N = \begin{pmatrix} 1 & \xi_k^1 & \xi_k^2 & \cdots & \xi_k^{N-1} \\ 1 & \theta\xi_k^1 & \theta^2\xi_k^2 & \cdots & \theta^{N-1}\xi_k^{N-1} \\ \vdots & \vdots & \vdots & & \vdots \\ 1 & \theta^{N-1}\xi_k^1 & \theta^{2(N-1)}\xi_k^2 & \cdots & \theta^{(N-1)(N-1)}\xi_k^{N-1} \end{pmatrix},$$

where $A_0^0 = \cdots = A_0^{N-1} = 1$ and $\xi_k^1, \ldots, \xi_k^{N-1}$ are any symbols taken from the constellation $\xi^{\mathbb{Z}_M}$. Naturally, the *generalized RSP vector* is iteratively defined by the formula

$$\mathbf{A}_k^N(z) = \mathbf{T}_k^N \mathbf{B}_k^N(z). \quad (6)$$

Each polynomial entry in $\mathbf{A}_k^N(z)$ is called a *generalied RSP*. Obviously, the generalized RSP degenerates to the ordinary RSP if $N = 2$. In this paper, we are interested in the case $N > 2$.

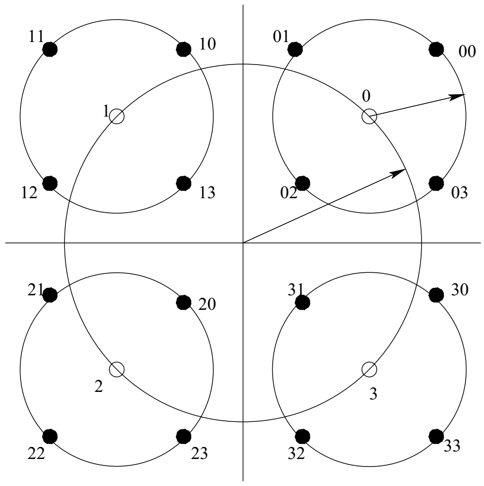

Fig. 1. Construction of 16-QAM from two QPSK.

B. PMEPR of 16*-QAM sequences by generalized RSP*

For a QPSK sequence Q constructed in the last subsection, we can write it as

$$Q = \alpha e^{j\pi/4} U + \beta e^{j\pi/4} V,$$

where $U = (\eta_0, \ldots, \eta_{n-1})$ and $V = (\zeta_0, \ldots, \zeta_{n-1})$ are the two QPSK sequences from generalized RSP (see Fig. 1).

Using the PMEPR of generalized RSP, one can obtain the PMEPR of 16-QAM sequences, which is summarized in the following theorem.

Theorem 1: For the 16-QAM sequence Q of length 4^k, constructed in the last subsection, the PMEPR is at most 7.2.
Proof: Let $Z = (1, z, \ldots, z^{n-1})^T$. Then the 16-QAM OFDM signal is $s_Q = QZ$. Since $Q = \alpha e^{j\pi/4} U + \beta e^{j\pi/4} V$, we have

$$s_Q(z) = \alpha e^{j\pi/4} UZ + \beta e^{j\pi/4} VZ.$$

If we can show $\mathrm{PMEPR}(U) \le B$ and $\mathrm{PMEPR}(V) \le B$, then it is easy to see

$$|s_Q(z)|^2 \le (\alpha + \beta)^2 Bn = \frac{9Bn}{5}.$$

In order to show $\mathrm{PMEPR}(Q) \le 7.2$, we only need to show $B = 4$. In the following, we will verify this result.

