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UNITARY MATRIX DESIGN VIA GENETIC SEARCH FOR DIFFERENTIAL SPACE-TIME MODULATION AND LIMITED FEEDBACK PRECODING

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ABSTRACT

Because of their orthogonality properties, unitary matrices are an important class of matrices that are used in mathematics, physics, control, communications and others. In multiple-input multiple-output (MIMO) communication systems, there are two main applications that use unitary matrices: differential space-time modulation (DUSTM) and precoding. DUSTM is used when the channel state information (CSI) is not available for both transmitter and receiver, while unitary precoding is used when complete or partial CSI is available for both sides. For DUSTM and limited feedback MIMO systems, a codebook of unitary matrices should be designed. Conventionally, design parameters are optimized based on a cost function depending on the application. This optimization is time consuming when the system dimension and/or codebook size are increased. In this paper, we propose to relax the design parameters to be real rather than integer and use a genetic algorithm to find the optimal solution based on the related cost function. This approach provides better codes than the codes extracted from exhaustive search over integer parameters. The code extraction is rapid even when the system dimensions are large.

I. INTRODUCTION

Multiple-input multiple-output (MIMO) wireless channels, created by deploying multiple antennas at both the transmitter and receiver, promise high capacity and high-quality wireless communication links [1], [2]. To realize the benefits of MIMO channels, space-time codes and receiver algorithms are required to provide a performance and complexity trade-off. The structure of a space-time code depends directly on the available information of the channel at the receiver and/or transmitter. Popular Space–time codes proposed for MIMO system are based on a common assumption that the channel gains are known to the receiver but not to the transmitter.

In some applications, particularly in fast fading environments, since the channel state information (CSI) is not available for both transmitter and receiver, differential unitary space-time modulation (DUSTM) uses a codebook of unitary matrices for differential modulation and demodulation [3], [4]. The unitary matrices [4] are constructed from a diagonal matrix and a rectangular sub-matrix of the Discrete Fourier Transform (DFT) matrix. The diagonal terms are some points on the unit circle in the complex plain where their angles are defined by some integers that should be optimized. To improve the performance of DUSTM, another class of unitary matrices has been proposed in [5] by appending a block-diagonal rotation matrix to the code structure proposed in [4] where the angle of rotation depends on an integer parameter. In both structures, the optimization process for the design parameters is based on a cost function related to the pairwise error probability of codewords.

On the other hand, communication systems that experience a slow fading environment, the transmitter may have complete or partial CSI. One way to exploit the channel information is precoding. The optimum precoder can be obtained with full channel state information at the transmitter since this allows the transmitted signal to be formed based on the eigen structure of the channel matrix [6], [7]. Due to the bandwidth limits of the feedback channel in many applications, however, full CSI is not available at the transmitter, and limited-feedback precoidng techniques are of interest [8], [9]. In [9], the authors propose to use a set of unitary precoders derived from Grassmannian subspace packing for limited feedback systems. The codebook of unitary precoders is known to both the transmitter and the receiver and for each channel realization only the index of the appropriate matrix (precoder) is sent back to the transmitter. The precoder structure is the same as the structure proposed in [3] for DUSTM where the optimization process for diagonal angles is based on a distance defined for subspace packing.

In summary, the unitary matrices proposed in the literature contain the design integer parameters to be extracted from an optimization problem. The cost function for the optimization depends on DUSTM or precoding. The optimum is obtained by exhaustive search over all possible integer design parameters, which is almost impossible even for moderate system dimension and/or codebook size. In this paper, we relax the design parameters to be real numbers rather than integers and introduce a genetic algorithm to solve the resulting optimization problem. Simulation results show that by relaxing the design parameters, we obtain better codes for DUSTM and precoding. Although there is no proof for the global optimality of the outputs of the genetic algorithm, it works much faster than exhaustive search with even better codes than optimal integer design.

