2006 IEEE International
Symposium on Signal Processing
and Information Technology

An Overview of Peak-to-Average Power Ratio Reduction Techniques for OFDM Systems

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***Abstract*— This paper reviews several Peak-to-Average Power Ratio (PAR) reduction techniques and the related optimization problems. Chipping-based PAR reduction techniques are related to convex optimization problems and the global optimum solutions are relatively easy to find. Probabilistic techniques result in discrete optimization. Although finding its global optima is difficult, moderate suboptimal solutions can be achieved with low computational cost. Coding is promising because of its inherit error-correcting property. However, its extremely low coding rate in cases of large number of subcarriers prevents its application. Many criteria involve in the selection of a PAR reduction technique, e.g., PAR reduction capacity, power increase, bit error rate increase, complexity, and throughput. A main consideration is that the cost of extra complexity for PAR reduction is lower than the cost of power inefficiency. Low complexity PAR reduction techniques may find application in mobile communications.**

I. Introduction

A main drawback of Orthogonal Frequency Division Multiplexing (OFDM) systems is the high Peak-to-Average Power Ratio (PAR) [1]. High PAR implies that the High Power Amplifier (HPA) in an OFDM wireless system must have an inefficiently used large linear range. Moreover, the nonlinearity of the HPA leads to inband distortion, which increases the bit-error ratio (BER), and out-of-band radiation, which interferes with communications in neighboring frequency bands [1]. Various techniques have been proposed to reduce the PAR, including, but not limited to, clipping and filtering [2], probabilistic techniques [3], and coding techniques [4]. Clipping and filtering clips the OFDM signal to a predefined threshold and uses a filter to eliminate out-of-band radiation. Probabilistic techniques use multiple candidates to represent the same information and select the one with lowest PAR for transmission. Although coding rates can be low, coding techniques guarantee a low PAR.

In this paper, we review several PAR reduction techniques available in the open literature, and discuss related optimization problems as well as the advantages and disadvantages of these techniques are also discussed. In general, clipping-based PAR reduction techniques can be formulated as convex optimization problems where a global optimum solution exists. On the other hand, probabilistic techniques lead to discrete optimization and global optima are difficult to find. PAR reduction is performed at the transmitter. Although the criteria for selecting a PAR reduction technique involve many aspects such as PAR reduction capacity, power increase, BER increase, complexity, and throughput, a main consideration is that the cost of extra complexity for PAR reduction is lower than the cost of power inefficiency. Low complexity PAR reduction techniques may find applications in mobile communications.

This paper is organized as follows: Section II briefly describes OFDM systems, the PAR distribution and the BER of a clipped OFDM signal. PAR reduction techniques are discussed in Section III. PAR reduction in Multiple-Input Multiple-Output (MIMO) OFDM systems is also briefly discussed in this section. Finally, Section IV concludes the paper.

II. OFDM Systems and the Peak-to-Average Power Ratio

In OFDM systems, the base-band equivalent time-domain signal $x(t)$ may be written as

$$x(t)=\frac{1}{\sqrt{N}}\sum_{k=-\frac{N}{2}}^{\frac{N}{2}-1}X_{\langle k+N\rangle}e^{j2\pi kt/T},\quad 0\le t\le T,\quad (1)$$

where $\langle k+N\rangle$ denotes $(K+N)$ modulo N, N is the number of subcarriers, T is the OFDM symbol period, and N data symbols X_k, $k=0,1,...,N-1$, form an OFDM symbol $\mathbf{X}=[X_0,X_1,...,X_{N-1}]$. The PAR of the OFDM symbol may be defined as [5]

$$\xi=\frac{\max_{t\in[0,T)}|x(t)|^2}{P_{\text{av}}},\quad (2)$$

where $P_{\text{av}}=2\sigma^2=E\{|x(t)|^2\}=E\{|X_k|^2\}$ is the average power. Since ξ is a random variable, a useful description is the complementary cumulative distribution function (CCDF), which is defined as the probability that PAR exceeds ξ_0, i.e., $\Pr[\xi>\xi_0]=P_c$.

