BER Analysis of Arbitrary QAM for MRC Diversity With Imperfect Channel Estimation in Generalized Ricean Fading Channels

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Abstract—Imperfect channel estimation (ICE) can severely degrade the bit error rate (BER) of digital modulations with maximum ratio combining (MRC) diversity reception. The resulting performance analysis problem in its most general setting has not been addressed before. In this paper, the effect of ICE on the BER of an arbitrary square/rectangular Gray-coded quadratic amplitude modulation (QAM) in generalized Ricean fading channels when MRC reception is employed is analyzed. A general expression for the bit error probability of an arbitrary square/rectangular QAM scheme is first derived. This general formula requires a number of conditional probabilities, which is derived in closed form for independent and nonidentically distributed (i.n.d.) Rayleigh-fading channels with MRC and ICE. An efficient numerical method is also presented to compute the conditional probabilities for i.n.d. and correlated Ricean fading. In addition, extensive Monte Carlo simulations that agree excellently with the analytical results are presented.

Index Terms—Channel estimation, maximum ratio combining (MRC), quadratic amplitude modulation (QAM), Ricean fading.

I. INTRODUCTION

QUADRATURE amplitude modulation (QAM), a highly spectrally efficient modulation scheme, is widely used in wireless-communication systems [1]. The bit error rate (BER) performance of M-ary QAM has been investigated by several authors. The exact BER for 16-QAM and 64-QAM in an additive white Gaussian noise (AWGN) channel is derived in [2]. Approximate expressions for the BER of an arbitrary M-ary square QAM using signal space concepts and recursive algorithms are given in [3] and [4], respectively. Cho and Yoon [5] derive a general expression for the BER of an arbitrary square/rectangular QAM in AWGN. In all these papers, the analysis assumes a perfectly known channel at the receiver.

For imperfect channel estimation (ICE), less work is available in the literature. Tang *et al.* [6] derive an approximate expression for the BER of 16-QAM and 64-QAM with ICE over Rayleigh-fading channels. In [7], a probability density

function (pdf) is introduced to determine the BER of maximum ratio combining (MRC) QAM with channel estimation error. However, the closed-form integrals are obtained only for a 16-QAM constellation with one and two diversity branches for Rayleigh fading and for one branch for Ricean fading. Cao and Beaulieu [8], [9] provide exact BER expressions for a 16-QAM MRC receiver in Rayleigh and Ricean fading channels with any number of diversity branches. Their results are only valid for independent and identically distributed (i.i.d.) fading channels. Furthermore, the results in [6]–[9] are presented only for a specific square QAM constellation.

Previously, the performance of M-ary phase shift keying (M-PSK) with MRC and ICE in generalized Ricean fading channels [10] and with general selection combining (GSC) and equal gain combining (EGC) receivers with Gaussian weighting errors for several types of i.i.d. fading channels [11] was investigated. In [12], we analyzed the BER performance of square/rectangular QAM with MRC and ICE in Rayleigh-fading channels. The results in [12] are extended here for the computation of the BER of an arbitrary square/rectangular QAM in generalized Ricean fading channels.

In this paper, we present a general expression for the BER of an arbitrary square/rectangular QAM. First, by analyzing the BER performance of 16 QAM, we observe a regular pattern for the bit error probability of the kth in-phase bit. From this pattern, we develop a general expression for the BER performance of an arbitrary M-ary square QAM receiver. This general expression is then extended to an arbitrary $I \times J$-ary rectangular QAM.

The general expressions obtained in this paper require a number of conditional probabilities, which depend on the channel statistics and the decision variable. For MRC with ICE in i.i.d. fading channels, our several conditional probabilities reduce to the formula provided in [13, Appendix B]. A special case (16 QAM) is considered in [9]. For MRC with ICE in independent and nonidentically distributed (i.n.d.) Rayleigh-fading channels, no BER expression is available in literature. Therefore, we derive a closed-form expression for the BER of an arbitrary square/rectangular QAM constellation with MRC and ICE in i.n.d. Rayleigh-fading channels. We also present an efficient numerical technique for the computation of the BER of any square/rectangular QAM in i.n.d. and correlated Ricean fading channels. This technique is based on a classic result due to Imhof [14], which provides a numerical algorithm for the distribution of a quadratic form in normal variables.

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This paper is organized as follows. The system and QAM signaling models and assumptions are described in Section II. In Section III, we derive a regular pattern for the BER of an arbitrary square QAM in terms of conditional error probabilities, and, in Section IV, the pattern is extended to an arbitrary rectangular QAM. A closed-form error-rate expression for an arbitrary square/rectangular QAM in i.n.d. Rayleigh-fading channels as well as a numerical technique for computing its BER in i.n.d. and correlated Ricean fading channels, all cases with MRC and ICE, are presented in Section V. In Section VI, numerical examples are given, and the validity of our analytical results is assessed using the Monte Carlo simulations. The conclusion is given in Section VII.

II. SYSTEM AND SIGNAL MODELS

A. System Model

The system model used here is identical to that considered in [7]. MRC is used at the receiver to combat the effects of fading. The received signal sample at each receiver antenna is given by

$$y_m = h_m x + n_m, \qquad m = 1, \ldots, L \tag{1}$$

where $x = x_I + jx_Q$, $j = \sqrt{-1}$, $x_I \in \{\pm d, \pm 3d, \ldots, \pm(I - 1)d\}$, $x_Q \in \{\pm d, \pm 3d, \ldots, \pm(J - 1)d\}$, is the data symbol chosen from an $I \times J$ rectangular QAM constellation, L is the number of diversity branches, and h_m and n_m denote the channel attenuation and AWGN of the mth branch, respectively. The channel h_m can be estimated by transmitting pilot symbols [15], [16]. At the receiver, the channel estimator outputs

$$g_m = \alpha h_m + s_m, \qquad m = 1, \ldots, L \tag{2}$$

where α is a complex number representing the bias in the estimate, and s_m is a circularly complex Gaussian random variable (RV) with mean zero and variance σ_S^2 representing the random noise in the estimate; note that s_m and h_m are uncorrelated. We assume that all branch noise variances are equal, i.e., $E|n_m|^2 = \text{constant}$ for $m = 1, \ldots, L$.