Notation: $(\cdot)^*$ denotes conjugate transpose. The trace, determinant and the Frobenius norm of matrix $\mathbf{A}$ are trace$(\mathbf{A})$, $\det(\mathbf{A})$ and $\|\mathbf{A}\|_F^2 = \mathrm{tr}(\mathbf{A}\mathbf{A}^H)$. A circularly complex Gaussian variable with mean μ and variance σ^2 is denoted by $z \sim \mathcal{CN}(\mu, \sigma^2)$. $\mathbf{A} \otimes \mathbf{B}$ denotes the Kronecker product of matrices $\mathbf{A}$ and $\mathbf{B}$. We also use $\mathbf{I}_N$ to represents the $N \times N$ unitary matrix.

II. DIFFERENTIAL UNITARY SPACE-TIME MODULATION

Consider a communication system with N_t transmit and N_r receive antennas operating over a Rayleigh flat fading MIMO channel that remains constant for $2T$ signaling intervals. In addition, the channel coefficients are assumed to be unknown

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to both the transmitter and receiver. In this case, the receive signal matrix can be modeled as:

$$\mathbf{Y}_\tau = \sqrt{\rho}\mathbf{S}_\tau \mathbf{H}_\tau + \mathbf{V}_\tau \tag{1}$$

where τ denotes the time index of block transmission, $\mathbf{Y}_\tau \in \mathcal{C}^{T\times N_r}$ and $\mathbf{S}_\tau \in \mathcal{C}^{T\times N_t}$ denote the complex received and transmitted matrix, respectively. Also, $\mathbf{H}_\tau \in \mathcal{C}^{N_t\times N_r}$ represents the channel matrix and $\mathbf{V}_\tau \in \mathcal{C}^{T\times N_r}$ stands for the additive noise matrix. Entries of $\mathbf{H}_\tau$ and $\mathbf{V}_\tau$ are independent and identically distributed (i.i.d.) $\mathcal{CN}(0,1)$ random variables. The sum of the average signal powers at each time instant is normalized to unity i.e. $E\{\mathrm{tr}(\mathbf{S}_\tau\mathbf{S}_\tau^*)\} = T$ to guarantee that ρ is the average signal-to-noise ratio (SNR) per receiver.

Suppose a data sequence of integers $d_1, d_2, \ldots$ with $d_\tau \in \{0,\ldots,L-1\}$ is to be transmitted. The positive integer $L \geqslant 2$ denotes the constellation size which is equal to $L = 2^{RN_t}$ with R representing the information rate in bits per second per hertz(b/s/Hz). Each d_τ is mapped to a matrix Φ_{d_τ} drawn from the set $\{\Phi_l | l = 0,\ldots,L-1\}$. In differential transmission, the initial signal matrix is $S_0 = \mathbf{I}$ where $\mathbf{I}$ is the identity matrix. Thereafter, at time τ to send Φ_{d_τ} from the constellation set of unitary matrices, the following matrix is transmitted:

$$S_\tau = \Phi_{d_\tau} S_{\tau-1} \qquad \tau = 1,2,\ldots. \tag{2}$$

Assuming the cannel coefficients are almost constant over two consecutive blocks i.e. $\mathbf{H}_\tau \approx \mathbf{H}_{\tau-1}$, it is shown in [4] that the maximum-likelihood decoder would be

$$\tilde{d_\tau} = \underset{0\leq l<L}{\operatorname{argmin}} \|\mathbf{Y}_\tau - \Phi_l \mathbf{Y}_{\tau-1}\|_F^2. \tag{3}$$