To facilitate the computation of ξ, $x(t)$ is sampled to obtain

$$x_n=\frac{1}{\sqrt{N}}\sum_{k=-\frac{N}{2}}^{\frac{N}{2}-1}X_{\langle k+N\rangle}e^{j2\pi\frac{nk}{JN}},\quad n=0,...,JN-1,\quad (3)$$

where J is the oversampling factor. The discrete time PAR is defined similar to (2), and is approximately equal to ξ when $J\ge 4$ [6]. The time-domain samples x_n can be calculated by applying a (JN)-point Inverse Discrete Fourier Transform (IDFT) on the extended input vector[1]

$$\mathbf{X}_{\text{ext}} = [X_0, ..., X_{\frac{N}{2}-1}, \underbrace{0, ..., 0}_{(J-1)N \text{ zeros}}, X_{\frac{N}{2}}, ..., X_{N-1}]. \quad (4)$$

A. PAR Distribution and BER Performance

When X_k are Phase Shift Keying (PSK) symbols, an upper bound of the PAR can be easily obtained as [7]

$$\xi \leq 1 + \frac{2}{N} \sum_{n=1}^{N-1} |R_X(n)|, \quad (5)$$

where $R_X(n)$ is the aperiodic autocorrelation function of X_k defined as

$$R_X(n) = \sum_{k=0}^{N-n-1} X_{k+n} X_k^*, \quad (6)$$

with $(\cdot)^*$ representing complex conjugate. With the assumption that X_k are i.i.d. random variables and based on the central limit theory, x_n can be approximated as (complex) Gaussian random variables when N is large. Then, $|x_n|$ is Rayleigh distributed. The PAR distribution can be approximated as [8]:

$$\Pr[\xi \leq \xi_p] \approx \left(1 - e^{-\xi_p/2\sigma^2}\right)^{\alpha N}, \quad (7)$$

where $\alpha = 2.8$ obtained from empirical experiments. More accurate approximations are also available [9]–[14]. In [9], by using the theory of level-crossing rate and normalizing $r(t) = |x(t)|$ such that $2\sigma^2 = 1$, the probability that all peaks are lower than r is

$$\Pr(\max[r(t)] < r) \approx \begin{cases} \left(1 - \frac{re^{-r^2}}{\bar{r}e^{-\bar{r}^2}}\right)^{\sqrt{\frac{\pi}{3}}N\bar{r}e^{-\bar{r}^2}}, & \text{for } r > \bar{r}, \\ 0, & \text{for } r \leq \bar{r}, \end{cases} \quad (8)$$

where $\bar{r}$ is empirically obtained as $\bar{r} = \sqrt{\pi}$ for QPSK and slightly lower for 16QAM. The PAR distribution can then be found by replacing r and $\bar{r}$ with $\sqrt{\xi_p}$ and $\sqrt{\bar{\xi}_p}$, respectively.

The effect of signal clipping on BER performance has been extensively studied. Such analysis mainly focuses on the Signal-to-Noise-plus-Distortion Ratio (SNDR) and BER after the passage of $x(t)$ through a Soft Limiter (SL). The input/output relationship of the SL can be written as

$$g(x(t)) = \begin{cases} Ae^{j\phi(t)}, & |x(t)| > A, \\ x(t), & \text{otherwise}, \end{cases} \quad (9)$$

where $A > 0$ represents the clipping threshold.

Clipping $x(t)$ by the SL introduces clipping noise $f(t) = x(t) - g(x(t))$, which includes in-band distortion and out-of-band radiation. Figure 1 illustrates the power spectral density (PSD) of unclipped and clipped OFDM signals. 9 dB clipping leads to relatively small (−51 dB) out-of-band radiation. However, deeper clipping, e.g. 6 dB and 3 dB clipping, significantly increases out-of-band radiation to −31 dB and −21 dB, which may be unacceptable in practical communications. In [15], it is shown that by applying Bussgang's theorem [16], the clipped signal can be written as

$$\hat{x}_n = \alpha x_n + d_n, \quad n = 0, ..., JN - 1, \quad (10)$$

where x_n and d_n are uncorrelated, and the attenuation factor α can be found as

$$\alpha = 1 - e^{-\gamma^2} + \frac{\sqrt{\pi}\gamma}{2}\text{erfc}(\gamma) \quad (11)$$

with $\gamma = A/\sqrt{P_i}$.