The MRC decision variable at the receiver is given in [7]

$$z_d = \frac{\sum\limits_{m=1}^{L} y_m g_m^*}{\sum\limits_{m=1}^{L} |g_m|^2}. \tag{3}$$

The superscript * represents the conjugate operation. Since we only consider two-dimensional (2-D) modulation schemes where the in-phase (I) and quadrature (Q) streams can be demodulated separately, we need the real and imaginary components of z_d

$$z_{d_I} = \text{Re}(z_d) = \frac{\sum\limits_{m=1}^{L} (y_m g_m^* + y_m^* g_m)}{2 \sum\limits_{m=1}^{L} |g_m|^2} \tag{4a}$$

and

$$z_{d_Q} = \text{Im}(z_d) = \frac{\sum\limits_{m=1}^{L} (y_m g_m^* - y_m^* g_m)}{2j \sum\limits_{m=1}^{L} |g_m|^2}. \tag{4b}$$

Define $\bar{W} = (g_1, y_1, g_2, y_2, \ldots, g_L, y_L)^t$ and $\bar{\mu} = E[\bar{W}] = [\mu_{g_1}, \mu_{y_1}, \ldots, \mu_{g_L}, \mu_{y_L}]^t$, where t represents the transpose operation, and

$$\mu_{g_m} = \alpha E(h_m)$$
$$\mu_{y_m} = x E(h_m). \tag{5}$$

We find the covariance matrix of $\bar{W}$ as follows:

$$C_W = E\left[(\bar{W} - \bar{\mu})(\bar{W} - \bar{\mu})^{\dagger}\right]$$

$$= \begin{pmatrix}
\sigma_{g_1 g_1}^2 & \sigma_{g_1 y_1}^2 & \sigma_{g_1 g_2}^2 & \sigma_{g_1 y_2}^2 & \cdots & \sigma_{g_1 g_L}^2 & \sigma_{g_1 y_L}^2 \\
\sigma_{y_1 g_1}^2 & \sigma_{y_1 y_1}^2 & \sigma_{y_1 g_2}^2 & \sigma_{y_1 y_2}^2 & \cdots & \sigma_{y_1 g_L}^2 & \sigma_{y_1 y_L}^2 \\
\vdots & \vdots & \vdots & \vdots & & \vdots & \vdots \\
\sigma_{g_L g_1}^2 & \sigma_{g_L g_1}^2 & \sigma_{g_L g_2}^2 & \sigma_{g_L y_2}^2 & \cdots & \sigma_{g_L g_L}^2 & \sigma_{g_L y_L}^2 \\
\sigma_{y_L g_1}^2 & \sigma_{y_L y_1}^2 & \sigma_{y_L g_2}^2 & \sigma_{y_L y_2}^2 & \cdots & \sigma_{y_L g_L}^2 & \sigma_{y_L y_L}^2
\end{pmatrix} \tag{6}$$

where the superscript † represents the conjugate transpose operation. The quantities $\sigma_{g_m g_n}^2$, $\sigma_{y_m y_n}^2$, and $\sigma_{g_m y_n}^2$ are defined as

$$\sigma_{g_m g_n}^2 = E\left((g_m - \mu_{g_m})(g_n - \mu_{g_n})^*\right)$$
$$= \begin{cases} |\alpha|^2 \sigma_{h_m h_m}^2 + E\left(|s_m|^2\right) & (m = n) \\ |\alpha|^2 \sigma_{h_m h_n}^2 & (m \neq n) \end{cases} \tag{7a}$$

$$\sigma_{y_m y_n}^2 = E\left((y_m - \mu_{y_m})(y_n - \mu_{y_n})^*\right)$$
$$= \begin{cases} |x|^2 \sigma_{h_m h_m}^2 + E\left(|n_m|^2\right) & (m = n) \\ |x|^2 \sigma_{h_m h_n}^2 & (m \neq n) \end{cases} \tag{7b}$$

$$\sigma_{g_m y_n}^2 = E\left((g_m - \mu_{g_m})(y_n - \mu_{y_n})^*\right) = \alpha x^* \sigma_{h_m h_n}^2 \tag{7c}$$

where $\sigma_{h_m h_n}^2 = E(h_m h_n^*) - \mu_{h_m} \mu_{h_n}^*$. When the branches m and n are uncorrelated, $\sigma_{h_m h_n}^2 = 0$. Also, note that $\sigma_{g_m y_n} = \sigma_{y_n g_m}^*$.

B. QAM Signaling

We consider $I \times J$ rectangular QAM consisting of two independent one-dimensional pulse amplitude modulation (PAM) signals. In the special case when $I = J$, the constellation represents an M-ary square QAM, where $M = I \times I$. We also assume that all the symbols are equally likely to be transmitted.

In a typical QAM modulator, the data stream is divided into the I and Q bit streams, each encoded onto a separate axis using identical Gray coding mapping blocks [1]. The in-phase and quadrature components are thus selected independently over the set of $\{\pm d, \pm 3d, \ldots, \pm(I - 1)d\}$ and $\{\pm d, \pm 3d, \ldots, \pm(J - 1)d\}$, respectively. Note that $2d$ is the Euclidean distance between two adjacent signal points. Denoting E_b as the bit energy, d can be written in terms of E_b, I and J as [5]

$$d = \sqrt{\frac{3 E_b \log_2(I \times J)}{I^2 + J^2 - 2}}. \tag{8}$$

For the case of M-ary square QAM, (8) becomes

$$d = \sqrt{\frac{3E_b \log_2 M}{2(M-1)}}. \tag{9}$$

At the receiver, the complex symbols are divided into I and Q components, and each component is demodulated independently against their respective decision boundaries.

We next derive a general BER expression for M-ary square and $I \times J$ rectangular QAM in terms of conditional probabilities.

III. GENERAL BER EXPRESSION FOR M-ARY SQUARE QAM

A general expression for the BER of an arbitrary square/rectangular QAM for AWGN channels is derived in [5]. However, this expression depends on the complementary error function and is only valid for AWGN channels. Here, we derive a general expression for the BER of M-ary QAM, which is valid not only in AWGN but also in any fading channel with/without diversity and with perfect/ICE.

Due to symmetry, the average BER of an M-ary square QAM is equal to the BER of either the I or the Q components. We assume that in the QAM constellation, the most significant bit is the in-phase bit. Then, the BER of an M-ary square QAM is given in [5, eq. (16)]

$$P_b = \frac{1}{\log_2 \sqrt{M}} \sum_{n=0}^{\log_2 \sqrt{M}-1} P_I(\log_2 M - 2n) \tag{10}$$

where $P_I(\log_2 M - 2n)$ denotes the bit error probability of the $(\log_2 M - 2n)$th bit in the in-phase components. Therefore, in order to obtain the average BER of an M-ary square QAM, one needs to determine $P_I(\log_2 M - 2n)$. In this section, we first determine $P_I(\log_2 M - 2n)$ for the two in-phase bits of a 16-QAM constellation. Using this example, we then obtain a general expression for the bit error probability of the $(\log_2 M - 2n)$th in-phase bit of an arbitrary M-ary square QAM. Then, using (10), the BER of an M-ary square QAM is determined.

