On Space-Time Block Codes from Coordinate Interleaved Orthogonal Designs

Dũng Ngọc Đào and Chintha Tellambura,
Department of Electrical and Computer Engineering, University of Alberta
Edmonton, Alberta T6G 2V4, Canada
Email: {dndung, chintha}@ece.ualberta.ca, Telephone: (1-780) 492 7228, Fax: (1-780) 492 1811

Abstract— **Space-time block codes (STBC) using coordinate interleaved orthogonal designs (CIOD) proposed recently by Khan and Rajan allow single-complex symbol decoding while offering higher data rates than orthogonal STBC. In this paper, we present the equivalent channels of CIOD codes. A new maximum likelihood metric is also derived, which is simpler than the one shown by Khan and Rajan. The exact symbol pair-wise error probability and a tight union bound on symbol error rate are derived. The tight union bound can be used to analyze the performance of CIOD codes with arbitrary constellations. We provide new optimal rotation angles based on minimizing the union bound for various constellations. Furthermore, a new signal design combining signal rotation and power allocation is presented for rectangular quadrature amplitude modulation.**

I. Introduction

Orthogonal space-time block codes (OSTBC) [1], [2], a special class of space-time block codes (STBC[1]) are one of the most attractive space-time coding techniques to exploit the spatial diversity of the multiple input multiple output (MIMO) fading channels. While OSTBC enable low-complexity maximum likelihood (ML) detection, their code rate is low when there are more than 2 transmit (Tx) antennas.

To improve the code rate of OSTBC and maintain single-symbol decoding complexity, some alternative code designs have been introduced recently. They are quasi-orthogonal space-time block codes (QSTBC) with minimum decoding complexity [3] and STBC using coordinate interleaved orthogonal designs (CIOD) [4]. Since the maximal rate of CIOD codes is higher than or equal to that of OSTBC, MDC-QSTBC [2]–[4], they will be the subject of our study.

While OSTBC achieve full diversity for any constellation, CIOD codes may not achieve full-diversity with the conventional constellations such as quadrature amplitude modulation (QAM) or phase shift keying (PSK). To achieve full diversity, modulation symbols may need to be rotated an angle α [4]. Proper choice of angle α will maximize the code diversity and also minimize the error performance. The authors in [4] use the coding gain parameter [5] to derive the optimal α for QAM. However, maximizing the coding gain amounts to minimizing the worst-case codeword pair-wise error probability (CPEP), which does not guarantee the minimization of the symbol error rate (SER). In general, how to find the optimal signal designs for QAM, PSK, and other constellation with good minimum Euclidean distance such as lattice of equilateral triangular (TRI) (also called hexagonal (HEX)) or amplitude PSK (APSK) [6] in terms of minimal SER is still unknown.

In this paper, we derive *equivalent channels* of CIOD codes. A *new ML metric* is also presented, which is more insightful than that presented in [4], yet offers lower detection complexity. A *closed form symbol pair-wise error probability* (SPEP) is derived. Hence the *union bound* on the symbol error rate (SER) can be easily evaluated. For all the tested cases, the union bound is within 0.1 dB of the simulated SER. Therefore, this bound can be used to *accurately analyze the performance* of CIOD codes, and moreover, to *optimize the signal rotation* for an arbitrary constellation. We furthermore present a new approach to design signal transformation for rectangular QAM (QAM-R) yielding better performance than the existing ones in [4], [7].

II. System Model

We consider data transmission over a quasi-static Rayleigh flat fading channel. The transmitter and receiver are equipped with M Tx and N receive (Rx) antennas. The channel gain h_{ik} $(i = 1, 2, \cdots, M; k = 1, 2, \cdots, N)$ between the (i, k)-th Tx-Rx antenna pair is assumed $\mathcal{CN}(0, 1)$[2]. We assume no spatial correlation at either Tx or Rx array. The channel gains are known perfectly at the receiver, but not the transmitter.

The ST encoder maps the data symbols into a $T \times M$ code matrix [3] $X = \left[c_{ti}\right]_{T\times M}$, where c_{ti} is the symbol transmitted from antenna i at time t. The average energy of code matrices is constrained such that $\mathbb{E}[||X||_F^2] = T$.

The received signals y_{lk} of the kth antenna at time t can be arranged in a matrix Y of size $T \times N$. Thus, one can represent the Tx-Rx signal relation as

$$Y = \sqrt{\rho} X H + Z \tag{1}$$

where $H = [h_{ik}]$, and $Z = [z_{ik}]_{T\times N}$, and z_{ik} are independently, identically distributed (i.i.d.) $\mathcal{CN}(0, 1)$. The Tx power

[1]Depending on the context, STBC stands for space-time block code/codes/coding.

