

# Transmitter Precoding for Orthogonal Space-Time Block-Coded OFDM in Transmit-Antenna and Path-Correlated Channels

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**Abstract**—Orthogonal space-time block-coded (OSTBC) orthogonal frequency-division multiplexing (OFDM) links for frequency-selective multiple-input multiple-output (MIMO) channels with correlated paths and transmit antennas are considered. In such systems, optimal precoding with only covariance feedback is derived using the minimum pair-wise error probability (PEP) criterion; linear and non-linear precoders are designed. The proposed precoding only needs the statistical knowledge of the channel at the transmitter, which significantly reduces the feedback requirements. Both linear and non-linear precoders substantially improve the system bit error rate (BER) for OSTBC OFDM in transmit-antenna and path-correlated channels. The proposed non-linear precoder outperforms the linear precoder.

**Index Terms**—OFDM, MIMO, optimized precoding, OSTBC

## I. INTRODUCTION

The multiple-input multiple-output (MIMO) paradigm, employing multiple antenna arrays at both the transmitter and the receiver, has recently emerged as a dominant technology for high-data-rate wireless communications. Using space-time signal processing techniques, MIMO systems effectively exploit spatial dimension inherent in multiple antennas, and obtain full diversity gains or capacity increases without bandwidth expansion or transmit power increase. Orthogonal space-time block coding (OSTBC) [1], [2] is an important space-time signal processing technique to exploit the total available spatial diversity in MIMO channels. Since OSTBC achieves full diversity with low decoding complexity, it is widely used and has been adopted in the third generation cellular standards [3], [4]. Orthogonal frequency-division multiplexing (OFDM) is a spectrally efficient transmission technique suitable for frequency-selective radio channels. OSTBC, although designed for flat-fading MIMO channels, can be immediately overlaid on OFDM by simply performing coding/decoding on a subcarrier basis. Therefore, orthogonal space-time block-coded (OSTBC) <sup>1</sup> OFDM achieves full diversity gain and facilitates the utilization of this gain on frequency-selective MIMO channels.

<sup>1</sup>OSTBC here stands for orthogonal space-time block-coded and orthogonal space-time block coding, depending on the context.

OSTBC has been originally designed for uncorrelated Rayleigh fading channels, where the channel gains are distributed as independent and identically distributed (i.i.d.) zero-mean complex Gaussian random variables. However, in practical systems, the MIMO channel may be spatially correlated due to poor scattering and/or insufficient transmit antenna spacing. The temporally-correlated multipath signals can lead to path correlations in each channel between the transmit and receive antenna pair. The path and antenna correlations make the received data streams correlated and lead to difficult stream separation and decoding. If conventional space-time processing techniques are directly used in correlated MIMO channels, the capacity and bit error rate (BER) performance can be degraded.

If channel state information (CSI) is available at the transmitter, precoding can exploit spatial diversity, offer higher link capacity, and reduce the complexity of MIMO transmission and reception. Transmitter precoding can increase throughput in spatially-multiplexed OFDM on spatially-correlated frequency-selective MIMO channels [5]. It also offers the flexibility of adapting OSTBC to spatially correlated flat-fading MIMO channels [6]–[8]. Similarly, directly applying OSTBC to OFDM in correlated frequency-selective channels leads to substantial BER increase [9]. Precoding in OSTBC OFDM systems adapts to channel conditions and pre-processes signals at the subcarrier level such that OSTBC designed for i.i.d. channels can also be used for correlated frequency-selective MIMO channels. Nevertheless, precoding for error-rate minimization in OSTBC OFDM with spatial correlations has not been considered yet.

In this paper, we develop linear precoding and non-linear Tomlinson-Harashima precoding (THP) for OSTBC OFDM in transmit-antenna and path-correlated frequency-selective channels to minimize the probability of error. With perfect CSI at the transmitter, a precoded system can achieve a significant capacity gain or BER reduction. However, the instantaneous and accurate CSI feedback is not realistic since the feedback capacity is usually very limited. Our proposed precoding approach only needs the channel statistical information (correlation matrices) to be available at the transmitter, i.e., the instantaneous values of the channel gains are not needed.