A. BER of 16 QAM

Fig. 1 shows Gray-coded bit mapping and the decision boundaries for square 16 QAM. The fourth[1] and the second bits are the in-phase bits while the third and the first bits are the quadrature bits. Due to symmetry, we only consider the bit error probability of the in-phase bits. Note that with ICE, the decision boundaries shown in Fig. 1 may no longer be optimal in the sense of minimizing the overall error rate. In [17] and [18], an optimal metric that takes ICE into account has been derived, the use of which is equivalent to modifying the decision boundaries

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[1]We refer to the first bit as the least significant bit and to the $(\log_2 M)$th bit as the most significant bit.
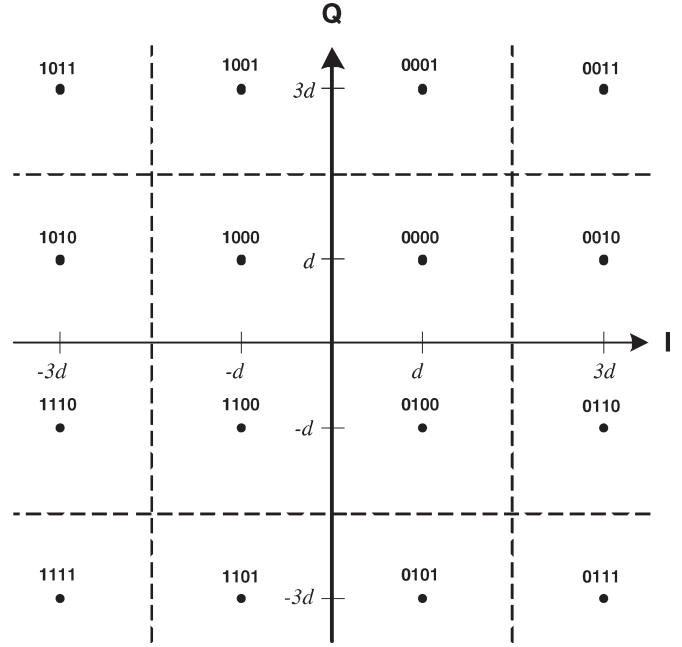

Fig. 1. Square 16-QAM constellation with gray encoding.

in Fig. 1. Our simulations reveal that the use of this metric has a negligible effect on the error performance of uncoded systems especially when the estimation noise variance is small compared with the channel noise variance. Consequently, we do not use the optimal metric derived in [17] and [18].

The Q-axis is the decision boundary for the fourth bit. Because of the symmetry between the right and left half sides of the plane, the probability that the fourth bit is in error is [2]

$$\begin{aligned} P_I(4) &= \frac{1}{2} \left[\Pr\left(z_{d_I} < 0|_{\mathrm{Re}(x)=d}\right) + \Pr\left(z_{d_I} < 0|_{\mathrm{Re}(x)=3d}\right) \right] \\ &= \frac{1}{4} \left[\Pr(z_{d_I} < 0|_{x=d+jd}) + \Pr(z_{d_I} < 0|_{x=d+j3d}) \right. \\ &\quad \left. + \Pr(z_{d_I} < 0|_{x=3d+jd}) + \Pr(z_{d_I} < 0|_{x=3d+j3d}) \right]. \end{aligned} \tag{11}$$

For the second bit, the decision boundaries are $2d$ and $-2d$. If the second bit is an "0," an error occurs when the real part of the decision variable z_d is out of the region $(-2d, 2d)$, and if the second bit is "1," an error occurs if the real part of z_d falls in $(-2d, 2d)$. Therefore, the error probability of the second bit is obtained as [2]

$$\begin{aligned} &P_I(2) \\ &= \frac{1}{2} \left[\Pr\left(|z_{d_I}| > 2d|_{\mathrm{Re}(x)=d}\right) + \Pr\left(|z_{d_I}| < 2d|_{\mathrm{Re}(x)=3d}\right) \right] \\ &= \frac{1}{4} [1 - \Pr(z_{d_I} < 2d|_{x=d+jd}) - \Pr(z_{d_I} < 2d|_{x=d+j3d}) \\ &\quad + 1 + \Pr(z_{d_I} < -2d|_{x=d+jd}) + \Pr(z_{d_I} < -2d|_{x=d+j3d}) \\ &\quad + \Pr(z_{d_I} < 2d|_{x=3d+jd}) + \Pr(z_{d_I} < 2d|_{x=3d+j3d}) \\ &\quad - \Pr(z_{d_I} < -2d|_{x=3d+jd}) - \Pr(z_{d_I} < -2d|_{x=3d+j3d})]. \end{aligned} \tag{12}$$

Substituting (11) and (12) into (10), the BER for 16 QAM is obtained.

B. General BER Expression for an M-ary Square QAM

From (11) and (12), a regular pattern can be found for the bit error probability of the $(\log_2 M - 2n, n \in \{0, 1, \ldots, \log_2 \sqrt{M} - 1\})$th bit as

$$P_I(\log_2 M - 2n)$$

$$= \begin{cases} \dfrac{4}{M} \left[\displaystyle\sum_{\substack{k=1 \\ k\,\mathrm{odd}}}^{\sqrt{M}-1} A_{k,0} \right] & (n = 0) \\[1em] \dfrac{4}{M} \left[\dfrac{M}{8} + \displaystyle\sum_{i=0}^{2^{n-1}} \sum_{\substack{k=F_1(i,n) \\ k\,\mathrm{odd}}}^{F_2(i,n)} (-1)^i A_{k,n} \right] & (n \neq 0) \end{cases} \quad (13\mathrm{a})$$

where $F_1(i,n) = \max[(\sqrt{M}/2^n)(2i-1)+1, 1]$, $F_2(i,n) = \min[(\sqrt{M}/2^n)(2i+1)-1, \sqrt{M}-1]$, and

$$A_{k,n}$$

$$= \begin{cases} \displaystyle\sum_{\substack{l=1 \\ l\,\mathrm{odd}}}^{\sqrt{M}-1} \left[\Pr(z_{dI} < 0|_{x=kd+jld}) \right] & (n=0) \\[1em] \displaystyle\sum_{\substack{l=1 \\ l\,\mathrm{odd}}}^{\sqrt{M}-1} \sum_{i=0}^{n-1} (-1)^i \big[\Pr(z_{dI} < -G_{i,n}|_{x=kd+jld}) \\ \qquad\qquad - \Pr(z_{dI} < G_{i,n}|_{x=kd+jld}) \big] & (n \neq 0) \end{cases} \quad (13\mathrm{b})$$

where $G_{i,n} = (\sqrt{M}(2i+1)/2^n)d$.

If the conditional probabilities are known, then the above expressions for $P_I(\log_2 M - 2n)$ can be used in (10) to obtain the BER of an M-ary square QAM.

To compare our result with the expression derived in [5], we consider an AWGN channel without fading, for which the decision variable z_d reduces to

$$z_d = x + n_0 \quad (14)$$

where n_0 is a zero-mean AWGN with a two-sided power spectrum density $N_0/2$. The bit error probability of the most significant bit ($n = 0$), for example, is obtained from (13) as

$$P_I(\log_2 M) = \frac{1}{\sqrt{M}} \left[\sum_{\substack{k=1 \\ k\,\mathrm{odd}}}^{\sqrt{M}-1} \mathrm{erfc}\left(\frac{kd}{\sqrt{N_0}} \right) \right] \quad (15)$$

which is equal to [5, eq. (15)].[2]

IV. GENERAL BER EXPRESSION FOR $I \times J$ RECTANGULAR QAM

The previous section derives a general expression for BER of M-ary square QAM. In this section, we extend this analysis to an arbitrary $I \times J$ rectangular QAM.

[2]Note that in [5] the most significant bit is referred to as first bit.

