Capacity-Approaching Semi-Orthogonal Space-Time Block Codes

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***Abstract*— A new class of full-diversity space-time block codes (STBC) called semi-orthogonal algebraic space-time block codes (SAST codes) with rate one symbol per channel use is proposed. The SAST codeword matrix has a generalized Alamouti code structure where the transmitted symbols are replaced by circulant matrices. Properties of rate-one linear threaded algebraic space-time (LTAST) codes are exploited to construct SAST codes with maximal coding gain. SAST codes attain nearly 100% of the Shannon capacity of open-loop multiple-input single-output (MISO) channels. Our theoretical analysis and simulation results show that SAST codes gain several dB over LTAST codes and also outperform other STBC.**

I. Introduction

Space-time block codes (STBC) have been studied extensively recently. One of the most well-known STBC is the Alamouti code, first designed for two transmit (tx) antennas [1] and later generalized as orthogonal STBC (OSTBC) [2]. Orthogonal designs enable minimal complexity maximum likelihood (ML) detection since the detection of symbols is decoupled. However, orthogonality results in low code rate [3]. A code rate of one symbol per channel use (pcu) is available for $m = 2$ Tx antennas only, and the code rate approaches 1/2 for a large number of Tx antennas [3]. To improve the code rate, quasi-orthogonal STBC (QSTBC) are proposed (see [4] and references therein). They achieve full diversity by signal constellation rotations, but pairs of symbols must be jointly detected. However, QSTBC also have low code rates because they are constructed from OSTBC.

On the other hand, full-diversity diagonal space-time (DST) codes are designed differently [5]–[7]. In this design, orthogonality is not considered and rate-one codes can be constructed for any number of Tx antennas. Optimal DST codes yield better coding gain compared to OSTBC for $m > 2$. Moreover, higher rate codes, called threaded algebraic space-time (TAST) codes (up to full-rate) can be derived from DST codes, for example, in [8]. However, DST and TAST codes exhibit high peak-to-average-power ratio (PAPR) and high complexity ML detection because all the data symbols of the transmitted codeword must be jointly detected. To reduce PAPR, linear TAST (LTAST) codes are proposed [9]. The rate-one LTAST codes have a circulant structure [10] and have the same PAPR with the input symbols. However, LTAST codes incur the same high complexity ML detection as TAST codes.

In this paper, we present a new class of full-diversity and rate-one STBC, namely semi-orthogonal algebraic space-time (SAST) codes. SAST codes reduce ML detection complexity significantly compared to DST or rate-one LTAST codes because the left-half columns of the codeword matrices are orthogonal to the right-half columns. This structure can be viewed as a generalization of the Alamouti code, in which each data symbol is replaced by a circulant matrix. To achieve full diversity, the input symbols are rotated. Optimal rotations for different constellations are found to be the ones designed for rate-one LTAST codes [9] based on algebraic number theory.

Table I compares existing space-time code designs (OSTBC, QSTBC and rate-one TAST/LTAST codes (or DST codes)) and proposed SAST codes for (m, n) MIMO systems where m, n are the number of Tx and receive (Rx) antennas. Compared parameters are diversity gain (G_d), coding gain (G_c), code rate (R, in symbol pcu), number of symbol to be jointly ML detected and degree of orthogonality (DO). The DO is defined as the minimum number of columns of the codeword matrix that a column is orthogonal with. SAST codes have higher coding gains compared to other codes that have the same or smaller code-rate. Simulation results corroborate the theoretical analysis.

TABLE I
Comparisons of several STBC.

Code	m	G_d	G_c	R	ML decoding	DO
OSTBC	4	$4n$	$\frac{1}{3}\,d^2_{\min}$	0.75	1 symbol	3
QSTBC	4	$4n$	$\frac{1}{4}\,d^2_{\min}$	1	2 symbols	2
LTAST	4	$4n$	$\frac{1}{4}\,d^2_{\min}$	1	4 symbols	0
SAST	4	$4n$	$\frac{1}{2}\,d^2_{\min}$	1	2 symbols	2
OSTBC	8	$8n$	$\frac{1}{5}d^2_{\min}$	0.625	1 symbol	7
QSTBC	8	$8n$	$\frac{1}{6}d^2_{\min}$	0.75	2 symbols	6
LTAST	8	$8n$	$\frac{1}{8}d^2_{\min}$	1	8 symbols	0
SAST	8	$8n$	$\frac{1}{4}d^2_{\min}$	1	4 symbols	4