Since U and V are two QPSK sequences constructed by generalized RSP. Without loss of generality, we can just investigate U. The derived result can be directly applied to V. Assume $U^1 Z, U^2 Z, U^3 Z$ together with $U^0 Z$ $(= UZ)$ form the generalized RSP vector. From (6), we have

$$\begin{aligned}\sum_{\ell=0}^{3} |s_{U^\ell}(z)|^2 &= \sum_{\ell=0}^{3} |A_m^\ell(z)|^2 \\ &= \left(\mathbf{A}_m^4\right)^\top \cdot \overline{\mathbf{A}_m^4},\end{aligned}$$

where $\left(\mathbf{A}_m^4\right)^\top$ is the transpose of the matrix $\mathbf{A}_m^4$. Since $\mathbf{T}_{m-1}^N$ is an orthogonal matrix, we have $\left(\mathbf{T}_{m-1}^4\right)^\top \overline{\mathbf{T}_{m-1}^4} = 4\mathbf{I}_4$ and

$$\begin{aligned}\sum_{\ell=0}^{3} |s_{U^\ell}(z)|^2 &= \left(\mathbf{B}_{m-1}^4(z)\right)^\top \left(\mathbf{T}_{m-1}^4\right)^\top \overline{\mathbf{T}_{m-1}^4 \mathbf{B}_{m-1}^4(z)} \\ &= 4\left(\mathbf{B}_{m-1}^4(z)\right)^\top \mathbf{I}_4 \overline{\mathbf{B}_{m-1}^4(z)} \\ &= 4\sum_{\ell=0}^{3} \left(\mathbf{B}_{m-1}^\ell(z)\right)^\top \overline{\mathbf{B}_{m-1}^\ell(z)} \\ &= 4\sum_{\ell=0}^{3} \left(\mathbf{A}_{m-1}^\ell(z)\right)^\top \overline{\mathbf{A}_{m-1}^\ell(z)} \\ &= \cdots \\ &= 4^m \sum_{\ell=0}^{3} \left(\mathbf{A}_0^\ell(z)\right)^\top \overline{\mathbf{A}_0^\ell(z)} \\ &= 4^{m+1} = 4n,\end{aligned}$$

where $\mathbf{I}_4$ is the identity 4×4 matrix. This clearly shows that $\mathrm{PMEPR}(U) = 4$. By the same reason, $\mathrm{PMEPR}(V) = 4$. Therefore $B = 4$, which completes the proof.

C. Code rate of 16*-QAM by generalized RSP*

For the generalized RSP of degree $4^k - 1$, there are $3k$ variables involved in $A_k^0(z)$, and each variable has 4 choices. Hence, one can construct 4^{3k} QPSK sequences by generalized RSP. Since for any $\xi \in \xi^{\mathbb{Z}_4}$, ξa is a generalized RSP if a is a generalized RSP, one can totally construct 4^{3k+1} distinct QPSK sequences by generalized RSP. Since we choose different QPSK sequences to construct the 16-QAM OFDM signals, we have totally $[4^{3k+1}]^2$ different choices. This gives the code rate of the 16-QAM by generalized RSP.

Theorem 2: For the 16-QAM codes of length 4^k by generalized RSP, the code rate is

$$\frac{6k+2}{4^k}.$$

Let $m = 2k$, the code rate is $(3m+2)/2^m$, which is higher than the constructions presented in [17], [18], [24]. Fig. 2 shows the code rate versus the code length for different constructions. One will see our performance is better than [18], [24] for the moduratelly large carriers. Since generalized RSP is a subset of Golay set, our curve is below that in [17]. But the generalied RSP based method in this paper is simple, efficient and practical..

D. Hamming distance of 16*-QAM by generalized RSP*

For two 16-QAM symbols $q_1 = q(\eta_1, \zeta_1)$ and $q_2 = q(\eta_2, \zeta_2)$, we can see $q_1 \ne q_2$ if either $\eta_1 \ne \eta_2$ or $\zeta_1 \ne \zeta_2$. Take a 16-QAM sequence $Q = \alpha e^{j\pi/4} U + \beta e^{j\pi/4} V$, where $U = (\eta_0, \ldots, \eta_{n-1})$ and $V = (\zeta_0, \ldots, \zeta_{n-1})$ are two QPSK sequences. Then the Hamming distance (HD) of Q is the smaller one between the Hamming distances of U and V. In the following theorem, the Hamming distance of 16-QAM sequences by generalized RSP is given.