If we assume that the most significant bit is the in-phase bit, then the BER of an $I \times J$ rectangular QAM is given by [5, eq. (22)]

$$P_b = \frac{1}{\log_2(I \times J)} \times \left[\sum_{n=0}^{\log_2 I - 1} P_I\left(\log_2(I \times J) - 2n\right) \right.$$
$$\left. + \sum_{n=0}^{\log_2 J - 1} P_Q\left(\log_2(I \times J) - (2n+1)\right) \right] \quad (16)$$

where $P_I(\log_2(I \times J) - 2n)$ and $P_Q(\log_2(I \times J) - (2n+1))$ denote the bit error probabilities of the $(\log_2(I \times J) - 2n)$th in-phase bit and $(\log_2(I \times J) - (2n+1))$th quadrature bit, respectively. The general expression for the BER of the in-phase bits is given in Section III. Note that for the case of $I \times J$ rectangular QAM, M and $\sqrt{M}$ in (13) should be replaced by $I \times J$ and I, respectively.

For the quadrature bits, the imaginary part of the decision variable should be considered. The regular pattern that can be found for the bit error probability of the $(\log_2 I \times J - (2n+1))$th bit is

$$P_Q\left(\log_2(I \times J) - (2n+1)\right)$$

$$= \begin{cases} \dfrac{4}{I \times J} \left[\displaystyle\sum_{\substack{k=1 \\ k\,\mathrm{odd}}}^{J-1} B_{k,0} \right] & (n = 0) \\[1em] \dfrac{4}{I \times J} \left[\dfrac{I \times J}{8} + \displaystyle\sum_{i=0}^{2^{n-1}} \sum_{\substack{k=E_1(i,n) \\ k\,\mathrm{odd}}}^{E_2(i,n)} (-1)^i B_{k,n} \right] & (n \neq 0) \end{cases}$$

$$(17\mathrm{a})$$

where $E_1(i,n) = \max[(J/2^n)(2i-1)+1, 1]$, $E_2(i,n) = \min[(J/2^n)(2i+1)-1, J-1]$, and

$$B_{k,n}$$

$$= \begin{cases} \displaystyle\sum_{\substack{l=1 \\ l\,\mathrm{odd}}}^{J-1} \left[\Pr(z_{dQ} < 0|_{x=ld+jkd}) \right] & (n=0) \\[1em] \displaystyle\sum_{\substack{l=1 \\ l\,\mathrm{odd}}}^{J-1} \sum_{i=0}^{n-1} (-1)^i \big[\Pr(z_{dQ} < -V_{i,n}|_{x=ld+jkd}) \\ \qquad\qquad - \Pr(z_{dQ} < V_{i,n}|_{x=ld+jkd}) \big] & (n \neq 0) \end{cases}$$

$$(17\mathrm{b})$$

where $V_{i,l} = (J(2i+1)/2^l)d$.

Once $P_I(\log_2(I \times J) - 2n)$ and $P_Q(\log_2(I \times J) - (2n+1))$ are defined from (13) and (17), the BER of an $I \times J$ rectangular QAM follows from (16).

The above analysis shows that the BER of an arbitrary square/rectangular QAM consists of the summation of the probability terms in the form of $\Pr(z_{d_I} < T|_{x=ad+jbd})$ and $\Pr(z_{d_Q} < t|_{x=ad+jbd})$, where $a \in \{1, 3, \ldots, (I-1)\}$, $b \in \{1, 3, \ldots, (J-1)\}$, and T depends on d. We next determine the expression for these probabilities when MRC with ICE is used at the receiver.

V. EXACT BER EXPRESSION FOR AN ARBITRARY SQUARE/RECTANGULAR QAM IN GENERALIZED RICEAN FADING CHANNELS

The previous sections show that the BER of an arbitrary square/rectangular QAM can be written as a sum of probabilities in the form of $\Pr(z_{d_I} < T|_{x=ad+jbd})$ and $\Pr(z_{d_Q} < T|_{x=ad+jbd})$. When MRC with ICE is employed, the decision variable z_d is defined as (3).

Using (4a), the $\Pr(z_{d_I} < T|_{x=ad+jbd})$ can be rewritten as

$$
\begin{aligned}
&\Pr(z_{dI} < T|_{x=ad+jbd}) \\
&= \Pr\left(\frac{\sum_{m=1}^{L}(y_m g_m^* + y_m^* g_m)}{2\sum_{m=1}^{L}|g_m|^2} < T|_{x=ad+jbd}\right) \\
&= \Pr\left(\sum_{m=1}^{L}\left(-2T|g_m|^2 + y_m g_m^* + y_m^* g_m\right) < 0|_{x=ad+jbd}\right) \\
&= \Pr(D_I < 0|_{x=ad+jbd})
\end{aligned}
\tag{18}
$$

where

$$
\begin{aligned}
D_I &= \sum_{m=1}^{L}\left(-2T|g_m|^2 + y_m g_m^* + y_m^* g_m\right) \\
&= \bar{W}^\dagger H_I \bar{W}
\end{aligned}
\tag{19}
$$

$$
\bar{W} = [g_1, y_1, g_2, y_2, \ldots, g_L, y_L]^t
\tag{20}
$$

and

$$
H_I = \begin{pmatrix}
-2T & 1 & 0 & 0 & \ldots & 0 & 0 \\
1 & 0 & 0 & 0 & \ldots & 0 & 0 \\
0 & 0 & -2T & 1 & \ldots & 0 & 0 \\
0 & 0 & 1 & 0 & \ldots & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & & \vdots & \vdots \\
0 & 0 & 0 & 0 & \ldots & -2T & 1 \\
0 & 0 & 0 & 0 & \ldots & 1 & 0
\end{pmatrix}.
\tag{21}
$$

Similarly, using (4b), $\Pr(z_{d_Q} < T|_{x=ad+jbd})$ can be expressed as

$$
\begin{aligned}
&\Pr(z_{dQ} < T|_{x=ad+jbd}) \\
&= \Pr\left(\frac{\sum_{m=1}^{L}(y_m g_m^* - y_m^* g_m)}{2j\sum_{m=1}^{L}|g_m|^2} < T|_{x=ad+jbd}\right) \\
&= \Pr\left(\sum_{m=1}^{L}\left(-2T|g_m|^2 - jy_m g_m^* + jy_m^* g_m\right) < 0|_{x=ad+jbd}\right) \\
&= \Pr(D_Q < 0|_{x=ad+jbd})
\end{aligned}
\tag{22}
$$

where

$$
D_Q = \sum_{m=1}^{L}\left(-2T|g_m|^2 - jy_m g_m^* + jy_m^* g_m\right) = \bar{W}^\dagger H_Q \bar{W}
\tag{23}
$$

and

$$
H_Q = \begin{pmatrix}
-2T & \frac{1}{j} & 0 & 0 & \ldots & 0 & 0 \\
-\frac{1}{j} & 0 & 0 & 0 & \ldots & 0 & 0 \\
0 & 0 & -2T & \frac{1}{j} & \ldots & 0 & 0 \\
0 & 0 & -\frac{1}{j} & 0 & \ldots & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & & \vdots & \vdots \\
0 & 0 & 0 & 0 & \ldots & -2T & \frac{1}{j} \\
0 & 0 & 0 & 0 & \ldots & -\frac{1}{j} & 0
\end{pmatrix}.
\tag{24}
$$

Note that D_I and D_Q are special cases of the general quadratic form. Following [19], we can immediately write the moment generating functions (MGF) of D_I and D_Q as follows[3]:

$$
\varphi_{D_I}(s) = \frac{1}{|I - sC_W H_I|}\exp\left\{\bar{\mu}^\dagger\left(s^{-1}H_I^{-1} - C_W\right)^{-1}\bar{\mu}\right\}
\tag{25a}
$$

and

$$
\varphi_{D_Q}(s) = \frac{1}{|I - sC_W H_Q|}\exp\left\{\bar{\mu}^\dagger\left(s^{-1}H_Q^{-1} - C_W\right)^{-1}\bar{\mu}\right\}
\tag{25b}
$$

where C_W is the covariance matrix defined in (6).