The exact pairwise error probability (PEP) has been derived in [4]. By assuming all messages are equally likely and $T = N_t$, the exact PEP is

$$P_{ll'} = p(\Phi_l \longrightarrow \Phi_{l'}) = \frac{1}{\pi}\int_0^{\frac{\pi}{2}} \prod_{i=1}^{N_t}\left(1 + \frac{\gamma\lambda_i}{4\sin^2(\theta)}\right)^{-N_r} d\theta \tag{4}$$

where $\gamma = \frac{\rho^2}{1+2\rho}$ and λ_i is the i-th eigenvalue of the matrix

$$\Delta_{ll'} = (\Phi_l - \Phi_{l'})(\Phi_l - \Phi_{l'})^*.$$

The Chernoff bound of PEP is derived in [4]

$$P_{ll'} \leq \frac{1}{2}\prod_{m=1}^{N_t}\left[1 + \gamma\sigma_m(\Phi_l - \Phi_{l'})\right]^{-N_r} \tag{5}$$

where $\sigma_m(\Phi_l - \Phi_{l'})$ is the m-th singular values of $(\Phi_l - \Phi_{l'})$.

It has been shown in [4] and [10] that under high SNR conditions to minimize PEP, the design criterion is to maximize the diversity product, ζ. The diversity product is expressed as

$$\zeta = \frac{1}{2}\min_{l\neq l'}\left(\prod_{m=1}^{N_t}\sigma_m(\Phi_l - \Phi_{l'})\right)^{\frac{1}{N_t}}. \tag{6}$$

In [11], based on the exact PEP, a union bound on the symbol error probability (SEP) of the DUSTM is derived. It has been shown [12] that a good approximation of the upper bound on SEP for any constellation sets (including both group or non-group DUSTM) is

$$P_{UB} \approx \frac{1}{18L}\sum_{l=0}^{L-1}\sum_{l\neq l'}^{L-1}\sum_{i=1}^{9}\frac{1}{\det[I + \frac{\gamma}{4x_i^2}\Delta_{ll'}]^{N_r}}, \tag{7}$$

where $x_i = \cos(2i-1)\pi/18$. Note from (7) that unlike (6), P_{UB} basically depends on the type of constellation sets $\{\Phi_l\}$ and number of receive and transmit antennas as well as the SNR. Therefore, for DUSTM, a set of unitary matrices $\{\Phi_l\}$ should be designed by maximizing (6) or minimizing (7).

III. Unitary Precoding

For unitary precoding, it is more convenient to use the following the linear transformation between the transmit and receive antennas

$$\mathbf{X} = \sqrt{\frac{\rho}{M}}\mathbf{HFS} + \mathbf{V} \tag{8}$$

where the matrix $\mathbf{X} \in \mathcal{C}^{N_r\times T}$ is the complex received matrix, $\mathbf{S} \in \mathcal{C}^{M\times T}(M \leqslant \min(N_t, N_r, T))$ is the transmitted matrix, $\mathbf{F} \in \mathcal{U}^{N_t\times M}$ is the unitary precoder matrix, $\mathbf{V} \in \mathcal{C}^{N_r\times T}$ is the additive noise matrix, $\mathbf{H} \in \mathcal{C}^{N_r\times N_t}$ is the channel matrix and ρ is the total transmit power at each signaling interval. Entries of $\mathbf{H}$ and $\mathbf{V}$ have the same characteristics as (1).

For each transmission, according to the put data, Q signals, $\{s_1,\ldots,s_Q\}$, are chosen from a signal constellation (for example QAM) with unit average energy. Then according to a space-time coding structure, the transmit matrix $\mathbf{S}$ with rate Q/T symbol per channel use is constructed to be sent over M virtual transmit antennas. $\mathbf{S}$ is precoded by $\mathbf{F}$ and sent over N_t transmit antennas.

By singular value decomposition (svd) of the channel matrix, $\mathbf{H} = \mathbf{V}_L\mathbf{\Sigma}\mathbf{V}_R^*$, where $\mathbf{V}_L$ and $\mathbf{V}_R$ are unitary matrices and $\mathbf{\Sigma}$ is an ordered diagonal matrix ($\sigma_i > \sigma_{i+1}$), it is shown [6] that the optimum precoder is $\bar{\mathbf{V}}_R = \mathbf{V}_R(:,1{:}M)$, i.e. a matrix constructed by the first M columns of $\mathbf{V}_R$. Since for limited feedback systems, it is impossible to send back $\bar{\mathbf{V}}_R$, in [8] and [9], a set ($\mathcal{W}$) of suboptimum unitary matrices, derived from Grassmannian subspace packing, has been proposed.