For large N and small A, the clipping noise can be approximated as a Gaussian process. For Nyquist-rate clipping (x_n is Nyquist-rate sampled), no out-of-band radiation exists. In this case, the SNDR for AWGN channel is given by [15]

$$\text{SNDR} = \frac{K_\gamma E_s/N_0}{(1 - K_\gamma)E_s/N_0 + 1}, \quad (12)$$

where E_s/N_0 is the ratio of the signal energy over noise PSD after clipping, and $K_\gamma = \alpha^2/(1 - e^{-\gamma^2})$. The BER of QPSK is then given by

$$P_b = Q(\text{SNDR}), \quad (13)$$

where $Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty e^{-t^2/2} dt$.

The assumption of Gaussian clipping noise only holds for small A. When A is large, the clipping noise is a series of pulses which can be approximated as parabolic arcs [17]. Based on this approximation, the BER of a real-valued time domain OFDM signal (which is used for Discrete Multitone (DMT) applications) can be calculated as [18]

$$P_b = \frac{8N(L-1)}{\sqrt{3}L} e^{-\gamma^2/2} Q\left(\left[\frac{3\pi\gamma^2}{\sqrt{8(L^2-1)}}\right]^{1/3}\right), \quad (14)$$

where a L^2 points square constellation with minimum distance of $2d$ is assumed.

In [19], the performance of OFDM with a strictly limited peak-power requirement is analyzed. The analytical results show that the clipping technique exhibits the lowest Input Back-Off (IBO) requirement compared to the probabilistic techniques, especially when N is large.

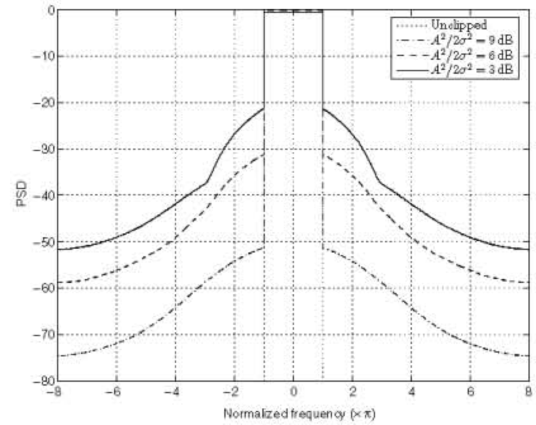

Fig. 1. Power spectral density of unclipped and clipped OFDM signals.

[1]This is the so called zero-insertion scheme. In literature, a zero-padding scheme is also used where the index of X_k is from $0, ..., N-1$, and $\mathbf{X}_{\text{ext}} = [X_0, X_1, ..., X_{N-1}, 0, ..., 0]$. The only difference between these two schemes is the position of the carrier frequency f_c.

III. PAR Reduction Techniques

Various techniques have been proposed to reduce the PAR, including clipping-based techniques , probabilistic techniques and coding techniques.

A. Clipping and Filtering

1) PAR Reduction: Clipping-based techniques [2], [20]–[26] clips the time domain signal to a predefined level. The out-of-band radiation is eliminated by filtering and the in-band clipping noise may be distributed on all or some subcarriers. The advantage of these techniques is that the optimization is usually a convex problem and a global optimum solution exists.

The clipping and filtering techniques [2], [20]–[22] allow clipping noise distributing on all subcarriers. The purpose of these technique is to eliminate the out-of-band radiation and to satisfy the spectral constraints, perhaps with the price of increased BER. Out-of-band radiation can be filtered either in time domain [2] using a lowpass filter or in frequency domain [20]–[22] by using an FFT/IFFT pair. By using frequency domain filtering, the clipping noise is converted to the frequency domain using oversampled FFT, and all out-of-band terms are set to zero. An IFFT is also required to convert the filtered clipping noise back to the time domain. Nevertheless, a side effect of filtering is the peak regrowth. Generally, peak regrowth can be combatted by iterative algorithms [22].

Tone reservation [23]–[25] partitions OFDM subcarriers into data tones (subcarriers) and low Signal-to-Noise Ratio (SNR) virtual tones not suitable for data transmission. The clipping noise is only allowed on virtual tones. Active constellation extension [26] allows the constellation be extended (by the clipping noise) so that the minimum Euclidian distance between any two constellation points does not increase. For example, the shaded areas in Fig. 2 are the feasible extension regions for the 16QAM constellation.