Equations (25a) and (25b) can also be written in another useful form. Since C_W is the covariance matrix, we can obtain its square root matrix denoted by $C_W^{1/2}$ such that $C_W = C_W^{1/2}C_W^{1/2}$. Then, if we define $\bar{V} = C_W^{-1/2}\bar{W}$, where $C_W^{-1/2}$ is the inverse of $C_W^{1/2}$, (19) and (23) can be written as

$$
D_I = \bar{V}^\dagger\left(C_W^{1/2}H_I C_W^{1/2}\right)\bar{V}
\tag{26a}
$$

$$
D_Q = \bar{V}^\dagger\left(C_W^{1/2}H_Q C_W^{1/2}\right)\bar{V}.
\tag{26b}
$$

Using an eigenvalue decomposition, we rewrite (26a) and (26b) as

$$
D_I = \bar{V}^\dagger\left(U_I \Lambda_I U_I^\dagger\right)\bar{V}
\tag{27a}
$$

$$
D_Q = \bar{V}^\dagger\left(U_Q \Lambda_Q U_Q^\dagger\right)\bar{V}
\tag{27b}
$$

where $\Lambda_I = \text{diag}(\lambda_{a_1}, \lambda_{a_2}, \ldots, \lambda_{a_{2L}})$, $\Lambda_Q = \text{diag}(\lambda_{b_1}, \lambda_{b_2}, \ldots, \lambda_{b_{2L}})$, and λ_{a_i} and λ_{b_i} are the eigenvalues of $C_W^{1/2}H_I C_W^{1/2}$ and $C_W^{1/2}H_Q C_W^{1/2}$, respectively.

Let $\bar{X}_I = U_I^\dagger C_W^{-1/2}\bar{\mu}$ and $\bar{X}_Q = U_Q^\dagger C_W^{-1/2}\bar{\mu}$. D_I and D_Q in (27a) and (27b) can now be written as a weighted sum of

---

[3]Note that we have defined the covariance matrix of a vector $\bar{W}$ as $E(\bar{W}\bar{W}^\dagger)$ rather than $(1/2)E(\bar{W}\bar{W}^\dagger)$ as in [19].

noncentral chi-square RVs. Using [13], the MGFs of D_I and D_Q are expressed as

$$\varphi_{D_I}(s) = \prod_{r=1}^{2L} \left(\frac{1}{1 - s\lambda_{a_r}} \right) \exp \left(\frac{s|x_{I_r}|^2 \lambda_{a_r}}{1 - s\lambda_{a_r}} \right) \qquad (28a)$$

$$\varphi_{D_I}(s) = \prod_{r=1}^{2L} \left(\frac{1}{1 - s\lambda_{b_r}} \right) \exp \left(\frac{s|x_{Q_r}|^2 \lambda_{b_r}}{1 - s\lambda_{b_r}} \right). \qquad (28b)$$

When repeated eigenvalues exist, the MGF expressions (28a) and (28b) are modified as [20]

$$\varphi_{D_I}(s) = \prod_{r=1}^{N} \left(\frac{1}{1 - s\lambda_{a_r}} \right)^{v_{i_r}} \exp \left(\frac{y_{I_r}^2 \lambda_{a_r} s}{1 - s\lambda_{a_r}} \right) \qquad (29a)$$

$$\varphi_{D_Q}(s) = \prod_{r=1}^{N} \left(\frac{1}{1 - s\lambda_{b_r}} \right)^{v_{q_r}} \exp \left(\frac{y_{Q_r}^2 \lambda_{b_r} s}{1 - s\lambda_{b_r}} \right) \qquad (29b)$$

where N is the total number of distinct eigenvalues, v_{i_r} and v_{q_r} are the multiplicity of the eigenvalues λ_{a_r} and λ_{b_r}, respectively, and

$$y_{I_r}^2 = \sum_{k \in \kappa_r} |x_{I_r}|^2$$

$$y_{Q_r}^2 = \sum_{k \in \kappa_r} |x_{Q_r}|^2$$

and κ_r denotes the set of k indexes associated with the rth distinct eigenvalue.

We now derive the BER expression for an arbitrary square/rectangular QAM in the generalized fading channel.

A. i.i.d. Fading Channels

With the assumption of i.i.d. fading statistics across the diversity branches, h_m and g_m in (3) are i.i.d. Gaussian RVs. With MRC in i.i.d. Rayleigh or Ricean fading channels, the conditional probabilities in (13) and (17) are reduced to a closed-form expression given in [13, eq. (B-21)]. Our general expression then can be used for the derivation of the BER of any square/rectangular QAM scheme with MRC and ICE in i.i.d. Rayleigh/ Ricean fading channels. A special case of 16 QAM is considered in [9].

B. i.n.d. Fading Channels

The fading statistics are different across the diversity branches. This occurs when different signals may propagate via completely different paths before arriving at the receiver.

For the i.n.d. case, we derive a closed-form error-rate expression for Rayleigh fading and present a numerical technique for Ricean fading to compute the BER performance of an arbitrary square/rectangular QAM with MRC and ICE.

1) Rayleigh Fading: In a Rayleigh-fading channel, we have $\bar{\mu} = 0$, and, therefore, the MGFs defined in (25a) and (25b) are simplified to

$$\varphi_{D_I}(s) = \frac{1}{|I - sC_W H_I|} = \prod_{k=1}^{2L} \frac{1}{1 - s\lambda_{I_k}} \qquad (30a)$$

$$\varphi_{D_Q}(s) = \frac{1}{|I - sC_W H_I|} = \prod_{k=1}^{2L} \frac{1}{1 - s\lambda_{Q_k}} \qquad (30b)$$

where λ_{I_L} and λ_{Q_L} are the lth eigenvalue of matrices $C_W H_I$ and $C_W H_Q$, respectively.