[2]The abbreviation $\mathcal{CN}(m, \sigma^2)$ stands for a mean-m and variance-σ^2 circularly complex Gaussian random variable. We also set common notations to be used through out the paper. $\Re(X)$ represents the real part of matrix X. Superscripts ^{T}, *, and † denote matrix transpose, conjugate, and transpose conjugate, respectively. An $n \times n$ identity and all-zero $m \times n$ matrices are denoted by $\boldsymbol{I}_n$ and $0_{m\times n}$, respectively. The diagonal matrix with elements of vector $\boldsymbol{x}$ on the main diagonal is denoted by $\mathrm{diag}(\boldsymbol{x})$. $\|X\|_{\mathrm{F}}$ denotes Frobenius norm of matrix X and $\otimes$ denotes Kronecker product. $\mathbb{E}[\cdot]$ denotes average.

[3]We use the term "codeword" and "code matrix" interchangeably.

is scaled by ρ so that the average SNR at each Rx antenna is ρ, independent of the number of Tx antennas.

The code matrix of an STBC can be represented in a general linear dispersion form [1], [8] as follows:

$$\mathcal{X} = \sum_{k=1}^{K} (a_k A_k + b_k B_k) \tag{2}$$

where A_k and B_k, $(k = 1, 2, \cdots, K)$ are $T \times M$ complex-valued constant matrices, a_k and b_k are the real and imaginary parts of the symbol s_k. We sometimes use the notation $\mathcal{X}_M(s_1, s_2, \ldots, s_K)$ to emphasize the transmitted symbols and the number of Tx antennas.

The code rate $\mathsf{R}_{\mathcal{X},M}$ of an ST code $\mathcal{X}$ designed for M Tx antennas is the ratio of data symbols transmitted in an ST code matrix and the number of channel uses T: $\mathsf{R}_{\mathcal{X},M} = K/T$.

III. Construction of CIOD Codes

The CIOD code for M Tx antennas is constructed from two OSTBC components, $\mathcal{O}_{M_1}$ and $\mathcal{O}_{M_2}$, where $M = M_1 + M_2$ [4]. The size of code matrices of $\mathcal{O}_{M_1}$ and $\mathcal{O}_{M_2}$ are $T_1 \times M_1$ and $T_2 \times M_2$, respectively; there are K_1 and K_2 complex symbols are embedded in $\mathcal{O}_{M_1}$ and $\mathcal{O}_{M_2}$, respectively. Additionally, the matrices $\mathcal{O}_{M_1}$ and $\mathcal{O}_{M_2}$ are scaled by constants κ_1 and κ_2 to satisfy the power constraint.

Let $\bar{K}$ be the least common multiple (lcm) of K_1 and K_2, $n_1 = \bar{K}/K_1, n_2 = \bar{K}/K_2$, $\bar{T}_1 = n_1 T_1, \bar{T}_2 = n_2 T_2$. A block of $K = 2\bar{K}$ data (information) symbols $s_i = a_i + \mathrm{j}\, b_i$ $(\mathrm{j}^2 = -1), i = 1, 2, \ldots, K$ is mapped to the intermediate symbols x_k $(k = 1, 2, \ldots, K)$ as follows:

$$x_k = \begin{cases} a_k + \mathrm{j}\, b_{k+\bar{K}}, & k = 1, 2, \ldots, \bar{K}; \\ a_k + \mathrm{j}\, b_{k-\bar{K}}, & k = \bar{K}+1, \bar{K}+2, \ldots, K. \end{cases} \tag{3}$$

By this encoding rule, the coordinates of the symbols $s_1, s_2, \ldots, s_{\bar{K}}$ are interleaved with the coordinates of the symbols $s_{1+\bar{K}}, s_{2+\bar{K}}, \ldots, s_{2\bar{K}}$. Now we construct n_1 OSTBC code matrices $\mathcal{O}_{M_1,i}$ $(i = 1, 2, \ldots, n_1)$ and n_2 OSTBC code matrices $\mathcal{O}_{M_2,j}$ $(j = 1, 2, \ldots, n_2)$ and arrange them in the intermediate matrices $\mathcal{C}_1$ and $\mathcal{C}_2$ as

$$\mathcal{C}_1 = \begin{bmatrix} \mathcal{O}_{M_1,1}(x_1, x_2, \ldots, x_{K_1}) \\ \mathcal{O}_{M_1,2}(x_{K_1+1}, x_{K_1+2}, \ldots, x_{2K_1}) \\ \vdots \\ \mathcal{O}_{M_1,n_1}(x_{(n_1-1)K_1+1}, x_{(n_1-1)K_1+2}, \ldots, x_{\bar{K}}) \end{bmatrix},$$

$$\mathcal{C}_2 = \begin{bmatrix} \mathcal{O}_{M_2,1}(x_{\bar{K}+1}, x_{\bar{K}+2}, \ldots, x_{\bar{K}+K_2}) \\ \mathcal{O}_{M_2,2}(x_{\bar{K}+K_2+1}, x_{\bar{K}+K_2+2}, \ldots, x_{\bar{K}+2K_2}) \\ \vdots \\ \mathcal{O}_{M_2,n_2}(x_{\bar{K}+(n_2-1)K_2+1}, x_{\bar{K}+(n_2-1)K_2+2}, \ldots, x_{2\bar{K}}) \end{bmatrix}.$$