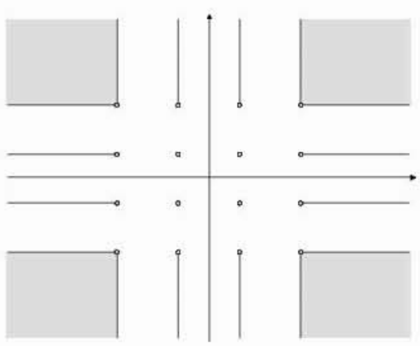

Fig. 2. Feasible extension region for 16-QAM constellation.

Since finding the global optimum solution may be computationally expensive, some fast algorithms are proposed in literature to obtain suboptimal solutions. The controlled clipper algorithm [23] obtains moderate PAR reduction. In this algorithm, an impulse-like signal is used to suppress the high peaks of OFDM signals. The convergence rate slows down after several iterations. Therefore, a tradeoff between PAR reduction and complexity has to be made to maintain reasonable computational complexity. [24] proposed an active-set approach similar to the iterative algorithm [20] but with a refined approach to find an optimal descent direction and an optimal step size. The optimization is based on the fact that an optimal solution may balance some peaks at the same magnitude. This algorithm is fast for DMT systems. However, for OFDM systems with complex signalling, this algorithm must approximate the circular boundary in the complex plane by a polygonal boundary such that the complex-numbered problem can be converted to a real-numbered problem. The complexity is then increased significantly. For example, an octagonal boundary leads to 4 times complexity as the real number case.

In [25], an adaptive-scaling algorithm is proposed for tone reservation. This algorithm utilizes the filtered clipping noise as the PAR reduction signal, and adaptively scales it to reduce the PAR. The objective function is

$$\min_{\beta} \sum_{|\hat{x}_n|>A} (|\hat{x}_n| - A)^2, \tag{15}$$

where $\hat{x}_n$ is the PAR-reduced signal written as

$$\hat{x}_n = x_n - \beta \hat{f}_n, \tag{16}$$

and f_n and $\hat{f}_n$ are the clipping noise before and after filtering, respectively. After simplifying (15) to

$$\hat{P} \approx \sum_{n\in\mathcal{S}_p} |f_n - \beta \hat{f}_n|^2, \tag{17}$$

where $\mathcal{S}_p$ is the index set of the peaks of f_n the optimal solution is found as

$$\beta^{(\mathrm{opt})} = \frac{\Re\mathfrak{e}[\sum_{n\in\mathcal{S}_p} f_n \hat{f}_n^*]}{\sum_{n\in\mathcal{S}_p} |\hat{f}_n|^2}, \tag{18}$$

where $\Re\mathfrak{e}[x]$ is the real part of x, and $(\cdot)^*$ represents complex conjugate. Since this algorithm only utilizes peak samples of clipping noise, its complexity is low.

2) Clipping Noise Cancelation: Clipping noise can be canceled at the receiver [27]–[30]. A powerful approach estimates clipping noise using the received signal [29], [30]. Using (10), the received samples after the FFT operation can be written as

$$Y_k = \alpha H_k X_k + H_k D_k + Z_k, \tag{19}$$

where Y_k, H_k, D_k and Z_k are the received signal, channel response, clipping noise, and AWGN noise on the k-th subcarrier, respectively. Assuming that H_k is known, a coarse estimation $\bar{X}_k$ can be obtained by making hard decision on Y_k/H_k. Converting $\bar{X}_k$ to time domain, clipping it with the same fashion as at the transmitter, and converting the clipped signal back to frequency domain, we have

$$\tilde{X}_k = \alpha \bar{X}_k + \bar{D}_k. \tag{20}$$

Assume that most $\bar{X}_k$ are correct. Then $\bar{D}_k \approx D_k$, and one can use $\bar{D}_k$ to obtain a better estimation of X_k, i.e.,

$$\hat{Y}_k = Y_k - H_k \bar{D}_k = \alpha H_k X_k + H_k(D_k - \bar{D}_k) + Z_k. \tag{21}$$

This procedure can be repeated to improve the estimation accuracy.

