A General Method to Decode ABBA Quasi-Orthogonal Space-Time Block Codes
Dũng Ngọc Đào, Student Member, IEEE, and Chintha Tellambura, Senior Member, IEEE

Abstract—ABBA codes, a class of quasi-orthogonal space-time block codes (STBC) proposed by Tirkkonen et al., allow low complexity pair-wise complex-symbol decoding. A refined version of ABBA codes with special signal mapping leads to pair-wise real-symbol decoding, i.e. with minimum decoding complexity (MDC) achievable by non-orthogonal STBC. Despite these advantages, a general simple closed-form method to decode ABBA and MDC-ABBA codes for an arbitrary number of transmit/receive antennas is not available. We thus derive an equivalent channel representation to address this issue. The maximum mutual information of ABBA/MDC-ABBA codes is shown to increase the rate of orthogonal space-time block codes (OSTBC) [2]. ABBA QSTBC also have low complexity pair-wise real-symbol decoding; they call such codes minimum decoding complexity (MDC) codes. Thus, not only is their code rate higher than that of OSTBC, but also their decoding complexity is not known in the most general case. Several decoders for ABBA codes have been proposed, but only for some well-known 8QAM constellations. We found that MDC-ABBA codes perform better than ABBA codes for two well-known 8QAM constellations.

Index Terms—Quasi-orthogonal space-time block codes.

I. INTRODUCTION

ABBA codes [1], a class of QSTBC, have been proposed to increase the rate of orthogonal space-time block codes (OSTBC) [2]. ABBA QSTBC also have low complexity pair-wise complex-symbol decoding and performs better than OSTBC [3]. ABBA codes have been widely studied for coherent and non-coherent transmissions, beamforming, and others. Recently, Yuen et al. (see [4] and references therein) have shown that the ABBA codes enable pair-wise real-symbol decoding; they call such codes minimum decoding complexity (PDWS) decoding; they call such codes minimum decoding complexity (MDC) codes. Thus, not only is their code rate higher than that of OSTBC, but also their decoding complexity is equal to that of OSTBC. In the following, we reserve the term ”ABBA” for the QSTBC proposed by Tirkkonen et al. [1] with pair-wise complex-symbol decoding and the term “MDC-ABBA” for the ABBA codes with PDWS decoding [4].

Despite extensive research on the ABBA QSTBC, a general decoding method for ABBA codes for arbitrary numbers of transmit (Tx) and receive (Rx) antennas is not available. One reason for this gap is that the equivalent channel for ABBA codes is not known in the most general case. Several decoders for ABBA codes have been proposed, but only for some specific cases, for example with 4 or 6 antennas in [5].

In this letter, we propose a general and closed-form method to decode ABBA/MDC-ABBA codes. We show how the ABBA space-time (ST) channel can be decoupled into parallel independent channels, each of which carries a pair of data symbols. The maximum mutual information (MMI) of QSTBC codes was calculated in [6] for 4 Tx antennas only. Thus, we derive a general MMI expression for ABBA/MDC-ABBA codes. Although the optimal coding gain of ABBA codes is higher than that of MDC-ABBA codes [4], we found that MDC-ABBA codes perform better than ABBA codes for two well-known 8QAM constellations.

II. DECODERS FOR ABBA AND MDC-ABBA CODES

We consider data transmission over a quasi-static Rayleigh flat fading channel. The transmitter and receiver are equipped with M Tx and N Rx antennas. The receiver, but not the transmitter, completely knows the channel gains.

From matrix representation theory, the mapping of a block of K data symbols \((s_1, s_2, \ldots, s_K)\) into a \(T \times M\) code matrix of an STBC can be represented in a general form [7] as follows:

\[
\mathbf{X}_M = \sum_{k=1}^{K} (s_k \mathbf{A}_k + s_k^* \mathbf{B}_k)
\]

where \(\mathbf{A}_k\) and \(\mathbf{B}_k\), \((k = 1, 2, \ldots, K)\) are \(T \times M\) constant basis matrices, superscript \(^*\) denotes conjugate \(^1\). The average energy of code matrices \(\mathbf{X} \in \mathbf{X}_M\) is constrained such that \(\mathbb{E}[\|\mathbf{X}\|^2] = T\). The code rate \(R_{\mathbf{X}_M}\) of a STBC \(\mathbf{X}_M\), in symbols per channel use (spcu), is defined by \(R_{\mathbf{X}_M} = M/K\).