Since correlation matrices change at a much slower rate than the channel gains or even do not change at all, the covariance feedback requires much lower capacity. We assume that the receiver has perfect CSI and uses maximum likelihood (ML) decoding. We derive both linear and non-linear precoding using the minimum pair-wise error probability (PEP) criterion. The proposed precoding remarkably reduces the system BER in OSTBC OFDM with path and transmit antenna correlations. Moreover, non-linear precoding outperforms linear precoding.

## II. SYSTEM MODEL

This section will introduce the system model of an  $N$ -subcarrier OFDM system with  $M_T$  transmit antennas and  $M_R$  receive antennas in the presence of transmit antenna and path correlations.

### A. Path and Transmit Antenna Correlations

We restrict our analysis to the downlink case, where correlations exist between the transmit antennas, and no correlations exist between receive antennas. Between the  $u$ -th transmit antenna and  $v$ -th receive antenna, a wideband frequency-selective fading channel with  $L$  resolvable paths is assumed. The  $l$ -th path gain is a zero-mean complex Gaussian random variable (Rayleigh fading) with variance  $\sigma_l^2$ , which can be represented by an  $M_R \times M_T$  matrix  $\mathbf{h}(l)$  with entries  $h_{u,v}(l)$ ,  $\forall l$ . We assume that the channel gains remain constant over several OFDM symbol intervals. The channel gain vector is  $\vec{\mathbf{h}} = [\text{vec}(\mathbf{h}(0))^T \dots \text{vec}(\mathbf{h}(L-1))^T]^T$ , where  $\text{vec}(\cdot)$  denotes the vectorization operator [10]. According to the model in [5], the transmit antenna correlation matrix can be represented by

$$\mathbf{R} = \mathbb{E}[\vec{\mathbf{h}}\vec{\mathbf{h}}^H] = \mathbf{R}_P \otimes \mathbf{R}_T^T \otimes \mathbf{I}_{M_R}, \quad (1)$$

where  $\otimes$  is the Kronecker product, and  $\mathbf{R}_P$  is the  $L \times L$  path correlation matrix with the  $\{m, n\}$ th entry

$$R_P(m, n) = \sigma_m \sigma_n p^{|m-n|} e^{j\theta_{m,n}}, \quad 0 < p \leq 1 \quad (2)$$

where  $p$  is the path correlation coefficient and the  $\theta_{m,n}$  is the phase of the path correlation between the  $m$ -th and the  $n$ -th path. If the paths between each transmit-receive antenna pair are uncorrelated, i.e.,  $p = 0$ , the  $\mathbf{R}_P = \text{diag}[\sigma_0^2 \dots \sigma_{L-1}^2]$  is only defined by the power delay profiles. The  $\mathbf{R}_T$  is the transmit antenna correlation matrix. From [10], the entries of  $\mathbf{R}_T$  are

$$R_T(m, n) = J_0(2\pi|m-n|\zeta_T), \quad (3)$$

where  $J_0$  is zero-order Bessel function of the first kind and  $\zeta_T = \Delta \frac{d_T}{\lambda}$ ;  $\lambda = c/f_c$  is the wavelength at the center frequency  $f_c$ ,  $\Delta$  is the angle of arrival spread, and the transmit antennas are spaced by  $d_T$ . As in [7], the  $M_R \times LM_T$  tap gain matrix can be obtained as

$$[\mathbf{h}(0) \dots \mathbf{h}(L-1)] = \mathbf{h}_w [\mathbf{R}_P^T \otimes \mathbf{R}_T]^{1/2} = \mathbf{h}_w [\mathbf{r}_P^T \otimes \mathbf{r}_T], \quad (4)$$

where  $\mathbf{h}_w$  is an  $M_R \times M_T L$  matrix of i.i.d zero mean complex Gaussian random variables with unit variance;  $\mathbf{r}_P = \sqrt{\mathbf{R}_P}$  and  $\mathbf{r}_T = \sqrt{\mathbf{R}_T}$ .