For a given $\mathbf{H}$, the only feedback parameter is $\mathcal{I}$ which is the index of $\mathbf{F}_\mathcal{I} \in \mathcal{W}$, obtained from one the following optimization problems:

- If $\mathbf{S}$ is an orthogonal space-time code, $\mathbf{F}_\mathcal{I}$ is the member of $\mathcal{W}$ that maximizes $\|\mathbf{HF}\|$. For this case, the members of $\mathcal{W}$ are designed such that the minimum *chordal distance* between each pair is maximized [8]. The chordal distance is defined as

$$d_{chord}(\mathbf{F}_i, \mathbf{F}_j) = \frac{1}{\sqrt{2}}\|\mathbf{F}_i\mathbf{F}_i^* - \mathbf{F}_j\mathbf{F}_j^*\|,\ 0 \leqslant i \neq j < L \tag{9}$$

where L is the size of the codebook $\mathcal{W}$.

- If $\mathbf{S}$ is a VBLAST code i.e. all elements of $\mathbf{S}$ are independently chosen (multiplexing), and the decoding is maximum likelihood (ML), then $\mathbf{F}_\mathcal{I}$ is the member of $\mathcal{W}$ that maximizes $\lambda_{\min}(\mathbf{HF})$. For this case, the members of $\mathcal{W}$ are designed such that the minimum *projection two-norm distance* between any pair is maximized [9]. The projection two-norm distance is defined as

$$d_{proj}(\mathbf{F}_i, \mathbf{F}_j) = \|\mathbf{F}_i\mathbf{F}_i^* - \mathbf{F}_j\mathbf{F}_j^*\|_2 = \sqrt{1 - \lambda_{\min}^2(\mathbf{F}_i^*\mathbf{F}_j)}. \quad (10)$$

- If $\mathbf{S}$ is a VBLAST code and the aim is to maximize the system capacity, then $\mathbf{F}_{\mathcal{I}}$ is the member of $\mathcal{W}$ that maximizes $\log_2 \det(\mathbf{I} + \frac{\rho}{M}\mathbf{F}^*\mathbf{H}^*\mathbf{HF})$. For this case, the members of $\mathcal{W}$ are designed such that the minimum *Fubini-study distance* between any pair is maximized [9]. The Fubini-study distance is defined as

$$d_{FS}(\mathbf{F}_i, \mathbf{F}_j) = \arccos |\det(\mathbf{F}_i^*\mathbf{F}_j)| \quad (11)$$

In summary, similar to unitary codebook design for DUSTM, several criteria apply for codebook design in unitary precoding. The common problem in DUSTM and precoding is to present a structured unitary matrix and optimize the related parameters for a given criterion.

IV. Unitary Matrix Design

Although different unitary constellations have been proposed in the literature in particular for DUSTM [4], [5] and [10], we introduce two of the most common ones in this paper: *cyclic group design* and *cyclic-rotated design*.

The cyclic group design is proposed in [4] where a diagonal unitary matrix is used for DUSTM. The cyclic matrix is expressed as

$$\Phi_l = \mathrm{diag}\,[e^{j\theta_L\mu_1 l}, e^{j\theta_L\mu_2 l}, \ldots, e^{j\theta_L\mu_{N_t} l}] \quad (12)$$

where $\theta_L = 2\pi/L$ and $l = 0, 1, \ldots, L-1$. The cyclic codes in (12) are determined by N_t parameters $\underline{\mu} = \{\mu_1, \mu_2, \ldots, \mu_{N_t} \mid 0 \leqslant \mu_i < L\}$. The design goal is to find a set of parameters $\underline{\mu}$ that are optimum given the different criteria mentioned in Sections II. and III..