Hence, the size of $\mathcal{C}_1$ and $\mathcal{C}_2$ are $\bar{T}_1 \times M_1$ and $\bar{T}_2 \times M_2$, respectively. The CIOD code matrix is formulated by

$$\mathcal{C} = \begin{bmatrix} \sqrt{\kappa_1}\,\mathcal{C}_1 & \mathbf{0}_{\bar{T}_1 \times M_2} \\ \mathbf{0}_{\bar{T}_2 \times M_1} & \sqrt{\kappa_2}\,\mathcal{C}_2 \end{bmatrix}. \tag{4}$$

where $\kappa_1 = \frac{1}{M_1 \mathsf{R}_{\mathcal{O},M_1}}, \kappa_2 = \frac{1}{M_2 \mathsf{R}_{\mathcal{O},M_2}}$ are the factors to normalize the Tx power. Note that κ is not always shown for notational brevity. The size of the CIOD code matrices are $T \times M$, where $T = \bar{T}_1 + \bar{T}_2 = n_1 T_1 + n_2 T_2$, $M = M_1 + M_2$.

IV. Equivalent Channels and ML Decoder

Since the mapping rule of the real and imaginary parts of symbols s_k are known, one can write explicitly the dispersion matrices of these symbols. For notational convenience, we reserve the letters A and B for the dispersion matrices of OSTBC and use the letters E and F for the dispersion matrices of CIOD codes; there are $K = 2\bar{K}$ pairs of such matrices E_k, F_k $(i = 1, 2, \ldots, K)$. Additionally, we can write $A_i(\mathcal{O}_{M_j})$ or $B_i(\mathcal{O}_{M_j})$ to denote the dispersion matrices of OSTBC $\mathcal{O}_{M_j}$ $(j = 1, 2)$. Since $A_i(\mathcal{O}_{M_j})$ or $B_i(\mathcal{O}_{M_j})$ are known, we can write E_k and F_k explicitly though they are quite lengthy.

We can write the CIOD codes using the dispersion form (2) as $\mathcal{C} = \sum_{k=1}^{K} (a_k E_k + b_k F_k)$, note that $K = 2\bar{K}$ and $\bar{K} = \mathrm{lcm}\{K_1, K_2\}$.

To simplify our analysis, we first consider the number of Rx antennas is $N = 1$ and generalize for $N > 1$ later.

Let the channel vector be $\boldsymbol{h} = \begin{bmatrix} h_1 & h_2 & \ldots & h_M \end{bmatrix}^{\mathsf{T}}$, the Rx vector be $\boldsymbol{y} = \begin{bmatrix} y_1 & y_2 & \ldots & h_T \end{bmatrix}^{\mathsf{T}}$, the data vector $\boldsymbol{d} = \begin{bmatrix} a_1 & b_2 & a_2 & b_2 & \ldots & a_K & b_K \end{bmatrix}^{\mathsf{T}}$, the additive noise vector be $\boldsymbol{z} = \begin{bmatrix} z_1 & z_2 & \ldots & z_T \end{bmatrix}^{\mathsf{T}}$. Let C be a CIOD code matrix, the Tx-Rx signals in (1) becomes

$$\begin{aligned} \boldsymbol{y} &= \sqrt{\rho} C \boldsymbol{h} + \boldsymbol{z} = \sqrt{\rho} \sum_{k=1}^{K} (a_k E_k \boldsymbol{h} + b_k F_k \boldsymbol{h}) + \boldsymbol{z} \\ &= \sqrt{\rho} \begin{bmatrix} E_1\boldsymbol{h} & F_1\boldsymbol{h} & E_2\boldsymbol{h} & F_2\boldsymbol{h} & \ldots & E_K\boldsymbol{h} & F_K\boldsymbol{h} \end{bmatrix} \boldsymbol{d} + \boldsymbol{z}. \end{aligned} \tag{5}$$

In (5), the scalars κ_1 and κ_2 are not included for brevity. We can rewrite (5) equivalently as

$$\begin{bmatrix} \boldsymbol{y} \\ \boldsymbol{y}^* \end{bmatrix} = \sqrt{\rho} \begin{bmatrix} E_1\boldsymbol{h} & F_1\boldsymbol{h} & \ldots & E_K\boldsymbol{h} & F_K\boldsymbol{h} \\ E_1^*\boldsymbol{h}^* & F_1^*\boldsymbol{h}^* & \ldots & E_K^*\boldsymbol{h}^* & F_K^*\boldsymbol{h}^* \end{bmatrix} \boldsymbol{d} + \begin{bmatrix} \boldsymbol{z} \\ \boldsymbol{z}^* \end{bmatrix}. \tag{6}$$