Theorem 3: For the 16-QAM sequences of length 4^k by generalized RSP, the minimum Hamming distance is 4^{k-1}.

Proof: Take the 16-QAM sequence $Q = \alpha e^{j\pi/4} U + \beta e^{j\pi/4} V$ of length 4^k. Since the hamming ditance of Q is

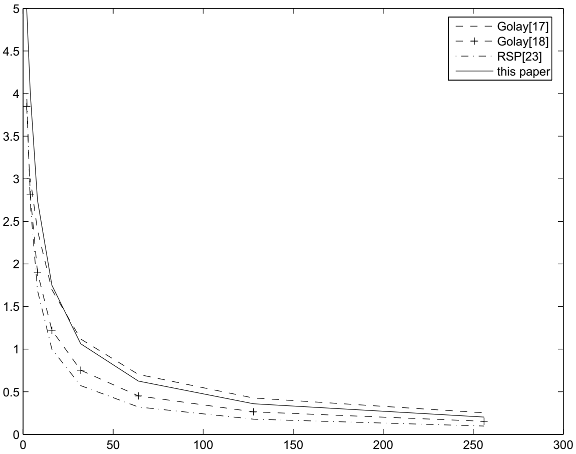

Fig. 2. The code rates of the constructions by [17], [18], [24] and this paper versus the length of the code. Our code rate is higher than those in [18], [24]. Our code rate is slightly below that in [17], but our mathod is simple, efficient and practical.

determined by those of U and V, we just need to find out the Hamming distances of U and V. In the following, we will use induction to show that the Hamming distances of U and V are 4^{k-1}.

For $k = 1$ we have $A_1^0(z) = 1 + \xi_0^1 z + \xi_0^2 z^2 + \xi_0^3 z^3$. For different choices of $\xi_0^1, \xi_0^2, \xi_0^3$, we obtain different codes. Then the Hamming distance of U is $1 = 4^{k-1}$.

For the case $k = m + 1$, we have

$$A_{m+1}^0 = A_m^0 + \xi_m^1 z^{4^m} A_m^1 + \xi_m^2 z^{2\cdot 4^m} A_m^2 + \xi_m^3 z^{3\cdot 4^m} A_m^3$$

Since the degrees of $A_m^0, \cdots, A_m^3$ are $4^m - 1$, the cofficients of $z^0, \cdot, z^{3\cdot 4^m}$ will not add to each other. For the different choices of ξ_m^ℓ, the derived codes are different at least at 4^m places, the length of A_m^ℓ. Therefore the Hamming distance of the code is $4^{(m+1)-1}$. For the case of fixed choice of ξ_m^ℓ, suppose the Hamming distance of the code by A_m^ℓ are 4^{m-1}. Since each A_m^ℓ includes all ξ_i^ℓ for $\ell = 0, 1, 2, 3$ and $0 \leq i < m$, there are must one ξ_i^ℓ changes. This implies that all A_m^ℓ are not fixed choices for two different A_{m+1}^0. Therefore the code rate of A_{m+1}^0 is $4 \times 4^{m-1} = 4^{(m+1)-1}$.

By induction, this proves the Hamming distance of U is 4^{k-1}. For the same reason, the Hamming distance of V is also 4^{k-1}. Therefore the Hamming distance of Q is 4^{k-1}, which completes the proof.

Let $2k = m$, the Hamming distance is actually 2^{m-2} for this kind of codes of length 2^m.

IV. CONCLUSIONS

In this paper, we introduce the generalized RSP, which can recursively produce a large number of QPSK sequences. Then we use these QPSK sequencs to construct 16-QAM codewords for OFDM signals. By this construction, we can obtain 4^{3m+2} 16-QAM codewordes of length 2^m, while controlling the PMEPR by 7.2. Moreover, we find that the Hamming distance is 2^{m-2} for this code of length 2^m. Since this construction is very simple and efficient, it is very practical.

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