We now review the main properties of OSTBC \(\mathcal{O}_M\) to be used later. The basis matrices of OSTBC satisfy [2]:

\[
\begin{align*}
\mathbf{A}_i^\dagger \mathbf{A}_i + \mathbf{B}_i^\dagger \mathbf{B}_i &= \mathbf{I}_M, & i = 1, 2, \ldots, K \\
\mathbf{A}_i^\dagger \mathbf{A}_j + \mathbf{B}_i^\dagger \mathbf{B}_j &= 0, & 1 \leq i < j \leq K \\
\mathbf{A}_i^\dagger \mathbf{B}_i + \mathbf{A}_j^\dagger \mathbf{B}_j &= 0, & i, j = 1, 2, \ldots, K.
\end{align*}
\]

Let the data symbols are drawn from a constellation with unit average power. To guarantee the average power constraint, the OSTBC matrices are multiplied by a constant \(\kappa = \frac{1}{\sqrt{M/T}}\).

A. General Decoder of ABBA Codes

Let \(\mathbf{A}_k\) and \(\mathbf{B}_k\) \((k = 1, 2, \ldots, K)\) be the \(t \times m\) basis matrices of an OSTBC \(\mathcal{O}_m\). Two blocks of data, each of \(K\) symbols, are mapped into two code matrices \(\mathbf{A}\) and \(\mathbf{B}\) of \(\mathcal{O}_m\) as \(\mathbf{A} = \sum_{k=1}^{K} (s_k \mathbf{A}_k + s_k^* \mathbf{B}_k)\), \(\mathbf{B} = \sum_{k=1}^{K} (s_k \mathbf{A}_k + s_k^* \mathbf{B}_k)\).

The ABBA code matrices for \(M = 2m\) Tx antennas are constructed from \(\mathcal{O}_m\) as \(\mathcal{Q}_M = \begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{B} & \mathbf{A} \end{bmatrix}\), or

\[
\mathcal{Q}_M = \sum_{k=1}^{K} (\mathbf{C}_k \otimes \mathbf{A}_k + \mathbf{C}_k^\dagger \otimes \mathbf{B}_k)
\]

\(^1\)From now on, superscripts \(^\top\) and \(^\dagger\) denote matrix transpose and transpose conjugate. The \(n \times n\) identity and all-zero matrices are denoted by \(\mathbf{I}_n\) and \(\mathbf{O}_n\), respectively. \(\|\mathbf{X}\|F\) denotes Frobenius norm of matrix \(\mathbf{X}\) and \(\otimes\) denotes Kronecker product. \(\mathbb{E}[\cdot]\) denotes average. A mean-\(m\) and variance-\(\sigma^2\) circularly complex Gaussian random variable is written by \(\mathcal{CN}(m, \sigma^2)\).

Manuscript received March 24, 2006. The associate editor coordinating the review of this letter and approving it for publication was Dr. Murat Uysal. This work is supported by The National Sciences and Engineering Research Council (NSERC) and Alberta Informatics Circle of Research Excellence (iCORE), Canada.

The authors are with the Dept. of Electrical and Computer Engineering, University of Alberta, Edmonton, Alberta T6G 2V4, Canada (email: [dndung, chintha]@ece.ualberta.ca).

Digital Object Identifier 10.1109/LCOMM.2006.060431.

1089-7798/06$20.00 © 2006 IEEE
where $C_k = \begin{bmatrix} s_k & s_{k+K} \\ s_{k+K} & s_k \end{bmatrix}$. Let $\Pi = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$, then

$$C_k = (s_k \Pi^0 + s_{k+K} \Pi). \quad (4)$$

For the sake of simplicity, we consider one Rx antenna in the following. Generalization for $N > 1$ is straightforward.

Let $h = [h_1 \ h_2 \ \cdots \ h_M]^T$ denote the channel vector with $h_i \sim CN(0,1)$. Let $Q \in \mathbb{Q}_M$ be a transmitted code matrix, the Rx signal vector is $y = \sqrt{\rho \kappa / 2} Q h + n$, where $n$ is noise vector with independently, identically distributed (i.i.d.) entries $\sim CN(0,1)$; $\rho$ is the average Rx signal-to-noise ratio (SNR).