The cyclic-rotated design proposed in [5] yields better performance than cyclic codes. The cyclic-rotated matrix is constructed of a cyclic matrix multiplied by a block diagonal rotation matrix $\mathbf{RF}$ given by [5]

$$\mathbf{RF}(k\theta_L) = \mathbf{I}_{N_t} \otimes \mathbf{R}_2(k\theta_L) \quad (13)$$

where

$$\mathbf{R}_2(k\theta_L) = \begin{bmatrix} \cos(k\theta_L) & \sin(k\theta_L) \\ -\sin(k\theta_L) & \cos(k\theta_L) \end{bmatrix}$$

and $0 \leqslant k < L$ is called rotation factor which should be optimized along with $\underline{\mu}$. Note that N_t should be an even number. The associated unitary matrix is expressed as

$$\Phi_l = \begin{bmatrix} e^{j\theta_L\mu_1} & \ldots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \ldots & e^{j\theta_L\mu_{N_t}} \end{bmatrix}^l .[\mathbf{RF}(k\theta_L)]^l. \quad (14)$$

In [12], it has been shown that the parameter k should not necessarily be the same for each rotation block (i.e. $\mathbf{R}_2(k_i\theta_L)$,$i = 1 \ldots N_t/2$ instead of $\mathbf{R}_2(k\theta_L)$). Although this assumption increases the search space in code design, resulting codes perform better than the codes with single rotation parameter.

The design parameters $\underline{\mu}$ and k are obtained based on the criteria mentioned in Sections (II.) and (III.) depending on the application. Without loss of generality, we assume that a set of parameters should be optimized by minimization of a cost function. In previous works, the design parameters are restricted to integers. Exhaustive computer search or random search for optimum parameters is employed since analytical determination of the optimum appears intractable. Moreover, due to the fact that the computational complexity increases exponentially with N_t and L, it is difficult to find the optimum parameters for large L and N_t with exhaustive search. With random search, there is no guarantee that it converges to even an acceptable neighborhood of the optimum parameters.

To handle these problems, we propose to employ a genetic algorithm to extract the optimal parameters. Although a genetic algorithm does not guarantee the global optimality of its answers, the cost of genetic solutions are better than the optimum values from exhaustive search. This seemingly contradictory result is obtained by relaxing the design parameters to be real rather than integer numbers in the genetic algorithm. This extension increases the set of search parameters, which allow us to improve the chance to obtain better codes. In the following sections, we explain how the genetic algorithm operates and provide experimental results.

V. Genetic Algorithms

The genetic algorithm [13] is an exceptional search technique for finding approximate solutions to optimization and search problems based on natural selection, the process that drives biological evolution. To use a genetic algorithm, we must find a method of representing a solution (encoding the solution) in such a form that it can be manipulated by the algorithm. Usually, solutions are represented in binary as strings of 0s and 1s but different encodings are also possible. Additionally, we require the fitness function (cost function) to measure the quality of any solution.

The algorithm begins by creating a random initial population and then making a sequence of new populations/generations. In each generation, the fitness of the whole population is evaluated and a score is assigned to each member of the current population. Each member with higher associated fitness value, gets higher score. A selection mechanism based on the given scores is applied to the population and the individuals strive for survival. The fitter individuals have more chance to be selected to produce the child generation by means of genetic transformations such as crossover and mutation. Because the entire population participates in the search, the genetic algorithm is less likely than many search procedures to get stuck at a local minimum. As the algorithm continues and newer and newer generations evolve, the quality of solutions improves.

In general, the next generation is composed of three types of

Table 1: Diversity products of DUSTM obtained by genetic algorithm and exhaustive search.