### B. Transmit Antenna and Path Correlations in OFDM

At the receiver, the channel on the  $k$ -th subcarrier can be represented as

$$\mathbf{H}[k] = \sum_{l=0}^{L-1} \mathbf{h}(l) e^{-j\frac{2\pi}{N}kl}. \quad (5)$$

With the  $l$ -th path gain matrix  $\mathbf{h}(l)$  satisfying (4), (5) can be written as

$$\mathbf{H}[k] = \mathbf{h}_w (\mathbf{r}_P^T \mathbf{F}[k] \otimes \mathbf{r}_T) = \mathbf{h}_w \mathbf{r}[k], \quad (6)$$

where  $\mathbf{F}[k] = [e^{-j\frac{2\pi}{N}k0} \dots e^{-j\frac{2\pi}{N}k(L-1)}]^T$  is an  $L$ -dimensional vector;  $\mathbf{r}[k] = \mathbf{r}_P^T \mathbf{F}[k] \otimes \mathbf{r}_T$  is an  $M_T L \times M_T$  matrix. The  $k$ -th received signal vector in spatially correlated OFDM channels (in which multiple paths are also correlated) thus can be given by

$$\mathbf{Y}[k] = \mathbf{H}[k]\mathbf{X}[k] + \mathbf{W}[k], \quad (7)$$

where  $\mathbf{Y}[k]$  is an  $M_R$ -dimensional vector and  $\mathbf{X}[k] = [X_1[k] \dots X_{M_T}[k]]^T$  is an input data vector;  $X_u[k]$  denotes an  $M$ -ary quadrature amplitude modulation (QAM) symbol on the  $k$ -th subcarrier sent by the  $u$ -th transmit antenna. The  $\mathbf{W}[k]$  is the noise vector where the entries  $W_v[k] = \sum_{u=1}^{M_T} W_{u,v}[k]$  are additive white Gaussian noise (AWGN) samples with zero mean and variance  $\sigma_W^2$ , and  $W_{u,v}[k]$ ,  $\forall k$ , are assumed i.i.d.

### C. OSTBC OFDM

Space-time codes improve power efficiency by maximizing spatial diversity. An OSTBC matrix is composed of linear combinations of constellation symbols and their conjugates, and encoding therefore only requires linear processing. The  $T \times M_T$  code matrix for orthogonal STBC satisfies

$$\mathbf{C}^H \mathbf{C} = \left( \sum_{t=1}^P |c_t|^2 \right) \mathbf{I}_{M_T}, \quad (8)$$

for all complex codewords  $c_t$ . The transmission (code) rate  $R_c$  is defined as  $P/T$ , where  $P$  represents the number of symbols transmitted over  $T$  time slots. OSTBC can be directly employed in OFDM at a subcarrier level to offer full spatial diversity gain, if there is no correlation between transmit antennas or different paths. For example, the full-rate Alamouti-coded OFDM transmits  $\begin{pmatrix} X_1[k] & -X_2^*[k] \\ X_2[k] & X_1^*[k] \end{pmatrix}$  onto the subcarrier  $k$ , i.e.,  $X_1[k]$  and  $X_2[k]$  are transmitted over the 1-st and 2-nd antenna at the first time slot, respectively; the  $-X_2^*[k]$  and  $X_1^*[k]$  are transmitted in the following time slot. Full-rate complex orthogonal designs do not exist for more than two transmit antennas.

The system performance of OSTBC OFDM can be analyzed using PEP, which is the probability that a transmitted signal vector  $\mathbf{X}[k]$  is erroneously decoded as a vector  $\hat{\mathbf{X}}[k]$ . We assume the ML decoder at the receiver uses the Euclidean distance decoding metric

$$\hat{\mathbf{X}}[k] = \arg \min_{\mathbf{X}[k]} \|\mathbf{Y}[k] - \mathbf{H}[k]\mathbf{X}[k]\|_F^2; \quad (9)$$

where  $\|\cdot\|_F$  is the Frobenius norm. The PEP on the  $k$ -th subcarrier can be upper bounded by [9]

$$P_e(\mathbf{X}[k] \rightarrow \hat{\mathbf{X}}[k]) \leq \exp\left(-\frac{\|\mathbf{H}[k]\boldsymbol{\Theta}\|_F^2}{4\sigma_W^2}\right), \quad (10)$$

where  $\boldsymbol{\Theta} = \mathbf{X}[k] - \hat{\mathbf{X}}[k]$  is the codeword difference vector. As in [11], by taking the expectation of (10) over the channel statistics, the average PEP can be bounded by