We assume that RVs g_m and y_m are i.n.d. Therefore, the eigenvalues are distinct and the characteristic functions can be partial factored as

$$\varphi_{D_I}(s) = \sum_{k=1}^{2L} \frac{a_{I_k}}{1 - s\lambda_{I_k}}, \quad a_{I_k} = \prod_{\substack{r=1 \\ r \neq k}}^{2L} \left(\frac{\lambda_{I_k}}{\lambda_{I_r} - \lambda_{I_k}} \right) \qquad (31a)$$

$$\varphi_{D_Q}(s) = \sum_{k=1}^{2L} \frac{a_{Q_k}}{1 - s\lambda_{Q_k}}, \quad a_{Q_k} = \prod_{\substack{r=1 \\ r \neq k}}^{2L} \left(\frac{\lambda_{Q_k}}{\lambda_{Q_r} - \lambda_{Q_k}} \right). \qquad (31b)$$

Using (31a) and (31b), we find closed-form expressions for the conditional probabilities in (18) and (22) as follows:

$$\Pr(D_I < 0|_{x=ad+jbd}) = \frac{-1}{2\pi j} \int_{\varepsilon - j\infty}^{\varepsilon + j\infty} \frac{\varphi_{D_I}(s)}{s} ds \qquad (32)$$

and

$$\Pr(D_Q < 0|_{x=ad+jbd}) = \frac{-1}{2\pi j} \int_{\varepsilon - j\infty}^{\varepsilon + j\infty} \frac{\varphi_{D_Q}(s)}{s} ds \qquad (33)$$

where ε is a small negative number to avoid the pole at $s = 0$.

Without loss of generality, we assume that $(\lambda_{I_1}, \ldots, \lambda_{I_t})$ and $(\lambda_{Q_1}, \ldots, \lambda_{Q_t})$ are the negative eigenvalues, and $(\lambda_{I_{t+1}}, \ldots, \lambda_{I_{2L}})$ and $(\lambda_{Q_{t+1}}, \ldots, \lambda_{Q_{2L}})$ are the positive eigenvalues. We now illustrate a common procedure for evaluating (32) and (33). Using Cauchy's residue theorem, we can write

$$\frac{1}{2\pi j} \int_{C_R} \frac{\varphi(s)}{s} ds + \frac{1}{2\pi j} \int_{\varepsilon - j\infty}^{\varepsilon + j\infty} \frac{\varphi(s)}{s} ds$$

$$= \sum_{\text{Re}(s_i) < 0} \text{Residue} \left(\frac{\varphi(s)}{s}, s = s_i \right) \qquad (34)$$

where C_r is a large semicircular arc so that the resulting closed contour encloses all the singularities s_I of $\varphi(s)$ that are located in the left-half plane (i.e., singular values whose real parts are negative). Using $s = \text{Re}^{j\theta}$ and $ds = j\text{Re}^{j\theta} d\theta$, we have

$$\left| \frac{1}{2\pi j} \int_{C_R} \frac{\varphi(s)}{s} ds \right| \leq \left| \frac{1}{2\pi} \int_{C_R} \varphi(\text{Re}^{j\theta}) d\theta \right| \leq \lim_{R \to \infty} \frac{C}{R^{2L}}. \qquad (35)$$

The last term follows from the fact that $\varphi(s)$ is given by either (31a) or (31b). Equation (35) shows that as $R \to \infty$, the first integral in (34) along the semicircular arc vanishes to zero; thus, the second integral in (34) is equal to the sum of the residues. Therefore, using (31a) and (31b), and considering the negative eigenvalues only, (32) and (33) can be written as

$$\Pr(D_I < 0|_{x=ad+jbd})$$
$$= -\sum_{l=1}^{t} \frac{a_{I_l}}{\lambda_{I_l}} \left(\frac{1}{2\pi j} \int_{\varepsilon-j\infty}^{\varepsilon+j\infty} \frac{ds}{s\left(s - \frac{1}{\lambda_{I_l}}\right)} \right)$$
$$= -\sum_{l=1}^{t} a_{I_l} \qquad (36)$$

and

$$\Pr(D_Q < 0|_{x=ad+jbd})$$
$$= -\sum_{l=1}^{t} \frac{a_{Q_l}}{\lambda_{Q_l}} \left(\frac{1}{2\pi j} \int_{\varepsilon-j\infty}^{\varepsilon+j\infty} \frac{ds}{s\left(s - \frac{1}{\lambda_{Q_l}}\right)} \right)$$
$$= -\sum_{l=1}^{t} a_{Q_l} \qquad (37)$$

where a_{I_l} and a_{Q_l} are defined in (31a) and (31b).

Now that the closed-form expressions for the conditional probabilities in (13) and (17) are obtained, the BER of an arbitrary square/rectangular QAM with MRC and ICE in i.n.d. Rayleigh fading follows from (10) and (16).

2) Ricean Fading: In the case of Ricean fading in i.n.d. channels, $\bar{\mu} \neq 0$, and a closed-form expression cannot be found. Employing the result for computing the distribution of a quadratic form [14], we can compute $\Pr(D_I < 0|_{x=ad+jbd})$ and $\Pr(D_Q < 0|_{x=ad+jbd})$ numerically using the following equations:

$$\Pr(D_I < 0|_{x=ad+jbd}) = \frac{1}{2} - \frac{1}{\pi} \int_0^\infty \frac{\sin\theta_I(t)}{t\rho_I(t)} dt \quad (38)$$

$$\Pr(D_Q < 0|_{x=ad+jbd}) = \frac{1}{2} - \frac{1}{\pi} \int_0^\infty \frac{\sin\theta_Q(t)}{t\rho_Q(t)} dt \quad (39)$$

where

$$\theta_I(t) = \sum_{k=1}^{2L} \left(\tan^{-1}(\lambda_{a_k}t) + \frac{|x_{i_k}|^2 \lambda_{a_k} t}{1 + \lambda_{a_k}^2 t^2} \right) \quad (40a)$$

$$\rho_I(t) = \prod_{k=1}^{2L} \left(1 + \lambda_{a_k}^2 t^2\right)^{\frac{1}{2}} \exp\left(\frac{|x_{i_k}|^2 \lambda_{a_k}^2 t^2}{1 + \lambda_{a_k}^2 t^2} \right) \quad (40b)$$

$$\theta_Q(t) = \sum_{k=1}^{2L} \left(\tan^{-1}(\lambda_{b_k}t) + \frac{|x_{q_k}|^2 \lambda_{a_k} t}{1 + \lambda_{b_k}^2 t^2} \right) \quad (41a)$$

$$\rho_Q(t) = \prod_{k=1}^{2L} \left(1 + \lambda_{b_k}^2 t^2\right)^{\frac{1}{2}} \exp\left(\frac{|x_{q_k}|^2 \lambda_{b_k}^2 t^2}{1 + \lambda_{b_k}^2 t^2} \right) \quad (41b)$$

where λ_{a_r} and λ_{b_r} are the eigenvalues of $C_W^{1/2} H_I C_W^{1/2}$ and $C_W^{1/2} H_Q C_W^{1/2}$, respectively. As mentioned in [14], the functions
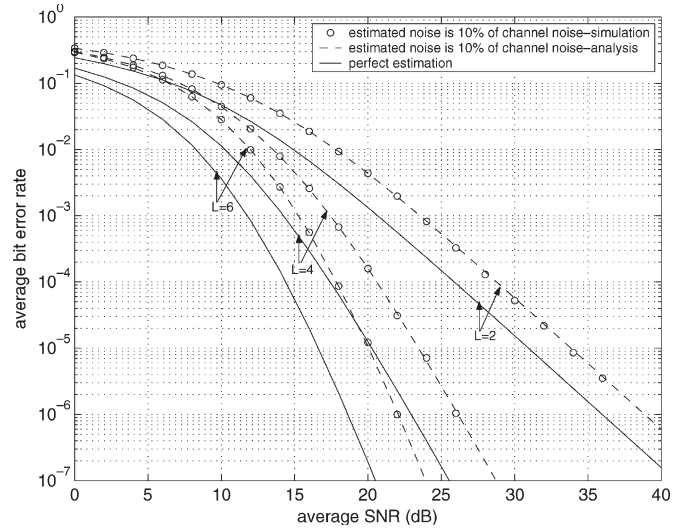

Fig. 2. Plot of BER of a 16 QAM with ICE Case 1 and $\beta = 0.5$ for three different diversity branches.