Let $\bar{\mathcal{H}}_k = \begin{bmatrix} E_k\boldsymbol{h} & F_k\boldsymbol{h} \\ E_k^*\boldsymbol{h}^* & F_k^*\boldsymbol{h}^* \end{bmatrix}$ for $k = 1, 2, \ldots, K$, it follows

$$\bar{\mathcal{H}}_k^\dagger \bar{\mathcal{H}}_k = \mathrm{diag}\left(\hat{h}_1, \hat{h}_2\right) \triangleq \hat{\mathcal{H}}_1, \quad \text{for } 1 \le k \le \bar{K}, \tag{7a}$$

$$\bar{\mathcal{H}}_k^\dagger \bar{\mathcal{H}}_k = \mathrm{diag}\left(\hat{h}_2, \hat{h}_1\right) \triangleq \hat{\mathcal{H}}_2, \quad \text{for } \bar{K} < k \le K, \tag{7b}$$

$$\bar{\mathcal{H}}_k^\dagger \bar{\mathcal{H}}_l = \mathbf{0}_{2\times 2}, \quad \text{for } k \ne l. \tag{7c}$$

where $\hat{h}_1 = 2\sum_{i=1}^{M_1} |h_i|^2, \hat{h}_2 = 2\sum_{i=1}^{M_2} |h_i|^2$.

Thus if the two sides of (6) are multiplied by $\bar{\mathcal{H}}_k^\dagger$, one gets

$$\underbrace{\bar{\mathcal{H}}_k^\dagger \begin{bmatrix} \boldsymbol{y} \\ \boldsymbol{y}^* \end{bmatrix}}_{\bar{\boldsymbol{y}}_k} = \sqrt{\rho} \hat{\mathcal{H}}_p \underbrace{\begin{bmatrix} a_k \\ b_k \end{bmatrix}}_{d_k} + \underbrace{\bar{\mathcal{H}}_k^\dagger \begin{bmatrix} \boldsymbol{z} \\ \boldsymbol{z}^* \end{bmatrix}}_{\bar{\boldsymbol{z}}_k}. \tag{8}$$

where $p = 1$ if $1 \le k \le \bar{K}$ and $p = 2$ if $\bar{K} < k \le K$.

The matrix $\bar{\mathcal{H}}_k^\dagger$ plays the role of the signature of the data vector d_k. Since the data vectors d_k can be completely decoupled, (8) can be used for ML detection. However, the noise vector $\bar{\boldsymbol{z}}_k$ is color with covariance matrix $\hat{\mathcal{H}}_p$, it needs to be whiten by a whitening matrix $\hat{\mathcal{H}}_p^{-1/2}$. After this whitening step, (8) becomes

$$\hat{\mathcal{H}}_p^{-1/2} \bar{\boldsymbol{y}}_k = \sqrt{\rho} \hat{\mathcal{H}}_p^{1/2} d_k + \hat{\mathcal{H}}_p^{-1/2} \bar{\boldsymbol{z}}_k. \tag{9}$$

We can conclude that the matrices $\mathcal{H}_1 = \hat{\mathcal{H}}_1^{1/2}$ and $\mathcal{H}_2 = \hat{\mathcal{H}}_2^{1/2}$ are the *equivalent channels* of CIOD codes.

The ML solution of (9) is

$$\hat{d}_k = \arg\min_{d_k}(\rho d_k^\top \hat{\mathcal{H}}_p d_k - 2\sqrt{\rho}\Re(\bar{\boldsymbol{y}}_k^\top) d_k). \tag{10}$$

The result in (10) can be generalized for multiple Rx antennas. To this end, we include the scalars κ_1 and κ_2 for completeness. We can show that $\hat{h}_1 = 2\kappa_1 \sum_{j=1}^{N}\sum_{i=1}^{M_1}|h_{i,j}|^2$, $\hat{h}_2 = 2\kappa_2 \sum_{j=1}^{N}\sum_{i=1}^{M_2}|h_{i,j}|^2$, $\bar{\boldsymbol{y}}_k = \sum_{j=1}^{N}\bar{\mathcal{H}}_{k,n}^\dagger \begin{bmatrix}\boldsymbol{y}_n \\ \boldsymbol{y}_n^*\end{bmatrix}$, where $\boldsymbol{y}_n$ is the Rx vector of nth antenna, $\bar{\mathcal{H}}_{k,n} = \begin{bmatrix}E_k\boldsymbol{h}_n & F_k\boldsymbol{h}_n \\ E_k^*\boldsymbol{h}_n^* & F_k^*\boldsymbol{h}_n^*\end{bmatrix}$, $\boldsymbol{h}_n$ is the nth column of the channel matrix H.