$$\log \bar{P}_e \leq -M_R \log \det(\mathbf{Q}[k]), \quad (11)$$

where  $\mathbf{Q}[k] = \frac{\boldsymbol{\Theta}\boldsymbol{\Theta}^H}{4\sigma_W^2}\mathbf{R}[k] + \mathbf{I}_{M_T}$ ;  $\mathbf{R}[k] = \mathbf{r}^H[k]\mathbf{r}[k]$  is an  $M_T \times M_T$  matrix. For an OSTBC structure,  $\boldsymbol{\Theta}\boldsymbol{\Theta}^H = d\mathbf{I}_{M_T}$  is a diagonal matrix [2], where  $d$  is the distance between codewords in pair. The  $d_{\min}$  is the minimum distance over all pairs of the codewords and dominates the error probability exponent and hence can be considered an indicator of the system performance. Obviously, the worst PEP primarily depends on  $\mathbf{R}[k]$ , which consists of  $\mathbf{r}_P$  and  $\mathbf{r}_T$ . Since the  $\mathbf{R}_P$  and transmit-antenna correlation matrix  $\mathbf{R}_T$  are constant, minimizing the worst average PEP is equivalent to maximizing

$$\mathcal{J}(\mathbf{Q}) = \log \det(\mathbf{Q}_{\min}[k]) = \log \det\left(\frac{d_{\min}}{4\sigma_W^2}\mathbf{R}[k] + \mathbf{I}_{M_T}\right). \quad (12)$$

### III. PRECODING FOR OSTBC OFDM WITH CORRELATIONS

We first show the impact of path correlations on the OSTBC OFDM system performance. Linear and non-linear precoders are then proposed to mitigate the performance degradation.

#### A. Impact of Path Correlations

The transmit antenna correlations will always degrade the BER performance in OSTBC OFDM systems [9]. We now show the impact of path correlations on the system performance.

To analyze the impact of  $\mathbf{r}_P$ , we separately decompose the two correlation matrices using singular value decomposition (SVD) as follows:

$$\begin{aligned} \mathbf{r}_P^T \mathbf{F}[k] &= \mathbf{U}_P \boldsymbol{\Gamma}_P \mathbf{V}_P^H \\ \mathbf{r}_T &= \mathbf{U}_T \boldsymbol{\Gamma}_T \mathbf{V}_T^H, \end{aligned} \quad (13)$$

where  $\mathbf{U}_T$  and  $\mathbf{V}_T$  are  $M_T \times M_T$  unitary matrices.  $\boldsymbol{\Gamma}_T$  is the singular value matrix of  $\mathbf{r}_T$ ; it is an  $M_T \times M_T$  diagonal matrix with real, non-negative entries  $\gamma_{T_u}$ ,  $u = 1, \dots, M_T$ , in descending order  $\gamma_{T_1} \geq \gamma_{T_2} \geq \dots \geq \gamma_{T_{M_T}} \geq 0$ . Since  $\mathbf{r}_P^T \mathbf{F}[k]$  is an  $L \times 1$  vector,  $\mathbf{U}_P$  is an  $L \times 1$  vector and  $\mathbf{U}_P^H \mathbf{U}_P = \mathbf{1}$ ;  $\mathbf{V}_P = \mathbf{1}$  and  $\boldsymbol{\Gamma}_P$  is a rank-one matrix with the only entry  $\gamma_{P_k} = \mathbf{F}^H[k]\mathbf{R}_P\mathbf{F}[k]$ . The matrix  $\mathbf{r}[k]$  in (6) therefore becomes

$$\begin{aligned} \mathbf{r}[k] &= \mathbf{r}_P^T \mathbf{F}[k] \otimes \mathbf{r}_T = (\mathbf{U}_P \gamma_{P_k}) \otimes (\mathbf{U}_T \boldsymbol{\Gamma}_T \mathbf{V}_T^H) \\ &= \gamma_{P_k} (\mathbf{U}_P \otimes \mathbf{U}_T) \boldsymbol{\Gamma}_T \mathbf{V}_T^H. \end{aligned} \quad (14)$$