II. Preliminaries

A. Notation

Superscripts ^{T}, * and † denote matrix transpose, conjugate and transpose conjugate operations. $E[\cdot]$ denotes statistical mean. A circularly complex Gaussian random variable with mean m and variance σ^2 is denoted by $\mathcal{CN}(m, \sigma^2)$. A signal constellation $\mathcal{S}$ is a finite set of possibly complex numbers. The minimum Euclidean distance of $\mathcal{S}$ is $d_{\min} = \min\{|s -$

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$\hat{s}|\forall s \neq \hat{s}; s, \hat{s} \in \mathcal{S}\}$. An element of $\mathcal{S}$ is called a signal or a symbol, and all the elements are equally likely to be transmitted. The order or size of $\mathcal{S}$ is the number of elements of $\mathcal{S}$. The average energy of a constellation is normalized such that $E[|s|^2] = 1$.

B. System Model

We consider data transmission over a quasi-static Rayleigh flat fading channel. The transmitter and receiver are equipped with m Tx and n Rx antennas. The channel gain $h_{ik}(i = 1, 2, ..., m; k = 1, 2, ..., n)$ between Tx-Rx antenna pair (i, k) is assumed $\mathcal{CN}(0, 1)$. We assume no spatial correlation at either Tx or Rx array, and the receiver, but not the transmitter, completely knows the channel gains.

The space-time encoder parses data symbols into space-time (ST) codewords $C = [c_{li}]$ of size $t \times m$ where c_{li} is the symbol transmitted from antenna i at time l $(1 \leq l \leq t)$. The average energy of a codeword is constrained such that $\sum_{i=1}^{m} \sum_{l=1}^{t} E[|c_{li}|^2] = t$.

The received signals y_{lk} of the kth antenna at time l can be arranged in a matrix Y of size $t \times n$. Thus, one can represent the Tx-Rx signal relation compactly as

$$Y = \sqrt{\rho} CH + Z \tag{1}$$

where $H = [h_{ik}]$, $Z = [zik]$ of size $t \times n$, z_{ik} are independently, identically distributed (i.i.d.) $\mathcal{CN}(0, 1)$. The Tx power is scaled by ρ so that the average signal-to-noise ratio (SNR) at each Rx antenna is ρ, independent of the number of Tx antennas.

The upper-bound of pair-wire error probability (PEP) derived by Tarokh $et\,al.$ [11] is as follows:

$$P(C \to \hat{C}) \leq \left(\prod_{i=1}^{\Gamma} \lambda_i \right)^{-n} \left(\frac{\rho}{4} \right)^{-\Gamma n} \tag{2}$$

where C and $\hat{C}$ are the transmitted and erroneous codewords, Γ is the minimum rank of a matrix Δ_C $(\Delta_C = C - \hat{C})$ for all $C \neq \hat{C}$, and $\lambda_1, \lambda_2, , ..., \lambda_\Gamma$ are non-zero eigenvalues of a product matrix $P_C = \Delta_C^\dagger \Delta_C$. The diversity gain or diversity order G_d and coding gain G_c of ST codes are defined as $G_d = \Gamma n$ and $G_c = \left(\prod_{i=1}^{\Gamma} \lambda_i \right)^{1/\Gamma}$, respectively. Since the $\operatorname{rank} \Delta_C = \operatorname{rank} P_C$, if Δ_C is of full rank m for all pairs of distinct codewords, the maximum achievable diversity order $d = mn$ is obtained and the coding gain is $G_c = \left[\det(\Delta_C^\dagger \Delta_C) \right]^{1/m}$.

For example, the coding gain of OSTBC [2] is given by

$$G_c^O = \frac{1}{mR_m} d_{\min}^2 \tag{3}$$

where R_m is the code-rate of OSTBC designed for m Tx antennas [4][1]. The maximal rate of OSTBC with the number of Tx antennas $m = 2a$ or $m = 2a - 1$ is given by $R = \frac{a+1}{2a}$ [3].

[1] In [4], the authors define a parameter ζ, namely diversity product. The coding gain can be calculated as $G_c = 4\zeta^2$.