B. Probabilistic Techniques

By modifying the phase, amplitude and/or subcarrier position of input symbols, these techniques use several candidate OFDM signals to represent the same information, and select the one with the lowest PAR. Side information may be required at the receiver for correct detection.

1) Phase Adjustment Techniques: A widely used technique is to modify the phase of input symbols to reduce the PAR [3], [31]. Let $\mathbf{X} = [X_0, ..., X_{N-1}]$ be an OFDM vector, and let $\mathbf{c} = [c_0, ..., c_{N-1}] = [e^{j\phi_0}, ..., e^{j\phi_{N-1}}]$ be a phase adjustment vector. Then, the objective function is

$$\min_{\mathbf{c}} (\text{PAR of } \mathbf{c} \odot \mathbf{X}), \tag{22}$$

where $\odot$ represents element-wise multiplication. In order to correctly decode $\mathbf{X}$, the optimal $\mathbf{c}$ may be correctly transmitted to the receiver as side information. Moreover, to minimize the resulting throughput loss, $\mathbf{c}$ must be quantized to some predefined values. For example, $c_k \in \{1, -1\}$ or $\{1, j, -1, -j\}$. It has been indicated that [31] a set of possible values of c_k that is larger than $\{1, j, -1, -j\}$ will only lead to minor improvement in PAR reduction.

Finding optimal $\mathbf{c}$ has exponential complexity with N. Several algorithms have been proposed to reduce the complexity.

Selective mapping (SLM) [3] reduces the complexity by using K predefined, randomly generated (therefore, uncorrelated) phase adjustment sequences $\mathbf{c}_1, ..., \mathbf{c}_K$. If the probability that $\mathbf{c}_i$ $(i = 1, ..., K)$ leads to a PAR larger than ξ is P_ξ, then the probability that all $\mathbf{c}_i$ leading to PAR larger than ξ is $P_\xi^K < P_\xi$. K IFFT are needed and the minimum side information is $\log_2 K$ bits.

Partial transmit sequence (PTS) [3] partitions $\mathbf{X}$ into K disjoint subblocks $\mathbf{X} = [\mathbf{X}_1, ..., \mathbf{X}_K]$. $\mathbf{c}$ is also partitioned into K corresponding subblocks $\mathbf{c} = [\mathbf{c}_1, ..., \mathbf{c}_K]$, where the elements within each $\mathbf{c}_i$ are the same, i.e., $\mathbf{c}_i = [c_i, c_i, ..., c_i]$. Then the number of phase adjustments is reduced from N to K. making an exhaustive search possible. Note that

$$\mathbf{x} = \text{IFFT}[\mathbf{c} \odot \mathbf{X}] = \sum_{k=1}^{K} c_k \text{IFFT}[\mathbf{X}_k], \tag{23}$$

where $\mathbf{x}$ is the vector of time domain signal. Since each $\text{IFFT}[\mathbf{X}_k]$ is only calculated once, K IFFT are needed. The minimum side information is $\log_2(K-1)$ bits (since $\mathbf{c}_1 \equiv 1$).

An alternative of PTS is based on the quantization of the continuously valued (sub)optimal solution. Rewrite Eqn.(23) as

$$\mathbf{x} = \underbrace{\begin{bmatrix} A_{1,1} & A_{1,2} & \cdots & A_{1,M} \\ A_{2,1} & A_{2,2} & \cdots & A_{2,M} \\ \vdots & \vdots & & \vdots \\ A_{JN,1} & A_{JN,2} & \cdots & A_{JN,M} \end{bmatrix}}_{\mathbf{A}} \underbrace{\begin{bmatrix} c_1 \\ c_2 \\ \vdots \\ c_M \end{bmatrix}}_{\mathbf{c}}, \tag{24}$$

where $[A_{1,m}, A_{2,m}, \cdots, A_{JN,m}]^T$ is the IFFT of $\mathbf{X}_m$. For each row of $\mathbf{A}$, an optimal $\mathbf{c}$ can be found either by sorting the elements in this row of $\mathbf{A}$ according to the decreasing order of their magnitudes and letting c_k alternatively be the phase and negative phase of the corresponding $A_{n,m}$ [32], or by projecting a predefined vector to the null space of this row of $\mathbf{A}$ [33]. Then, the optimal continuous valued $\mathbf{c}$ is quantized to discrete values. Eventually JN candidates of $\mathbf{c}$ are found, and the one leading to the lowest PAR is selected.