Because  $\mathbf{U}_T$  is a unitary matrix,  $\tilde{\mathbf{U}} = \mathbf{U}_P \otimes \mathbf{U}_T$  is an  $LM_T \times M_T$  unitary matrix, i.e.,  $\tilde{\mathbf{U}}^H \tilde{\mathbf{U}} = \mathbf{I}_{M_T}$ . The correlation matrix  $\mathbf{R}[k]$  can thus be given by

$$\mathbf{R}[k] = \mathbf{r}^H[k]\mathbf{r}[k] = \gamma_{P_k}^2 \mathbf{V}_T \boldsymbol{\Gamma}_T^H \tilde{\mathbf{U}}^H \tilde{\mathbf{U}} \boldsymbol{\Gamma}_T \mathbf{V}_T^H = \gamma_{P_k}^2 \mathbf{V}_T \boldsymbol{\Gamma}_T^2 \mathbf{V}_T^H. \quad (15)$$

The performance degradation due to path correlations is shown in Fig. 1. There is no correlation between the transmit antennas, i.e.,  $\mathbf{R}_T = \mathbf{I}_{M_T}$ . We provide three groups of BER for different values of  $p$  at SNR=5 dB, 10 dB and 15 dB, respectively. In each group, when  $\theta_{m,n} = 0$ , the BER monotonously increases as  $p$  grows. Compared with the BER of zero phase, the random phase can mitigate the impact of path correlations, especially at the high path-correlation ( $p \rightarrow 1$ ).

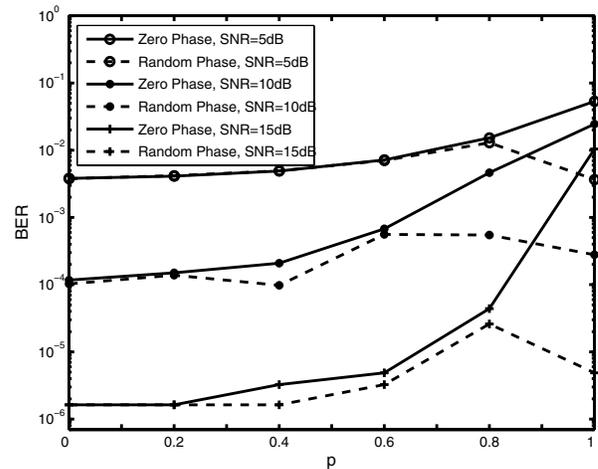


Fig. 1. BER as a function of the path correlation coefficient for different values of SNR for a  $2 \times 2$  QPSK Alamouti-coded OFDM system.

#### B. Optimal Precoding for OSTBC OFDM with Correlations

We design optimal precoding on a subcarrier basis to overcome the performance degradation due to path and transmit antenna correlations. Both linear and non-linear TH precoding is considered. We first consider linear precoding. With an  $M_T \times M_T$  precoding matrix  $\mathbf{E}[k]$  on the  $k$ -th subcarrier, the transmitted signal vector on the  $k$ -th subcarrier is  $\mathbf{E}[k]\mathbf{X}[k]$ , instead of  $\mathbf{X}[k]$ . The  $\mathcal{J}(\mathbf{Q})$  in (12) becomes

$$\mathcal{J}(\mathbf{Q}) = \log \det\left(\frac{d_{\min}\mathbf{E}[k]\mathbf{E}^H[k]}{4\sigma_W^2}\mathbf{R}[k] + \mathbf{I}_{M_T}\right). \quad (16)$$

Our optimal precoding matrix thus can be given by

$$\mathbf{E}[k]_{\text{opt}} = \arg \max_{\text{tr}(\mathbf{Z}[k])=M_T} \log \det(\xi \mathbf{Z}[k]\mathbf{R}[k] + \mathbf{I}_{M_T}), \quad (17)$$

where  $\xi = \frac{d_{\min}}{4\sigma_W^2}$ ,  $\mathbf{Z}[k] = \mathbf{E}[k]\mathbf{E}^H[k]$ , and  $\text{tr}(\cdot)$  denotes the trace of a matrix. Substituting (15) into (17) and applying the determinant identity, we have