The optimal coding gain of QSTBC with even m Tx antennas can be derived as [4]

$$G_c^Q = \frac{1}{mR_{m/2}} d_{\min}^2. \tag{4}$$

C. Circulant Matrices

The circulant matrix [10] structure is proposed for rate-one LTAST codes in [9]. However, some interesting properties of circulant matrix are not considered. Therefore, we first review several necessary properties of a circulant matrix that will be directly applied to SAST codes. A matrix $\mathcal{C} = [c_{ik}]$ is called a circulant matrix of order m

$$\mathcal{C} = \begin{bmatrix} x_1 & x_2 & \dots & x_m \\ x_m & x_1 & \dots & x_{m-1} \\ \vdots & \vdots & & \vdots \\ x_2 & x_3 & \dots & x_1 \end{bmatrix}. \tag{5}$$

Therefore, $c_{ik} = x_{(k-i+1) \bmod m}$. The basic properties of circulant matrices are given below.

- P1 $\mathcal{C}$ is a circulant if and only if $\mathcal{C}^\dagger$ is a circulant.
- P2 if $\mathcal{A}$ and $\mathcal{B}$ are ciculants of order m, and α_1 and α_2 are two scalars, then the matrices $\mathcal{A}^T$, $\alpha_1\mathcal{A} + \alpha_2\mathcal{B}$, and $\mathcal{AB}$ are circulants.
- P3 All circulants of the same order commute, i.e. $\mathcal{AB} = \mathcal{BA}$.
- P4 Let F be Fourier transform matrix with $f_{i,k} = \frac{1}{\sqrt{m}} e^{-j2\pi(i-1)(k-1)}$, where $j^2 = -1$. If C is a circulant, it is diagonalized by F: $\mathcal{C} = F^\dagger \Lambda F$, where $\Lambda = \operatorname{diag}[\lambda_1, \lambda_2, \dots, \lambda_m]$, and λ_i are the eigenvalues of $\mathcal{C}$.
  Thus, any order-m circulant always has m distinct eigenvectors, which are the column of Fourier transform matrix. However, the number of eigenvalues may be less than m.

Since there are only m independent entries in a circulant, which are the elements of vector $\boldsymbol{x} = [x_1, x_2, ..., x_m]$, we can write $\mathcal{C}(\boldsymbol{x})$ to emphasize that the first row of the circulant $\mathcal{C}$ is exactly the row vector $\boldsymbol{x}$. The ith row $(i = 2, 3, ..., m)$ is obtained by circular shifts to the right $(i-1)$ times vector $\boldsymbol{x}$.

Another type of circulant matrix is left circulant. We can denote these by $\mathcal{C}^L(\boldsymbol{x})$ where the ith row is obtained by circular shifts $(i-1)$ times to the left vector $\boldsymbol{x}$.

$$\mathcal{C}^L(\boldsymbol{x}) = \begin{bmatrix} x_1 & x_2 & \dots & x_m \\ x_2 & x_3 & \dots & x_1 \\ \vdots & \vdots & & \vdots \\ x_m & x_1 & \dots & x_{m-1} \end{bmatrix}. \tag{6}$$

Let a permutation Π on an arbitrary matrix X be such that the $(m - i + 2)$th row is permuted with the ith row for $i = 2, 3, ..., \lceil \frac{m}{2} \rceil$, where $\lceil (\cdot) \rceil$ is the ceiling function. One can verify that

$$\Pi(\mathcal{C}^L(\boldsymbol{x})) = \mathcal{C}(\boldsymbol{x}). \tag{7}$$

This useful operator will be used for our derivations.

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D. Linear Threaded Algebraic Space-Time Constellations

We briefly review the idea, code construction and properties of rate-one LTAST codes. For a full treatment of TAST and LTAST codes, the reader is referred to [8], [9] and references therein. For brevity, we use the term LTAST codes to denote the rate-one LTAST codes when there is no ambiguity.

Modulation symbols are drawn from a constellation $\mathcal{S}$ with the minimum Euclidean distance $d_{\min}$ and arranged in a vector $\boldsymbol{s} = [s_1, s_2, \ldots, s_m]^T$. The transmitted vector $\boldsymbol{u}$ is given by

$$\boldsymbol{u} = R\boldsymbol{s} \tag{8}$$

where $R = \mathrm{diag}[\,1,\ \phi^{1/m},\ \cdots,\ \phi^{(m-1)/m}\,]$ and ϕ is called a Diophantine number [9]. LTAST codewords are circulants given by

$$D = \mathcal{C}(\boldsymbol{u}^T)\,. \tag{9}$$

The upper bound of coding gain is as follows.

Proposition 1 [9, eq. (7)]: *The coding gain of the rate-one LTAST codes is upper-bounded by* $G_c^L \le \frac{1}{m} d_{\min}^2$.