From (8), the decoding of the real symbols a_k and b_k can be decoupled. However, since the symbols a_k and b_k are not transmitted over M channels, full diversity cannot be achievable. Hence, we need to spread out these symbols over M channels by applying a real unitary rotation R_p as

$$R_p = \begin{bmatrix}\cos(\alpha_p) & \sin(\alpha_p) \\ \sin(\alpha_p) & -\cos(\alpha_p)\end{bmatrix}, \quad (p = 1, 2),$$

to the data vectors d_k [4], [7]. Including the rotation matrix to (9) and (10), we have

$$\hat{\mathcal{H}}_p^{-1/2}\bar{\boldsymbol{y}}_k = \sqrt{\rho}\hat{\mathcal{H}}_p^{1/2}R_p d_k + \hat{\mathcal{H}}_p^{-1/2}\bar{\boldsymbol{z}}_k. \tag{11}$$

and

$$\hat{d}_k = \arg\min_{d_k}(\rho d_k^\top R_p^\top \hat{\mathcal{H}}_p R_p d_k - 2\sqrt{\rho}\Re(\bar{\boldsymbol{y}}_k^\top) R_p d_k). \tag{12}$$

Akin to the decoding of OSTBC, the decoding metric (12) of CIOD codes does not involve the dispersion matrices [9]. This fact greatly reduces the decoding complexity compared with the one proposed in [4, eq. (84)], where the dispersion matrices of symbols are required. In the next section, we will investigate the performance of CIOD codes with different types of constellations by exploiting the special structure of the equivalent channels.

V. SER Union Bound and Optimal Signal Designs

We first consider the data vectors $d_k = [a_k \; b_k]^\top$ for $1 \leq k \leq \bar{K}$. These data vectors are sent over the same equivalent channel $\hat{\mathcal{H}}_1^{1/2}$ and therefore they have the same error probability; we thus drop the subindex k for short. Let $d = [a \; b]^\top$ and $\hat{d} = [\hat{a} \; \hat{b}]^\top$ be the transmitted and the erroneous detected vectors, let $\delta_1 = a - \hat{a}$, $\delta_2 = b - \hat{b}$, $\Delta = [\delta_1 \; \delta_2]^\top$. From (11), the SPEP of the symbol pair d_k and $\hat{d}_k$ can be expressed by the Gaussian tail function as [10]

$$P(d \to \hat{d}|\hat{\mathcal{H}}_1) = Q\left(\sqrt{\frac{\rho|\hat{\mathcal{H}}_1 R_1 \Delta|^2}{4N_0}}\right) \tag{13}$$

where $N_0 = 1/2$ is the variance of the real part of the elements of the white noise vector $\hat{\mathcal{H}}_p^{-1/2}\bar{\boldsymbol{z}}$ in (11). Let

$$\begin{bmatrix}\beta_1 \\ \beta_2\end{bmatrix} = R_1\Delta = \begin{bmatrix}\cos(\alpha_1) & \sin(\alpha_1) \\ \sin(\alpha_1) & -\cos(\alpha_1)\end{bmatrix}\begin{bmatrix}\delta_1 \\ \delta_2\end{bmatrix}. \tag{14}$$

Using the Craig's formula [11] to derive the conditional SPEP in (13), one has

$$\begin{aligned}P(d \to \hat{d}|\hat{\mathcal{H}}_1) &= Q\left(\sqrt{\frac{\rho(\beta_1^2 h_1 + \beta_2^2 h_2)}{2}}\right) \\ &= \frac{1}{\pi}\int_0^{\pi/2}\exp\left(\frac{-\rho(\beta_1^2 h_1 + \beta_2^2 h_2)}{4\sin^2\theta}\right)d\theta \\ &= \frac{1}{\pi}\int_0^{\pi/2}\prod_{j=1}^{N}\left(\prod_{i=1}^{M_1}\exp\left(-\frac{\rho\kappa_1\beta_1^2|h_{i,j}|^2}{4\sin^2\theta}\right)\right. \\ &\qquad \left.\prod_{i=1}^{M_2}\exp\left(-\frac{\rho\kappa_2\beta_2^2|h_{i,j}|^2}{4\sin^2\theta}\right)\right)d\theta. \end{aligned} \tag{15}$$

We can apply a method based on the moment generating function (MGF) [12], [13] to obtain the unconditional SPEP in the following:

$$\begin{aligned}&P_1(d \to \hat{d}) \\ &= \frac{1}{\pi}\int_0^{\pi/2}\left(1 + \frac{\rho\kappa_1\beta_1^2}{4\sin^2\theta}\right)^{-M_1 N}\left(1 + \frac{\rho\kappa_2\beta_2^2}{4\sin^2\theta}\right)^{-M_2 N}d\theta.\end{aligned} \tag{16}$$