$$\begin{aligned} \mathbf{E}[k]_{\text{opt}} &= \arg \max_{\text{tr}(\mathbf{Z}[k])=M_T} \log \det(\xi \gamma_{P_k}^2 \boldsymbol{\Gamma}_T \mathbf{V}_T^H \mathbf{Z}[k] \mathbf{V}_T \boldsymbol{\Gamma}_T + \mathbf{I}_{M_T}) \\ &= \arg \max_{\text{tr}(\tilde{\mathbf{Z}}[k])=\xi M_T} \log \det(\tilde{\boldsymbol{\Gamma}}_T \tilde{\mathbf{Z}}[k] \tilde{\boldsymbol{\Gamma}}_T + \mathbf{I}_{M_T}), \end{aligned} \quad (18)$$

where  $\tilde{\boldsymbol{\Gamma}}_T = \gamma_{P_k} \boldsymbol{\Gamma}_T$  and  $\tilde{\mathbf{Z}}[k] = \xi \mathbf{V}_T^H \mathbf{Z}[k] \mathbf{V}_T$ . The water-filling solution can be derived from (18) [12]. The optimal main-diagonal entries in  $\tilde{\mathbf{Z}}[k]_{\text{opt}}$  will then be

$$\tilde{Z}_{k_{uu}} = (\mu - \tilde{\gamma}_{uu}^{-2})_+, \quad u = 1, \dots, M_T, \quad (19)$$

where  $(a)_+$  denotes  $\max(a, 0)$ ; the parameter  $\mu$  is chosen to satisfy  $\text{tr}(\tilde{\mathbf{Z}}[k]) = \xi M_T$ . Hence, the optimal precoding matrix can be obtained by

$$\mathbf{E}[k]_{\text{opt}} = \sqrt{\mathbf{Z}[k]_{\text{opt}}} = \sqrt{\frac{1}{\xi} \mathbf{V}_T \tilde{\mathbf{Z}}[k]_{\text{opt}} \mathbf{V}_T^H}. \quad (20)$$

The precoding is designed using the singular values of the transmit antenna correlation matrix and has the waterfilling solution. With the precoding matrix, the effective channel becomes  $\mathbf{H}[k]\mathbf{E}[k]$ . After reception the receiver performs ML decoding on the  $k$ -th subcarrier in an OSTBC OFDM system. The proposed precoding only needs the correlation matrices  $\mathbf{r}_P$  and  $\mathbf{r}_T$ , i.e., only covariance feedback is needed for our precoding design.

### C. Non-Linear Precoding

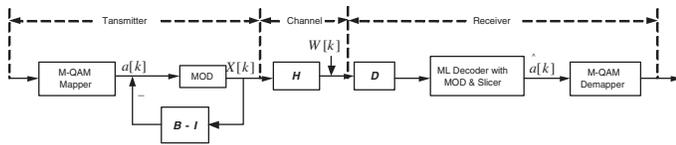


Fig. 2. Tomlinson-Harashima precoding in OSTBC OFDM.

In this subsection, we propose a non-linear TH precoder. The structure of the proposed precoder is illustrated in Fig. 2. The receiver side consists of a diagonal scaling matrix  $\mathbf{P}[k]$ , an ML decoder and a modulo arithmetic device. The transmitter side includes a modulo arithmetic feedback structure employing the matrix  $\mathbf{B}[k]$ , by which the transmitted symbols  $X[k]$  are successively calculated for the data symbols  $a[k]$  drawn from the initial signal constellation. Without the modulo device, the feedback structure is equivalent to  $\mathbf{B}^{-1}[k]$ , which can be optimally designed as in (20),  $\mathbf{B}[k]_{\text{opt}} = \mathbf{E}^{-1}[k]_{\text{opt}}$ . The effective channel is  $\mathbf{H}[k]\mathbf{E}[k]_{\text{opt}}$  and ML decoding is used at the receiver. The diagonal scaling matrix  $\mathbf{P}$  is to keep the average transmit power constant. THP employs modulo operation at both the transmitter and the receiver. The modulo  $2\sqrt{M}$  reduction at the transmitter, which is applied separately to the real and imaginary parts of the input, is to restrict the transmitted signals into the boundary of  $\Re\{X[k]\} \in (-\sqrt{M}, \sqrt{M}]$  and  $\Im\{X[k]\} \in (-\sqrt{M}, \sqrt{M}]$ . If the input sequence  $a[k]$  is a sequence of i.i.d. samples, the output of the modulo device is also a sequence of i.i.d. random variables, and the real and imaginary parts are independent, i.e., we can assume  $\mathbf{E}[X[k]X^H[k]] = E_s \mathbf{I}_{M_T}$ ,  $\forall k$  [13]. At the receiver, the filtered noise vector becomes  $\mathbf{W}' = \mathbf{P}\mathbf{W}$ , where the  $k$ -th entry  $W'[k]$  has individual variance  $\sigma_{W'_k}^2$ . A slicer, which applies the same modulo operation as that at the transmitter, is used. After the ML decoding and discarding the modulo congruence, the unique estimates of the data symbols  $\hat{a}[k]$  can be generated. The details of THP operation are further discussed in [13].