To ensure the codes achieve full diversity, the Diophantine number is chosen as $\phi = e^{j\alpha}$. Thus the ith symbol s_i is rotated by an angle $\frac{i-1}{m}\alpha$. The optimal values of ϕ that maximize the coding gain are specified in [9, Theorem 2].

Proposition 2: *For* $m = 2^r, r \ge 1$, *the optimal coding gain of rate-one LTAST codes, i.e.* $G = \frac{1}{m} d_{\min}^2$, *can be obtained by choosing the Diophantine number* $\phi = j$ ($j^2 = -1$) *and constellations* $\mathcal{S}$ *carved from the ring of Gaussian integers, and for* $m = 2^{r_0} 3^{r_1}, r_0, r_1 \ge 0$ *by choosing* $\phi = e^{2j\pi/6}$ *and constellations* $\mathcal{S}$ *carved from the ring of Einstein integers.*

Note that the constellations carved from the ring of Gaussian integers include QAM constellations, while the constellations carved from the ring of Einstein integers include hexagonal constellations [12]. [9, Theorem 1] also suggests how to select ϕ for PSK constellations; however, computer search is required to find the ϕ that maximizes the coding gain. Additionally, for $m \ne 2^r$ or $m = 2^{r_0} 3^{r_1}$, only local maxima of the coding gain are guaranteed by computer search.

To achieve full diversity, the decoding of LTAST codes requires ML joint detection of m transmitted symbols by a sphere decoder with complexity roughly cubic in m [13]. Therefore, the decoding complexity of the LTAST codes is much higher than that of OSTBC for $m > 1$. Additionally, Proposition 1 shows that the coding gain reduces when m increases. In the next section, we will present our new SAST codes using a circulant structure and rotations of TAST codes (but not limited to). SAST codes exploit efficiently the commutativity of circulant matrices and hence yield significant improvements over LTAST codes.

III. SAST Code Construction and Properties

A. Encoder

We consider the number of Tx antennas to be $M = 2m$. Two input data vectors $\boldsymbol{s}_1$ and $\boldsymbol{s}_2$, both consisting of m information symbols, are first rotated such that $\boldsymbol{u}_1 = R\boldsymbol{s}_1$ and $\boldsymbol{u}_2 = R\boldsymbol{s}_2$. Then the rotated vectors are used to generate two LTAST codewords of size m, $A = \mathcal{C}(\boldsymbol{u}_1^T)$ and $B = \mathcal{C}(\boldsymbol{u}_2^T)$. The SAST codeword is formulated as follows:

$$W = \begin{bmatrix} A & B \\ -B^\dagger & A^\dagger \end{bmatrix}. \tag{10}$$

At the moment, we do not restrict ourself to the optimal rotations of LTAST codes given in Proposition 2. The only requirement is that LTAST codes of size m have full diversity.

The structure of SAST codes is fundamentally different from QSTBC proposed by Jafarkhani [14], where A and B are OSTBC codewords of the same size, and A and B are conjugate, but not conjugate transpose as our SAST codes.

B. Properties of SAST Codes

As the primary important design criterion, we first analyze the diversity gain of SAST codes.

Lemma 1: *SAST block codes achieve full diversity for quasi-static channels.*

Proof: Let W and $\hat{W}$ be two distinct codewords. The product matrix $P_W = \Delta_W^\dagger \Delta_W$ is given by

$$P_W = \Delta_W^\dagger \Delta_W = \begin{bmatrix} P_A + P_B & \mathbf{0}_m \\ \mathbf{0}_m & P_A + P_B \end{bmatrix} \tag{11}$$

where $P_A = \Delta_A^\dagger \Delta_A$, $P_B = \Delta_B^\dagger \Delta_B$, $\Delta_A = A - \hat{A}$, $\Delta_B = B - \hat{B}$, and $\mathbf{0}_m$ denotes the $m \times m$ all-zero matrix. The second line of (11) comes from the property P3 of circulant matrices.

For two distinct codewords, at least either $A \ne \hat{A}$ or $B \ne \hat{B}$. Thus, at least one of the matrices P_A and P_B is of full rank or they both are full rank because A and B are the LTAST codewords. Equivalently, at least one of P_A or P_B or both are positive definite matrices. Therefore, $P_A + P_B$ is positive definite, and consequently, $\det(P_W) = [\det(P_A + P_B)]^2 \ge [\det(P_A)]^2 > 0$.