N_t	L	ζ (genetic)	ζ(exhaustive)	cyclic
6	16	0.6602	0.6083	0.5066
	32	0.5678	0.5069	0.448
8	16	0.6601	0.6153	0.5623
	32	0.5827	0.5453	0.5221

children as follows:

Elite Children: Children in the current generation are selected for the next generation based on their fitness values. Since the selection rule here is probabilistic not deterministic, fitter solutions (measured by a fitness function) are typically more likely to be selected. Non determined rule helps to keep the diversity of the population large and also avoids convergence to a poor solution as well. *Crossover Children*: This type of children is created by combining pairs of parents in the current population. Generally, the crossover operation recombines selected solutions (parents) by swapping parts of them for producing divergent solutions to explore the search space. Many crossover techniques exist to produce a child of a pair parents. However, all of them are surprisingly simple to implement, involving random number generation and some partial string exchange. Scattered crossover is a technique that is usually used in crossover generation. This method first creates a random binary vector with the same size of parents. Then if the ith bit is 0, corresponding gene is selected from the first parent, otherwise it is selected from the second parent. Ultimately the all selected genes are combined to form the child.

Mutation Children: The algorithm generates mutation children by randomly changing the bits (genes) of individual parent in the current system. This process can be done by adding a random vector from a Gaussian distribution to the parent. The aim of mutation in genetic algorithm is to allow the algorithm to avoid local optima by preventing the population from becoming too similar to each other, thus slowing or even stopping evolution.

As a result, new mutated members along with new crossed over members and the rest of those selected from the previous population form the new generation. The genetic algorithm uses the following conditions to terminate:

- A solution is found that satisfies the criteria(Fitness limit).
- Allocated time is reached (Time limit).
- The specified number of generations is reached.
- There is no improvement in the objective function for a specific number of successive iterations.

Table 1 shows the parameter search results and their corresponding diversity product for signal constellation (14) for $L = 16$ and 32 and $N_t = 6$ and 8, obtained from genetic algorithm. For comparison, the diversity product of the obtained codes in [5] and [4], obtained from exhaustive integer search, are included in Table 1. Table 2 shows the parameter results and their corresponding chordal distance for cyclic design (12) for $L = 8, 16$ and 32 and $N_t = 4, 5$ and 6, obtained from genetic algorithm. For comparison, the chordal distances obtained from exhaustive integer search, are also included. Both tables show that by parameter relaxation and genetic search, almost all result are better than exhaustive search.

Table 2: Chordal distance of unitary precoders obtained by genetic algorithm and exhaustive search.

N_t	L	d_{chord}(genetic)	d_{chord}(exhaustive)
4	8	1.0606	1.000
	16	1.000	1.000
	32	0.8101	0.7947
5	8	1.1520	1.0105
	16	1.0708	1.0208
	32	1.0082	.9962
6	8	1.2237	1.0431
	16	1.1685	1.0459
	32	1.0939	1.0398

Remark: In some cases, the extracted parameters from exhaustive search seem to be the global optimum answer. However, there is no proof for this in the literature to our knowledge.

VI. Simulation Results and Comparisons

For comparison, we include performance results for DUSTM codes and precoders from genetic algorithms and exhaustive integer search.

Fig. 1 displays the SEP for the proposed constellations in [4] and [5] with integer parameters obtained from exhaustive search and with real parameters obtained from genetic search. In our simulations, we use the Rayleigh fading channel Jakes' model with a normalized fade rate of $f_d T_s = 2.5 \times 10^{-3}$, where f_d is the Doppler frequency and T_s is the symbol duration. The performance is for a MIMO system with $N_t = 6$ and $N_r = 1$ for $L = 16$ and 32. Fig. 1 clearly shows that codes extracted by genetic search outperform the previous results in the literature obtained by exhaustive search. The performance improvement is about 0.4 dB for cyclic group design and 0.6 dB for cyclic-rotated design, at 10^{-5} error rate.