It has been proved [34] that with $c_k \in \{1, -1\}$ $(k = 1, ..., N)$ the PAR can be reduce to less than $c \log N$ where c is a constant. By using derandomization theory, an iterative approach is proposed in [35]. This approach outperforms other probabilistic techniques. However, its computational complexity is still high since it requires the calculation of $2JN$ trigonometric functions.

2) Amplitude/Phase Adjustment Techniques: The tone injection technique [23] expands the constellation to a larger constellation, e.g., expanding 4QAM to 16QAM. Then each data symbol can be represented by several points in the larger constellation. PAR can be reduced by carefully choosing candidate constellation points. The side effects of this technique are an increasing average power and loss in throughput. The latter can be alleviated by using convolutional codes [36].

3) Scrambling and interleaving: Data permutation can also be used to randomly generate K independent candidates for PAR reduction [5]. The permutation can be performed either bit-wise or symbol-wise. In [37], a selective scrambling approach is proposed where 4 scramblers are used for picking up a candidate with lowest PAR, and 2 bits (00,01,10, or 11) are concatenated with information bits to indicate which scrambler is used. Note that, by setting some subcarriers to zero, PAR can also be reduced, which leads to zero-padded PSK [38]. This approach can also be viewed as modifying the signal constellation such that zero is included in the constellation.

C. Coding Techniques

Coding techniques only transmit the codewords having low PAR [39]–[41]. A look-up table may be used if N is small. On the other hand, Golay complementary sets may be used to generate polyphase sequences with low PAR [4].

K sequences $\mathbf{X}_1, ..., \mathbf{X}_K$ are said to be complementary if the sum of their aperiodic autocorrelation functions satisfies

$$\sum_{k=1}^{K} R_{\mathbf{X}_k}(n) = \delta(n) \sum_{k=1}^{K} R_{\mathbf{X}_k}(0), \tag{25}$$

where $\delta(n)$ is the Kronecker delta function and $R_{\mathbf{X}_k}(n)$ is defined in (6). Golay complementary sets become the Golay complementary pairs when $K = 2$. We can show that

$$\text{PAR}_{\mathbf{X}_k} \leq K, \quad k = 1, ..., K. \tag{26}$$

The Rudin-Shapiro sequence [4] is a special case of Golay complementary pair, which can be generated recursively as follows:

$$p_{n+1} = p_n + e^{j\theta_n} z^{2^n} q_n, \tag{27}$$

$$q_{n+1} = p_n - e^{j\theta_n} z^{2^n} q_n, \tag{28}$$

where $p_0 = q_0 = 1$, $z = e^{j\omega_0 t}$, and $e^{j\theta_n}$ are PSK symbols. The PAR of p_n or q_n is no larger than 2.

Golay complementary sequences can also be generated by concatenating or interleaving two short complementary sequences [41]. It has been shown [4] that most Golay complementary sets are related to first order Reed-Muller codes. Therefore, a Golay complementary sequence can be generated by

$$\mathbf{X} = [u_0, u_1, ..., u_m, c_1, c_2, ..., c_K]\mathbf{G}, \quad (29)$$

where $u_i \in \{0, ..., M-1\}$ ($i = 0, ..., m$, and M is an even number) are phase indices of M-PSK symbols, $c_k \in \{0, M/2\}$ ($k = 1, ..., K$, and $K = \binom{m}{2}$) defines the second order coset, $\mathbf{G}$ is the generator matrix of the second order Reed-Muller code with elements of 0 or 1 and dimension of $(m+K+1) \times 2^m$, and the addition and multiplication operations in matrix multiplication is defined over modulo-M. The corresponding M-PSK OFDM input symbol is $e^{j\mathbf{X}/M}$.

For given c_k, the minimum Hamming distance is $d_{\min} = 2^{m-1}$, and the coding rate is

$$R = \frac{m+1}{2^m} = \frac{\log_2 N + 1}{N}, \quad (30)$$

where N is the length of coded codewords. With N increasing, R goes to zero.

Note that there are many choices of c_k that lead to codewords with PAR less than 2. If all these codewords are used, the minimum Hamming distance is $d_{\min} = 2^{m-2}$, and the increased coding rate is bounded as

$$R \leq \frac{m+1+\binom{m}{2}}{2^m}. \quad (31)$$

However, R is close to zero when N is very large.