From (3) and (4), we have

$$y = \sqrt{\rho \kappa / 2} \sum_{k=1}^{K} \left[ [(\Pi^{-i} \otimes A_k) h_s] s_{k+(i-1)K} + (\Pi^{-i} \otimes B_k) h_s^* s_{k+(i-1)K} \right] + n. \quad (5)$$

Let $e_{ki} = (\Pi^{-i} \otimes A_k) h, E_k = [e_{k1} \ e_{k2} \ \cdots \ E_k]$, and $s_k = [s_k \ s_{k+K}]^T$, (5) can be rewritten as

$$y = \sqrt{\rho \kappa / 2} \left[ E_1 \ F_1 \ E_2 \ F_2 \ \cdots \ E_K \ F_K \right] \times \left[ s_1^T \ s_1^T \ \cdots \ s_K^T \ s_K^T \right]^T + n. \quad (6)$$

Using a trick in [8], (6) is written equivalently as

$$\begin{bmatrix} y \\ y^* \end{bmatrix} = \sqrt{\rho \kappa / 2} \begin{bmatrix} E_1 \ F_1^* \\ E_2 \ F_2^* \\ \vdots \\ E_K \ F_K^* \end{bmatrix} \begin{bmatrix} s_1^T \\ s_1^T \\ \vdots \\ s_K^T \ s_K^T \end{bmatrix}^T + \begin{bmatrix} n \\ n^* \end{bmatrix}. \quad (7)$$

It can be shown that the columns of matrix $W$ are orthogonal.

Proof: We will show that the following equations hold:

$$\begin{bmatrix} E_k \\ F_k^* \end{bmatrix}^T \begin{bmatrix} E_l \\ F_l^* \end{bmatrix} = E_k^T E_l + F_k^T F_l^* = 0_2 \quad \text{for} \ k \neq l, \quad (8a)$$

$$\begin{bmatrix} E_k \\ F_k^* \end{bmatrix}^T \begin{bmatrix} E_l \\ F_l^* \end{bmatrix} = E_k^T F_l + F_k^T E_l^* = 0_2 \quad \forall k, l. \quad (8b)$$

We just provide the proof for (8a); (8b) can be shown similarly.

Let $Z_{kl} = (E_k^T E_l + F_k^T F_l^*)$, its element can be calculated as

$$[Z_{kl}]_{ij} = e_{ki} e_{lj} + f_{ki} f_{lj} = h_1^* [(\Pi^{-i} \otimes A_k) h] + h_1^* [(\Pi^{-j} \otimes B_k) h] = \begin{cases} 0, & k \neq l; \\ h_1^* [(\Pi^{-i} \otimes A_k) h], & k = l. \end{cases} \quad (9)$$

Thus, $Z_{kl} = 0_2$ if $k \neq l$.

Since for $k = l$, the matrices $Z_{kk} = Z \forall k$, where the entries of $Z$ are $z_{ij} = h_1^* [(\Pi^{-i} \otimes I_m) h]$. In particular, $z_{1,1} = z_{2,2} = \|h\|^2, z_{1,2} = z_{2,1} = \sum_{k=1}^{K} (h_k^* h_{k+1} + h_k^* h_{k+K})$. Therefore, $Z$ is a circulant real matrix and can be represented as

$$Z = \sum_{i=1}^{m} H_i^j H_i \quad (10)$$

where $H_i = \left[ \begin{array}{cc} h_i & h_{i+m} \\ h_{i+m} & h_i \end{array} \right]$. To separate the transmitted vector $s_k (k = 1, 2, \ldots, K)$ at the receiver, we can multiply the two sides of (7) with $E_k^T F_k^T$ to get

$$E_k^T y + F_k^T y^* = \sqrt{\rho \kappa / 2} Z s_k + (E_k^T n + F_k^T n^*) \frac{n_k}{\sqrt{\rho \kappa / 2}}. \quad (11)$$

The vector $E_k^T F_k^T$ plays the role of the spatial signature of data vector $s_k$. However, the noise $n_k$ is color with covariance matrix $V = \mathbb{E}[n_k n_k^*] = Z \neq I_M$. This color noise can be whitened by multiplying the two sides of (11) with a whitening matrix $Z^{-\frac{1}{2}}$. The received signal with whitened noise is

$$Z^{-\frac{1}{2}}(E_k^T y + F_k^T y^*) = \sqrt{\rho \kappa / 2} Z^{-\frac{1}{2}} s_k + \mathbb{E}[n_k n_k^*] \frac{n_k}{\sqrt{\rho \kappa / 2}} \quad (12)$$

Thus, (12) is the general and compact detection equation for ABBA codes. Since this decoder decouples the ABBA ST channels into $K$ parallel channels, one needs to apply $K$ times (12) to decode ABBA codes for any $M = 2m$ Tx antennas.