In our proposed linear and non-linear precoder, the transmitter does not require explicit channel gain information and only the channel correlation matrices are delivered to it. Since the correlation matrices may change much slower than the

channel response or even may not change at all, the covariance feedback significantly reduces the feedback load. In most applications, transmit antenna spacing can be estimated at the transmitter, i.e., no feedback for  $\mathbf{R}_T$  is needed. The feedback requirement can hence be further reduced. Furthermore, due to the non-linear property, THP avoids power efficiency loss present in linear precoding. Therefore, low BER can be expected for the proposed non-linear precoder.

## IV. SIMULATION RESULTS

In this section, our simulation results show how the proposed linear and non-linear precoders improve the system performance in OSTBC OFDM with path and transmit-antenna correlations. The transmitter knows only the correlation matrices  $\mathbf{R}_T$  and  $\mathbf{R}_P$  with  $\zeta_T = \Delta \frac{d_T}{\lambda}$  and path correlation coefficient  $p$ , respectively; the phase correlation coefficients  $\theta_{m,n}$  in (2) are assumed zero,  $\forall m, n$ . We assume the angle of arrival spread is  $12^\circ$ , i.e.,  $\Delta \approx 0.2$ . Perfect channel information is assumed to be available only at the receiver and ML decoding is used.

### A. Flat-Fading MIMO Channels

We first consider the special case with  $N = 1$  and  $L = 1$ , where the channel model (6) is reduced to a flat-fading MIMO channel. Only transmit antenna correlations need to be considered. The BERs of 16-QAM  $4 \times 2$  systems with  $\zeta_T = 0.25$  and  $4 \times 4$  systems with  $\zeta_T = 0.5$  are shown in Fig. 3. The transmission rate  $R_c$  of the orthogonal STBC matrix is  $1/2$  as in [2]. The BER for uncorrelated MIMO channels is shown as reference. Evidently, in an OSTBC MIMO system, the transmit-antenna correlations significantly degrade the BER performance. As transmit-antenna correlation becomes high ( $\zeta_T$  decreases), the degradation becomes severe. Both the linear and non-linear TH precoders mitigate the detrimental impact of the antenna correlations. The TH precoder almost completely eliminates the degradation due to correlations. At a BER of  $10^{-4}$ , the linear precoding obtains 0.5 dB gain in  $4 \times 4$  systems and 1 dB gain in  $4 \times 2$  systems; THP achieves gains of 0.8 dB and 1.8 dB, respectively.

### B. OSTBC OFDM

We now consider 64-subcarrier QPSK OSTBC OFDM. The vehicular B channel specified by ITU-R M. 1225 [14] is used where the channel taps are zero-mean complex Gaussian random processes with variances of  $-4.9$  dB,  $-2.4$  dB,  $-15.2$  dB,  $-12.4$  dB,  $-27.6$  dB, and  $-18.4$  dB relative to the total power.

In Fig. 4,  $2 \times 2$  and  $2 \times 4$  Alamouti-coded OFDM systems are considered. The paths are uncorrelated, i.e.,  $p = 0$  and  $\zeta_T = 0.25$ . Similarly, both linear and non-linear precoding suppress the BER increase due to transmit-antenna correlations. Non-linear TH precoding outperforms linear precoding.