The above SPEP is given for symbols s_k sent over the equivalent channel $\mathcal{H}_1$. For the symbols s_k $(\bar{K} < k \leq K)$ transmitted over the equivalent channel $\mathcal{H}_2$, the SPEP can be found similarly:

$$\begin{aligned}&P_2(d \to \hat{d}) \\ &= \frac{1}{\pi}\int_0^{\pi/2}\left(1 + \frac{\rho\kappa_2\bar{\beta}_1^2}{4\sin^2\theta}\right)^{-M_2 N}\left(1 + \frac{\rho\kappa_1\bar{\beta}_2^2}{4\sin^2\theta}\right)^{-M_1 N}d\theta\end{aligned} \tag{17}$$

where

$$\begin{bmatrix}\bar{\beta}_1 \\ \bar{\beta}_2\end{bmatrix} = \begin{bmatrix}\cos(\alpha_2) & \sin(\alpha_2) \\ \sin(\alpha_2) & -\cos(\alpha_2)\end{bmatrix}\begin{bmatrix}\delta_1 \\ \delta_2\end{bmatrix}. \tag{18}$$

Assume that d_i, d_j, d_m, d_n, $(i, j, m, n = 1, 2, \ldots, L)$, are signals drawn from a constellation $\mathcal{S}$ of size L. From the SPEP expression (15) and (17), we can find the union bound on SER of CIOD codes with constellation $\mathcal{S}$ as

$$P_u(\mathcal{S}) = P_{u,1}(\mathcal{S}) + P_{u,2}(\mathcal{S}) \tag{19}$$

where

$$P_{u,1}(\mathcal{S}) = \frac{1}{L}\sum_{i=1}^{L-1}\sum_{j=i+1}^{L}P(d_i \to d_j), \tag{20}$$

$$P_{u,2}(\mathcal{S}) = \frac{1}{L}\sum_{m=1}^{L-1}\sum_{n=i+1}^{L}P(d_m \to d_n). \tag{21}$$

For a fixed SNR, the union bound $P_u(\mathcal{S})$ depends on the constellation $\mathcal{S}$ and the rotation angles α_1 and α_2. Thus one can find the optimal values of α_1 and α_2 to minimize the union bound on SER. Note that α_1 and α_2 can be optimized separately. We can run computer search to find the optimal values of α_1 and α_2.

In practice, signal constellation $\mathcal{S}$ is usually symmetric via either horizontal or vertical axis of the Cartesian coordinate system. We can assume that $\mathcal{S}$ is symmetric via the vertical axis. If $\mathcal{S}$ is symmetric via the horizontal axis, we can always rotate the whole constellation an angle of $\pi/2$ to make it symmetric via the vertical axis.

Assume that $\alpha_2 = \pi/2 - \alpha_1$. Then, for each pair of symbols $(d_i, d_j) = \left([a_i,\ b_i]^\mathsf{T}, [a_j,\ b_j]^\mathsf{T}\right)$, we can find one and only one pair $(d_m, d_n) = \left([a_i,\ -b_i]^\mathsf{T}, [a_j,\ -b_j]^\mathsf{T}\right)$ so that $P_1(d_i \to d_j) = P_2(d_m \to d_n)$. Therefore, $P_{u,1}(\mathcal{S}) = P_{u,2}(\mathcal{S})$; and if α_{opt} is the optimal value of α_1, then $\pi/2 - \alpha_{opt}$ is optimal for α_2. Hence, we just write the value of α_1 and imply that the value of $\alpha_2 = \pi/2 - \alpha_1$.

The union bound on SER is plotted in Fig. 1 for a CIOD code for $M = 4$ Tx antennas $(M_1, M_2) = (2, 2)$. For the three examined constellations (4QAM, 8QAM-R, and 16QAM), and $\alpha_1 = 31.7175°$ [4], the union bound becomes tight when SER $< 10^{-1}$ and is less than 0.1 dB apart from the simulated SER at high SNR.

Since the union bound is tight for SER < 10^{-2}, it can be used to optimize the values of rotation angles α_1 and α_1. The new optimal signal rotations for the popular constellations based on minimizing the SER union bound are summarized in Table I. Only the optimal values α_{opt} of α_1 are listed, the optimal values of $\alpha_2 = \pi/2 - \alpha_{opt}$. The geometrical shapes of 8-ary constellations are sketched in Fig. 2. The best 8TRI in terms of minimum Euclidean distance (carved from the lattice of equilateral triangular) is selected [6].

Numerical Results

We compare the SER union bounds of CIOD code with several constellations using new optimal signal designs in Fig. 3 for $(M_1, M_2) = (2, 4)$. Obviously, QAM signals yield the best performance compared with other constellations of the same size. On the other hand, TRI constellations have the best minimum Euclidean distance; however, their performance is inferior to that of QAM signals.