$t\rho_I(t)$ and $t\rho_Q(t)$ in (38) and (39) increase monotonically toward ∞. Therefore, the integrals in (38) and (39) can be truncated over a finite range, with negligible error. After the computation of the conditional probabilities in (13) and (17), the BER of an arbitrary square/rectangular QAM with MRC and ICE in i.n.d. Ricean fading follows from (10) and (16).

C. Correlated Fading

When the diversity branches are correlated, the analysis can proceed in a similar manner to that of i.n.d Ricean fading. Note that for the correlated branches m and n, the covariance $\sigma_{g_m y_n}$ in the covariance matrix (6) is not zero. Using the covariance and correlation matrices, the conditional probabilities in (13) and (17) can be evaluated using (38) and (39).

VI. SIMULATIONS

All the simulations in this section are implemented in Matlab. Two ICE cases are considered as in [7]: Case 1, $\alpha = 1$ and $\sigma_S^2 = 0.1\sigma_n^2$, which allows for the estimation noise to be 10% of the channel noise; Case 2, in addition to the estimation noise, an irreducible error in the estimation of the channel exists, i.e., $\sigma_S^2 = 0.001\sigma_h^2 + 0.1\sigma_n^2$.

For i.n.d. fading channels, we consider an exponential multipath intensity profile (MIP) with the decay parameter β, i.e., $\gamma_l = \gamma_1 \exp\{-\beta(l-1)\}$, $L = 1, 2, \ldots, L$.

For analytical results, we used (31) and (32) in (13) and (17) and then in (10) and (16) to obtain the BER performance of MRC QAM with ICE. In Fig. 2, the BER performance of 16 QAM with MRC for Case 1 and for three different diversity branches is presented. The MIP decay parameter β is set to 0.5. For both perfect and imperfect estimations, the BER performance improves when the number of diversity branches increases. Also, for Case 1, ICE does not change the slope of the BER curves. Interestingly, as the diversity order increases, the SNR difference between perfect channel estimation and ICE increases. For example, at a BER of 10^{-4}, the SNR differences between perfect and imperfect estimations for
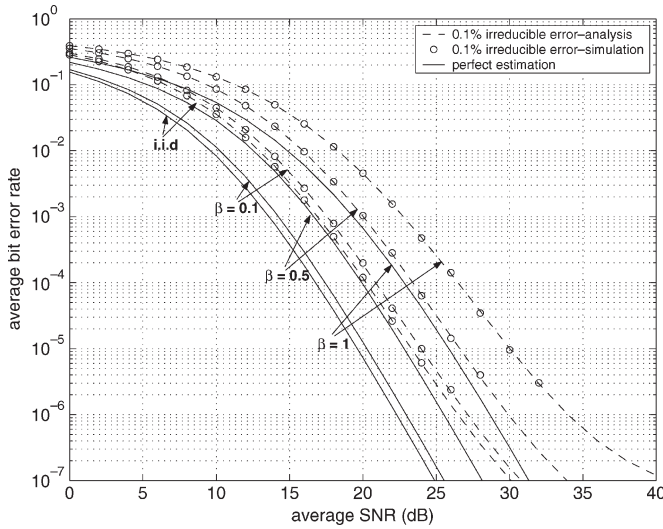
Fig. 3.   Plot of BER of an 16 QAM with four diversity branches and ICE Case 2 for four different channel MIPs.
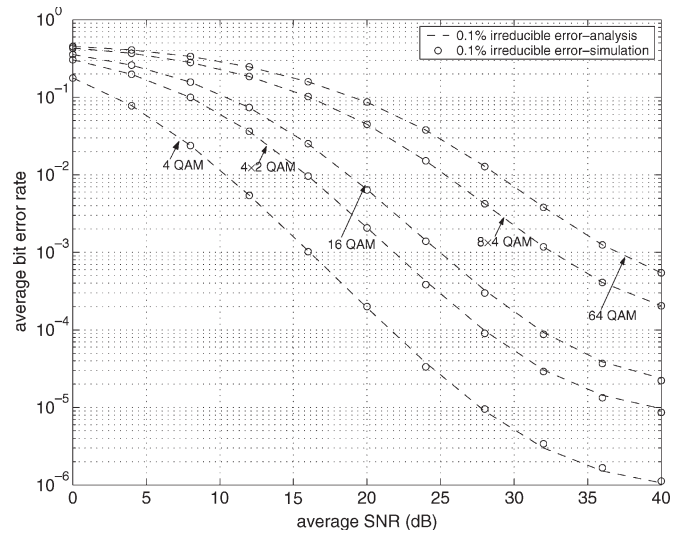

Fig. 5.   Plot of BER of several QAM constellations with two diversity branches and ICE Case 2 and $\beta = 0.1$.
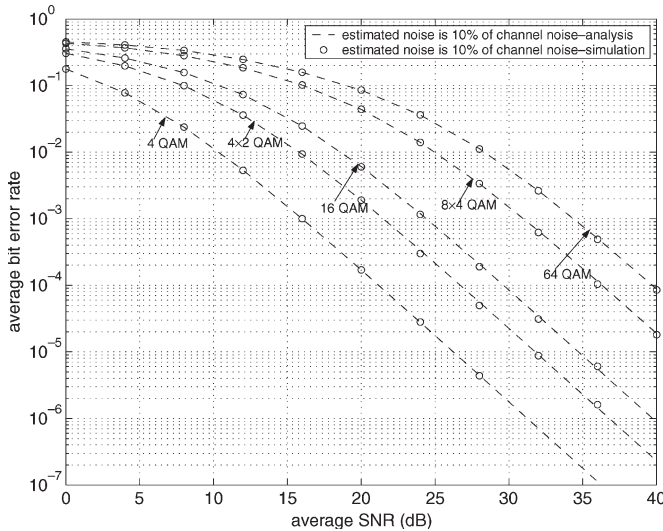

Fig. 4.   Plot of BER of several QAM constellations with two diversity branches and ICE Case 1 and $\beta = 0.1$.
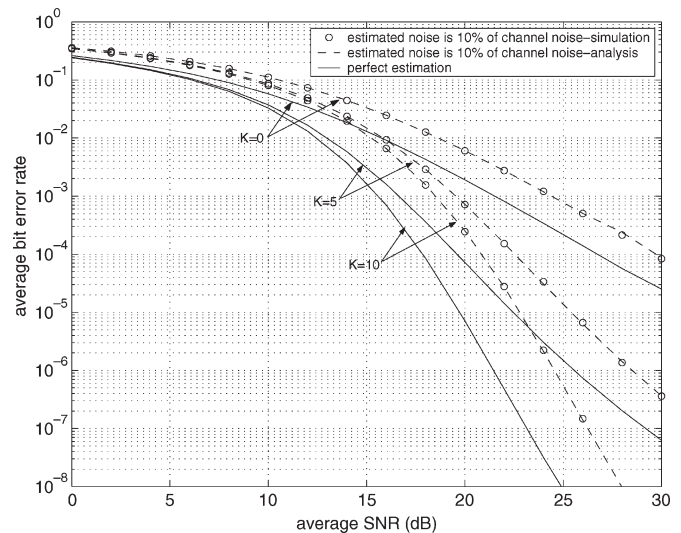

Fig. 6.   Plot of BER of a 16 QAM with two diversity branches, ICE Case 1, and $\beta = 0.5$ for three different Ricean factors.