To achieve full diversity, $K$ data symbols $s_{k+K}$ ($k = 1, 2, \ldots, K$) must be rotated by an angle $\alpha [3]$. So the general detection equation of ABBA codes with complex symbol rotation is:

$$\tilde{y}_k = \sqrt{\rho \kappa / 2} \tilde{H} \text{diag}(1, e^{j\alpha}) s_k + \tilde{n}_k, \quad j^2 = -1. \quad (13)$$

B. General Decoder of MDC-ABBA Codes

Since $Z$ is a $2 \times 2$ normal circulant matrix, its two eigenvalues $\lambda_1$ and $\lambda_2$ are non-negative; $Z$ can be diagonalized by a $2 \times 2$ (real) Fourier transform matrix $F_2 = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$ as $Z = F_2^T \text{diag}(\lambda_1, \lambda_2) F_2$. If $\tilde{H}^2 = Z$, then $\tilde{H} = F_2^T \text{diag}(\sqrt{\lambda_1}, \sqrt{\lambda_2}) F_2$. Thus $\tilde{H}$ is real.

We can rewrite (12) by decoupling the real and imaginary parts of the two sides of (12) as

$$\begin{bmatrix} \Re(y_k) \\ \Im(y_k) \end{bmatrix} = \sqrt{\rho \kappa / 2} \begin{bmatrix} \tilde{H} & 0_2 \end{bmatrix} \begin{bmatrix} \Re(s_k) \\ \Im(s_k) \end{bmatrix} + \begin{bmatrix} \Re(n_k) \\ \Im(n_k) \end{bmatrix}. \quad (14)$$

The real and imaginary parts of the data vectors (of two complex symbols) can be separately detected as below:

$$\begin{bmatrix} \Re(y_k) \\ \Im(y_k) \end{bmatrix} = \sqrt{\rho \kappa / 2} \tilde{H} \Re(s_k) + \Im(n_k), \quad (15a)$$

$$\begin{bmatrix} \Re(y_k) \\ \Im(y_k) \end{bmatrix} = \sqrt{\rho \kappa / 2} \tilde{H} \Im(s_k) + \Re(n_k), \quad (15b)$$

There are only two real symbols in (15) to be jointly detected, thus (15a) and (15b) are the general detection equations for the MDC-ABBA codes.

To achieve full diversity, a real linear signal transformation $R$ is required for the real data vector $[\Re(s_k), \Im(s_k)]^T$. Moreover, $R$ must be designed so that the equivalent channel matrix (either in the form $\tilde{H} R$ or $R^T \tilde{H} R$) is still block diagonal. For example, transformation $R$ in [4] meets this requirement.
IV. Simulation Results

Performances of the ABBA code $Q_4$ [3] and the MDC-ABBA code $D_4$ (with signal rotation in [4]) using the proposed decoders are presented in Fig. 2 for a system with $M = 4, N = 1$. Performances of OSTBC $Q_4$ [2] and rate-one linear TAST code $C_4$ [9] are also presented for comparison.

With 4QAM and 16QAM (2 and 4 bits pcu, respectively), the performance of $D_4$ closely approaches that of $Q_4$. $D_4$ also performs much better than $C_4$ (with higher decoding complexity, joint decoding of 4 complex symbols).

Using an 8QAM-1 constellation $\{\pm 1, \pm j, \pm 1 \pm j\}$ (3 bits pcu), surprisingly, $D_4$ outperforms both $Q_4$ and $Q_4$ (with 16QAM), about 0.25 and 0.5 dB, respectively. With another 8QAM-2 $\{\pm 3, \pm j, \pm 1 \pm j\}$, $D_4$ also performs better than $Q_4$ but slightly worse than $Q_4$.

V. Conclusion

We have presented a general and simple method to decode ABBA and MDC-ABBA QSTBC by deriving the equivalent channel representation. It has also been used to derive maximum mutual information of these codes. Simulations show that MDC-ABBA codes closely approach the performance of ABBA code with 4- and 16QAM, and even perform better with two 8QAM constellations. Compared with OSTBC, the MDC-ABBA codes attain higher portion of channel capacity and perform better. Therefore, MDC-ABBA codes may be a better choice than OSTBC when there are more than 2 Tx antennas. The results of our paper can be developed in different aspects. For example, when a certain form of channel state information is available at the transmitter, the equivalent channel can be used to determine the optimal channel parameters to be fed back to improve the performance of QSTBC.

REFERENCES