Reed-Muller codes can be written as a boolean function $f(x_1, ..., x_m)$, where $[x_m, ..., x_1]^T$ forms a m by 2^m matrix with columns, from left to right, being the binary representation of $1, 2, ..., 2^m$, respectively. The second order Reed-Muller code only contains the terms of x_i and $x_i x_j$ ($\forall i$ and j). [4] shows that, for any permutation π of the symbols $\{1, 2, ..., m\}$ and for any u, $u_k \in \mathbb{Z}_{2^h}$, where h is an integer,

$$a(x_1, ..., x_m) = \sum_{k=1}^{m} u_k x_k + 2^{h-1} \sum_{k=1}^{m-1} x_{\pi(k)} x_{\pi(k+1)} + u, \quad (32)$$

is a Golay complementary sequence over $\mathbb{Z}_{2^h}$ of length 2^m. Note that the second term determines the value of c_k in Eqn.(29). This equation gives $m!/2$ cosets of first order Reed-Muller codes. Therefore, the coding rate is

$$R = \frac{m+1+\lfloor \log_2(m!/2) \rfloor}{2^m}. \quad (33)$$

In [42], it is proved that (32) forms a path on a graph $G(Q)$ with vertices of $x_1, x_2, ...x_m$. If deleting k vertices of the graph results in a path, then all codeword of the coset $Q + \mathrm{RM}_q(1, m)$ (q is an even number) has PAR no larger than 2^{k+1}. Therefore, a tradeoff is allowed between the coding rate and PAR. Similarly, [43] proposed multiple shift codes which also make a tradeoff between the coding rate and PAR. The main property of multiple shift codes is

$$R_{\mathbf{X}}(n) + R_{\mathbf{Y}}(n) = 0, \quad \text{for } 1 \leq n \leq N-1 \text{ and } n \bmod L = 0, \quad (34)$$

where $L \in \{1, 2, ..., N-1\}$. The PAR of $\mathbf{X}$ or $\mathbf{Y}$ is then no larger than L.

Ordinary Golay complementary sequences are restricted to PSK modulation. Recently, methods of constructing complementary sequences on high order QAM constellations have been proposed [44], [45]. These methods use two 4QAM on the complex plane. By properly choosing the offsets of the two 4QAM, complementary sequences can be constructed on 16QAM or 64QAM. For more detail, see [45] and references therein.

D. PAR Reduction for MIMO OFDM systems

Interest has been growing to apply OFDM into multiple antenna systems and use Space-Time Block Codes (STBC) or Space-Frequency Block Codes (SFBC) to improve the system capacity. MIMO OFDM systems also suffer from the high PAR. Generally, PAR reduction techniques for conventional Single-Input Single-Output (SISO) OFDM systems can be directly applied to MIMO OFDM systems. Some modifications exploiting the structure of MIMO systems are also proposed in the literature.

Instead of optimizing each antenna separately as in SISO OFDM, most modifications for PAR reduction in MIMO OFDM focus on optimization over all antennas to reduce the amount of side information and/or the complexity (with slight loss of PAR reduction performance). For example, when SLM is used, each phase adjustment vector is multiplied to all antennas. The phase adjustment vector leading to the lowest PAR on all antennas is selected [46]. Cross-antenna rotation and inversion (CARI) method also adjusts the phase of data symbols and swaps data symbols between two antennas to reduce the PAR [47]. When SFBC is used in MIMO OFDM, CARI must be modified such that the SFBC code structure is not violated [48].

IV. CONCLUSIONS

In this paper, we reviewed various PAR reduction techniques and discussed related optimization problems as well as the advantages and disadvantages of these techniques. While clipping-based PAR reduction techniques can be formulated as convex optimization problems where the global optimum solution exists, probabilistic techniques leads to discrete optimization and global optima are difficult to find. Many criteria involve in the selection of a PAR reduction technique, e.g., PAR reduction capacity, power increase, BER increase, complexity, and throughput. A main consideration is that the cost of extra complexity for PAR reduction is lower than the cost of power inefficiency. Low complexity PAR reduction techniques may find application in mobile communications.

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