$L = 2, \ 4,$ and 6 are 2.89, 3.19, and 3.46 dB, respectively. This shows that as the number of diversity branches increases, the ICE degradation increases rapidly.

Fig. 3 shows the BER of four branch reception of 16 QAM for four different channel MIPs for Case 2. The BER with perfect estimation for each β is also presented for comparison. The i.i.d. channel case has the best BER performance, and it degrades as the decay exponent β increases. For Case 2, an error floor occurs at high values of SNR.

We use (13) and (17) to plot the BER performances of several square and rectangular QAM constellations. We assume a two-branch MRC receiver, and $\beta = 0.1$. The analytical results along with Monte Carlo simulations are shown in Figs. 4 and 5. For comparison between the two estimation models, we use Case 1 and Case 2 in Figs. 4 and 5, respectively, for ICE. These two figures show that to transmit an extra bit per dimension, the SNR must be increased by 3–6 dB. Also, for Case 1 (Fig. 4), both analytical and simulation results show a constant decrease

while in Fig. 5 (Case 2), an error floor occurs at high values of SNR.

Fig. 6 shows the average BER of 16 QAM with two diversity branches and ICE Case 1 for different Ricean factors. Analytical bit error probability is calculated using (38) and (39) numerically. The finite range used in calculating (38) and (39) numerically needs to be increased as SNR increases. Note that the SNR difference between perfect channel estimation and ICE remains constant and does not change with the increase of the Ricean factor. As expected, the system performs better as the Ricean factor increases.

For correlated channels, we assume that the Ricean factor parameter K is identical for all branches. For simplicity, we also consider equally correlated channels with the fading correlation factor ρ. Fig. 7 shows the BER of a two correlated branches 16 QAM with ICE Case 1 for five different correlation factors. For simulations, we have used the technique presented in [21], and (38) and (39) have been used to obtain the average BER
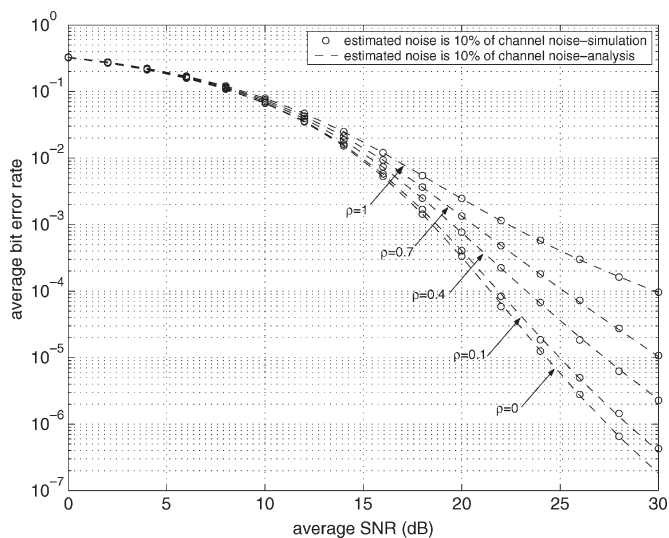
Fig. 7. Plot of BER of a 16-QAM constellations with two correlated diversity branches, ICE Case 1, and $K = 5$ for five different correlation factors.
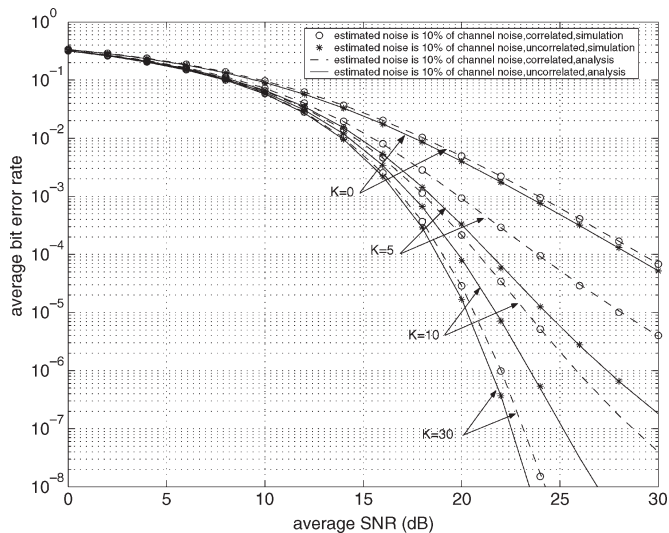

Fig. 8. Plot of BER of a 16-QAM constellations with two correlated/uncorrelated diversity branches and ICE Case 1 for four different Ricean factors.

analytically. We observe that as ρ increases, the performance of the system degrades. This is more pronounced at higher SNRs.

Fig. 8 plots the BER curves for both correlated ($\rho = 0.5$) and uncorrelated ($\rho = 0$) two diversity branches of 16 QAM with ICE Case 1, for Ricean factors $k = 0$, 5, 10, and 30. Note that as SNR increases, the SNR difference between correlated and uncorrelated cases increases as well. Also, for K > 0 and a specific average BER, the SNR difference between the correlated and uncorrelated cases decreases as K increases.

In Figs. 2–8, both simulation and analytical results are presented, and they are in excellent agreement.

VII. CONCLUSION

Channel estimation is an essential element of coherent digital demodulation, and ICE can severely degrade the BER of digital modulations with MRC diversity reception. The most general

case involves nonidentical and possibly correlated fading, arbitrary QAM, and Ricean fading, which includes Rayleigh fading as a special case. This problem has not been comprehensively addressed before. In this paper, we analyzed the effect of ICE on the BER of an arbitrary square/rectangular Gray-coded QAM in generalized Ricean fading channels, when MRC reception is employed. We derived a general expression for the bit error probability of an arbitrary square/rectangular QAM scheme. We also derived the necessary conditional probabilities in closed form for independent and i.n.d. Rayleigh-fading channels, with MRC and ICE. An efficient numerical method has also been developed to compute the conditional probabilities for i.n.d. and correlated Ricean fading. In addition, we presented extensive Monte Carlo simulations that agreed excellently with our analytical results.

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