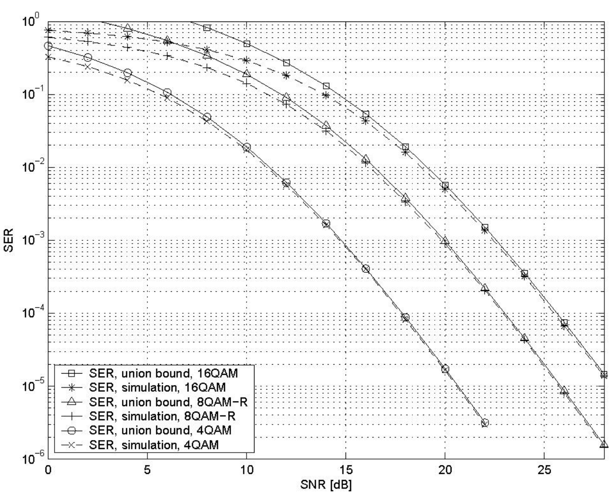

Fig. 1. Comparison of the simulated SER and the union bound of a rate-one CIOD code for 4 Tx antennas $((M_1, M_2) = (2, 2))$, using 1 Rx antennas.

TABLE I

OPTIMAL ROTATION ANGLES OF POPULAR CONSTELLATIONS

Signal	(2, 1)	(2, 2)	(2, 3)	(2, 4)	(3, 3)
4QAM	28.939°	30.417°	29.698°	29.003°	30.778°
4TRI	20.142°	13.883°	71.739°	68.687°	75.836°
8PSK	37.690°	39.216°	38.808°	38.534°	39.857°
8APSK	10.316°	11.528°	11.181°	11.000°	12.015°
8TRI	20.309°	45.000°	11.061°	9.430°	45.000°
8QAM-R	33.037°	31.834°	29.658°	28.626°	31.737°
8QAM-SR	12.234°	13.036°	12.925°	12.701°	13.173°
16PSK	3.485°	2.570°	2.832°	2.964°	2.200°
16TRI	19.236°	45.000°	47.116°	70.690°	45.000°
16QAM	31.436°	31.677°	31.557°	31.462°	31.704°

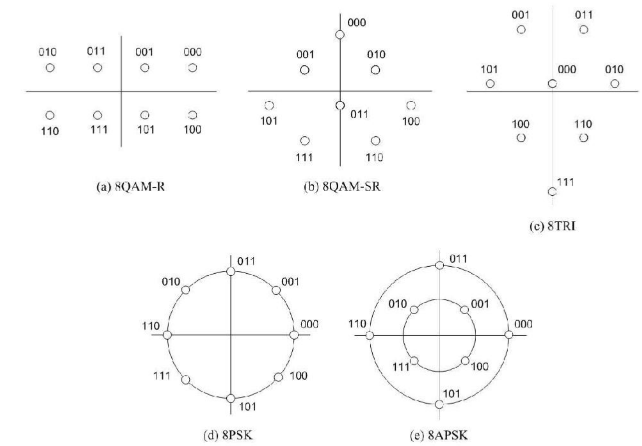

Fig. 2. Geometrical shapes of 8-ary constellations.

Compared with the optimal rotation angles for QAM in terms of coding gain [4], our newly proposed rotation angles result in marginal performance improvement. However, we will next present a new signal design method combining power allocation and signal rotation such that the new signal designs outperform the ones in [4] significantly for QAM-R.

VI. OPTIMAL SIGNAL ROTATION WITH POWER ALLOCATION

For QAM-R, e.g. 8QAM-R in Fig. 2, the average powers of the real and imaginary parts of the signal points are different. We may change the power allocation to the real and imaginary parts of QAM-R signals to get better overall SER.

To change the power allocation, the real and imaginary of QAM-R signals are first multiplied by constants σ_1 and σ_2, respectively, then they are rotated by unitary matrix R_1, R_2. For example, let $\mathcal{S}$ be a constellation with signal set $\mathcal{S} = \{d \mid d = a + \mathrm{j}\,b, a, b \in \mathbb{R}\}$, the new constellation with new power allocation is $\bar{\mathcal{S}} = \{\bar{d} \mid \bar{d} = \sigma_1 a + \mathrm{j}\,\sigma_2 b; a, b \in \mathbb{R}\}$. The average energy of the constellation $\bar{\mathcal{S}}$ is kept the same as that of $\mathcal{S}$, i.e. unitary. For example, the 8QAM-R with signal points $\{(\pm 3 \pm \mathrm{j}, \pm 1 \pm \mathrm{j})/\sqrt{48}\}$ has constraint equation for coefficients σ_1 and σ_2 as $5\sigma_1^2 + \sigma_2^2 = 6$. Hence, if the value of σ_1 is given, the value of σ_2 is known explicitly.