Fig. 2 shows the symbol error rate for a MIMO system with $N_t = 6$ and $N_r = 1$, where a 6×2 unitary precoder is used to transmit the Alamouti's code over the channel. We used three and four bits of information for feedback, corresponding to $L = 8$ and 16 codebook size. This figure clearly shows that the precoders extracted by genetic search outperform the previous results in [8] obtained by exhaustive search. The performance improvement is about 0.5 dB at 10^{-5} error rate. The interesting point is that the performance of the 4 bit precoder with exhaustive search is almost the same as the performance of the 3 bit precoder with genetic search. Thus by using the genetic based precoder, we can save on the number of feedback bits.

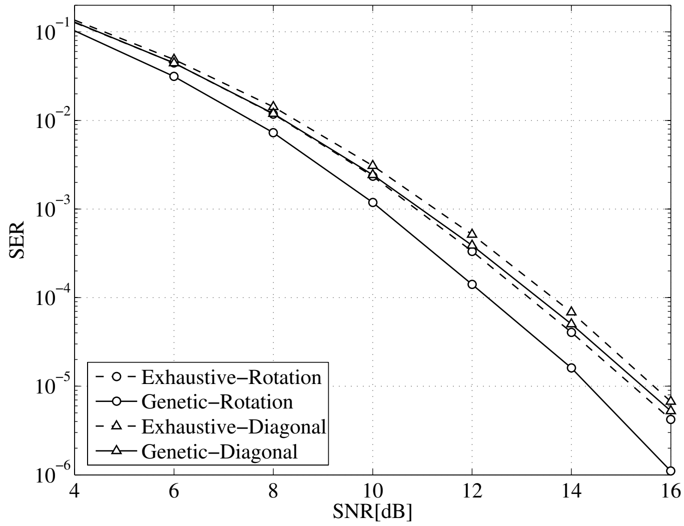

Figure 1: Symbol Error Rate performance of cyclic and cyclic rotated design when $L = 16$, $N_t = 6$ and $N_r = 1$. The dashed line curves are for exhaustive search and solid lines are for genetic search.

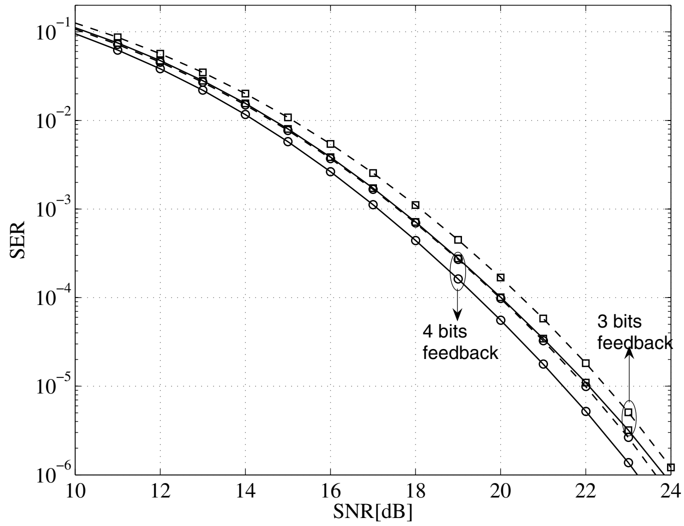

Figure 2: Symbol Error Rate of a precoded OSTBC with $N_t = 6$, $N_r = 1$. The dashed curves are for exhaustive search and the solid lines are for genetic search.

VII. CONCLUSION

In this paper, we improved the efficiency of the rotation and diagonal matrix designs for DUST modulation and unitary precoding by using a genetic algorithm. It allows us to relax the design parameters to be real numbers rather than integers. Consequently, the extracted DUSTM codes and precoders outperform the previous published codes in the literature where the design parameters are found by exhaustive search. Our simulation results confirmed this statement. Exhaustive search is almost impossible when the dimension of the MIMO system and/or codebook size is high, but genetic algorithm based search is easy and yields good codebooks.

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