Thus, the matrix P_W is always full rank for all input symbols, and SAST codes achieve full diversity. □

Now we consider the coding gain of SAST block codes. In the worst case, $\min \det(P_W) = \min\,[\det(P_A)]^2$. With reference to Proposition 1, we have the following Lemma on the coding gain of SAST codes.

Lemma 2: *The coding gain of SAST block codes for* $M = 2m$ *transmit antennas equals the coding gain of rate-one LTAST codes for* m *transmit antennas and is upper-bounded by* $G_c^W \le \frac{2}{M} d_{\min}^2$.

Lemma 2 shows that one can use optimal rotations of LTAST codes as optimal rotations for SAST codes. Moreover, the optimal coding gain of SAST codes is larger than that of optimal LTAST codes, an important improvement of SAST codes over LTAST codes. The next section shows that this improvement comes at no extra decoding complexity, and, better still, decoding complexity reduces significantly.

C. Decoder

We consider the number of Rx antennas $N = 1$. If $N > 1$, one can perform maximal ratio combining (MRC) for received signals of each Rx antenna, and the decoder for 1 Rx antenna can be employed.

Let $Y = [Y_1^T \quad Y_2^T]^T$, $Y_1 = [y_1, y_2, ..., y_m]^T$, $Y_2 = [y_{m+1}, y_{m+2}, ..., y_M]^T$, $H = [\boldsymbol{h}_1^T \quad \boldsymbol{h}_2^T]^T$, $\boldsymbol{h}_1 = [h_1, h_2, ..., h_m]^T$; $\boldsymbol{h}_2 = [h_{m+1}, h_{m+2}, ..., h_{2m}]^T$, $Z = [Z_1^T \quad Z_2^T]^T$, $Z_1 = [z_1, z_2, ..., z_m]^T$; $Z_2 = [z_{m+1}, z_{m+2}, ..., z_{2m}]^T$, we obtain

$$\begin{bmatrix} Y_1 \\ Y_2 \end{bmatrix} = \begin{bmatrix} A & B \\ -B^\dagger & A^\dagger \end{bmatrix} \begin{bmatrix} \boldsymbol{h}_1 \\ \boldsymbol{h}_2 \end{bmatrix} + \begin{bmatrix} Z_1 \\ Z_2 \end{bmatrix} \tag{12}$$

An equivalent form of (12) is

$$\begin{bmatrix} Y_1 \\ Y_2^* \end{bmatrix} = \begin{bmatrix} X_1 & X_2 \\ X_3 & X_4 \end{bmatrix} \begin{bmatrix} \boldsymbol{u}_1 \\ \boldsymbol{u}_2 \end{bmatrix} + \begin{bmatrix} Z_1 \\ Z_2^* \end{bmatrix} \tag{13}$$

where $X_1 = \mathcal{C}^L(\boldsymbol{h}_1^T), X_2 = \mathcal{C}^L(\boldsymbol{h}_2^T), X_3 = \mathcal{C}^\dagger(\boldsymbol{h}_2^T)$, and $X_4 = -\mathcal{C}^\dagger(\boldsymbol{h}_1^T)$.

At this point, using (13), two vectors $\boldsymbol{u}_1$ and $\boldsymbol{u}_2$ can be jointly decoded, for example, by a sphere decoder. However, the joint detectors result in the same complexity of LTAST code decoders. We will show how to reduce the decoding complexity in sequence.

Applying permutation Π (defined in Section II-B) for the column matrix Y_1, we obtain

$$\begin{bmatrix} \bar{Y}_1 \\ \bar{Y}_2 \end{bmatrix} = \begin{bmatrix} \Pi(Y_1) \\ Y_2^* \end{bmatrix} = \underbrace{\begin{bmatrix} H_1 & H_2 \\ H_2^\dagger & -H_1^\dagger \end{bmatrix}}_{H} \begin{bmatrix} \boldsymbol{u}_1 \\ \boldsymbol{u}_2 \end{bmatrix} + \begin{bmatrix} \bar{Z}_1 \\ \bar{Z}_2 \end{bmatrix} \tag{14}$$

where $\bar{Y}_1 = \Pi(Y_1)$, $\bar{Y}_2 = Y_2^*$, $H_1 = \mathcal{C}(\boldsymbol{h}_1^T)$, $H_2 = \mathcal{C}(\boldsymbol{h}_2^T)$, $\bar{Z}_1 = \Pi(Z_1)$, and $\bar{Z}_2 = Z_2^*$. The elements of $\bar{Z}_1$ and $\bar{Z}_2$ have the same statistics, $\mathcal{CN}(0,1)$, as elements of Z_1 and Z_2.