We still use (15) to calculate the union bound on SER of CIOD codes with signal rotation and power re-allocation; (16)

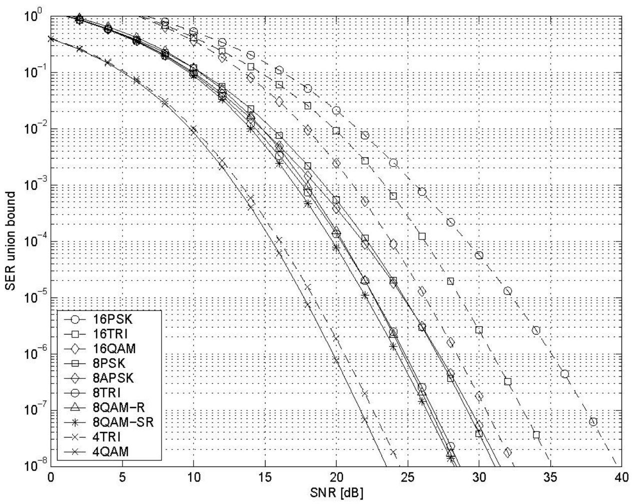

Fig. 3. SER union bound a CIOD code with rate of 6/7 symbol pcu for 6 Tx antennas ($(M_1, M_2) = (2, 4)$), using 1 Rx antennas.

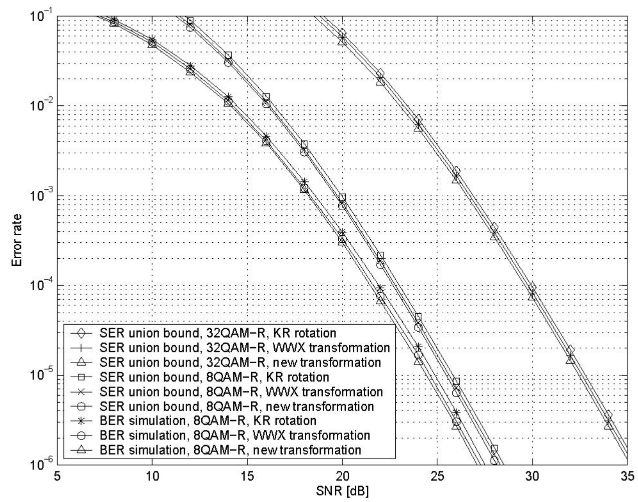

Fig. 4. BER and Union bound on SER of the rate-one CIOD code with 8QAM-R and 32QAM-R for 4 Tx/1 Rx antennas ($(M_1, M_2) = (2, 4)$).

can be rewritten to include the power re-allocation as

$$\begin{bmatrix}\beta_1\\ \beta_2\end{bmatrix} = \underbrace{\begin{bmatrix}\cos(\alpha_1) & \sin(\alpha_1)\\ \sin(\alpha_1) & -\cos(\alpha_1)\end{bmatrix}\begin{bmatrix}\sigma_1 & 0\\ 0 & \sigma_2\end{bmatrix}}_{\bar{R}_1}\begin{bmatrix}\delta_1\\ \delta_2\end{bmatrix}. \qquad (22)$$

The total effect of signal rotation and power re-allocation is the non-unitary signal transform $\bar{R}_1$. Now the minimization of the union bound is based on two variables: σ_1 (or σ_2) and α_1. We run exhaustive computer search to find the optimal values of σ_1 and α_1. In fact, there is only single value of σ_1 so that the union bound is minimized; this value of σ_1 is the global solution of the union bound minimization. The optimal values $(\sigma_1, \sigma_2, \alpha_{opt}) = (0.9055, 1.3784, 45.0°)$ for 8QAM-R and $(\sigma_1, \sigma_2, \alpha_{opt}) = (0.8972, 1.3487, 43.0°)$ for 32QAM-R.

The union bounds on SER of 8QAM-R and 32QAM-R using signal rotation of Khan-Rajan with $\alpha_1 = 31.7175°$ [4], signal transformation of Wang-Wang-Xia [7, Theorem 6], and our new signal transformation for CIOD codes with $M = 4$ $(M_1 = 2, M_2 = 2), N = 1$ are compared in Fig. 4. At SER = 10^{-6}, our new signal transformation yields 0.2 dB and 0.4 dB gains compared with the signal designs of Wang-Wang-Xia and Khan-Rajan, respectively. The BER of 8QAM-R also confirms the improvement of our newly proposed transformation over the existing ones.

VII. Conclusion

We have presented the equivalent channels for CIOD codes, enabling their decoding readily. The exact union bound on SER has been calculated. This bound is within 0.1 dB of the simulated SER at medium and high SNR. Thus, it can be used to analyze the performance of CIOD codes and, more importantly, to optimize the signal rotation for any constellation with an arbitrary geometrical shape. Performances of CIOD codes with different constellations such as QAM, PSK, TRI have been compared; among these constellations, QAM yields the best performance. We further present a new approach to design signal transformation for signal with uneven powers of the real and imaginary parts such as QAM-R. The new signal designs for QAM-R outperform the existing ones. The results of this paper can be extended in different aspects such as antenna selection and beamforming.

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