In Fig. 5, we assume the path correlation coefficient  $p = 0.9$ . The  $\zeta_T = 0.25$  and  $\zeta_T = 0.5$  are considered. The BER is substantially degraded due to path correlations. Both the linear and non-linear precoders mitigate the impact of correlations.

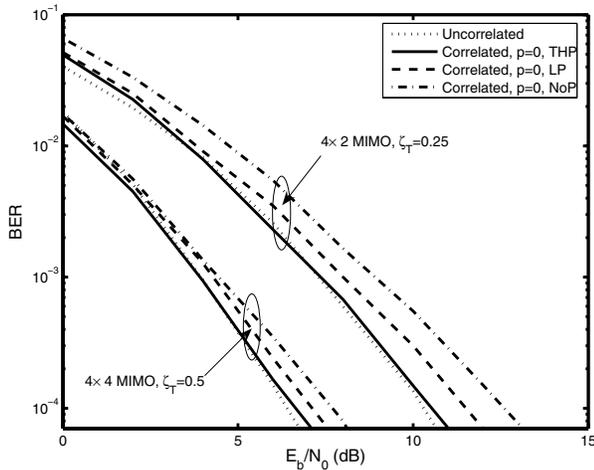


Fig. 3. BER with linear precoding (LP), THP and no precoding (NoP) as a function of the SNR for different values of the normalized transmit antenna spacing for  $4 \times 2$  and  $4 \times 4$  16-QAM  $1/2$ -rate OSTBC systems.

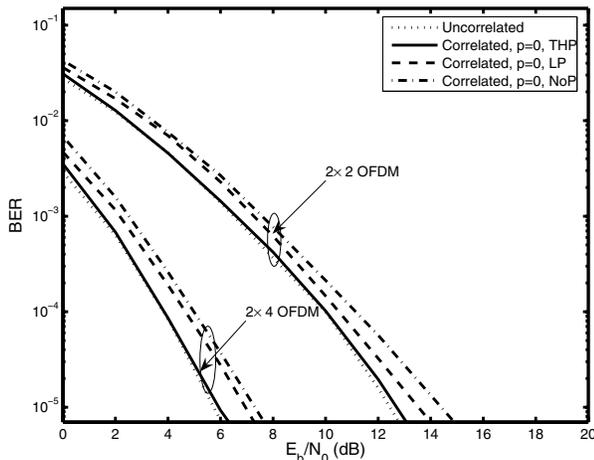


Fig. 4. BER with linear precoding (LP), THP and no precoding (NoP) as a function of the SNR for  $2 \times 2$  and  $2 \times 4$  QPSK Alamouti-coded OFDM systems,  $\zeta_T = 0.25$ .

## V. CONCLUSIONS

We have derived PEP-optimal linear and non-linear precoding with only covariance feedback for OSTBC OFDM systems in the presence of transmit-antenna and path correlations. Not only are the feedback requirements reduced, since our precoding needs only the statistical knowledge of the channel at the transmitter, but also the system BER is reduced in transmit-antenna and path-correlated channels. The proposed non-linear precoding outperforms linear precoding.

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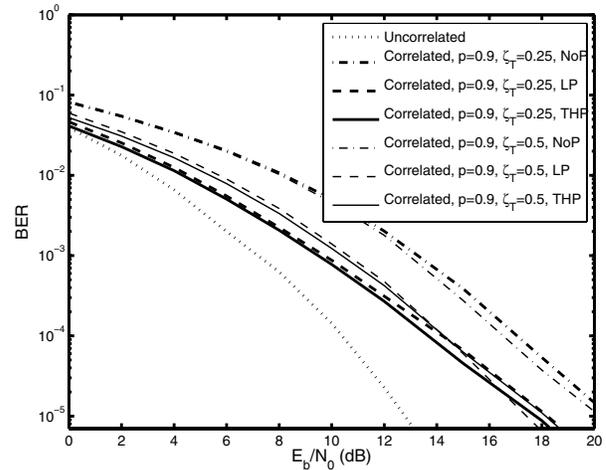


Fig. 5. BER with linear precoding (LP), THP and no precoding (NoP) as a function of the SNR for different values of the path correlation coefficient and the normalized transmit antenna spacing for  $2 \times 2$  QPSK Alamouti-coded OFDM systems.

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