We now perform MRC by left multiplying $H^\dagger$ to both sides of equation (14) [15]. Let $\hat{H} = H_1^\dagger H_1 + H_2^\dagger H_2$, we get

$$\begin{aligned} \begin{bmatrix} \hat{Y}_1 \\ \hat{Y}_2 \end{bmatrix} &= H^\dagger \begin{bmatrix} \bar{Y}_1 \\ \bar{Y}_2 \end{bmatrix} = \begin{bmatrix} \hat{H} & \mathbf{0}_m \\ \mathbf{0}_m & \hat{H} \end{bmatrix} \begin{bmatrix} \boldsymbol{u}_1 \\ \boldsymbol{u}_2 \end{bmatrix} + H^\dagger \begin{bmatrix} \bar{Z}_1 \\ \bar{Z}_2 \end{bmatrix} \\ &= \begin{bmatrix} \hat{H} & \mathbf{0}_m \\ \mathbf{0}_m & \hat{H} \end{bmatrix} \begin{bmatrix} \boldsymbol{u}_1 \\ \boldsymbol{u}_2 \end{bmatrix} + \underbrace{\begin{bmatrix} \hat{Z}_1 \\ \hat{Z}_2 \end{bmatrix}}_{\hat{Z}}. \end{aligned} \tag{15}$$

The covariance matrix of the additive noise vector $\hat{Z}$ is

$$E[ZZ^\dagger] = \begin{bmatrix} \hat{H} & \mathbf{0}_m \\ \mathbf{0}_m & \hat{H} \end{bmatrix} \tag{16}$$

Therefore, noise vectors $\hat{Z}_1$ and $\hat{Z}_s$ are uncorrelated and have the same covariance matrix $\hat{H}$. Thus, $\boldsymbol{u}_1$ and $\boldsymbol{u}_2$ can be decoded separately using $\hat{Y}_i = \hat{H}\boldsymbol{u}_i + \hat{Z}_i$, $i = 1, 2$.

The noise vectors $\hat{Z}_1$ and $\hat{Z}_2$ can be whitened by the same whitening matrix $(\hat{H})^{-1/2}$. Finally, we obtain

$$\check{Y}_i = \hat{H}^{-1/2}\hat{Y}_i = \underbrace{\hat{H}^{1/2}R}_{\check{H}}\boldsymbol{s}_i + \underbrace{\hat{H}^{-1/2}\hat{Z}_i}_{\check{Z}_i}, \quad i = 1, 2. \tag{17}$$

Any decoders for LTAST codes can also be used to decode SAST codes. However, two data vectors $\boldsymbol{s}_1$ and $\boldsymbol{s}_2$ can be decoded in parallel. The complexity of ML detection by a sphere decoder for SAST codes is roughly $\mathcal{O}(m^3)$, instead of $\mathcal{O}(8m^3)$ as with LTAST codes. Hence, on average, the decoding complexity of SAST codes reduce significantly compared with that of LTAST codes with the same rate of one symbol pcu.

D. Maximum Mutual Information

We now study the maximum mutual information of SAST codes over $(M, 1)$ MISO channels. The equivalent channel H given in (14) can be used to calculate the maximum mutual information [16], [17] of SAST codes as

$$\mathsf{C}_S = \frac{1}{M} E\left[\log_2 \det\left(I_M + \frac{\rho}{M} HH^\dagger\right)\right]. \tag{18}$$

The normalizing factor $\frac{1}{M}$ indicates that only M symbols are repeatedly sent over M channel uses.

Let $\boldsymbol{h} = [\boldsymbol{h}_1^T \ \boldsymbol{h}_2^T]^T$ and $H_L = \mathcal{C}^L(\boldsymbol{h}^T)$. The maximum mutual capacity of LTAST codes can be shown to be

$$\mathsf{C}_L = \frac{1}{M} E\left[\log_2 \det\left(I_M + \frac{\rho}{M} H_L H_L^\dagger\right)\right]. \tag{19}$$

Fig. 1 plots the maximum mutual information of SAST and LTAST codes together with the capacity of MISO channels for $M = 2, 4, 8, 16$. The maximum mutual information of SAST codes is constant with respect to the number of Tx antennas. While this behavior is similar to LTAST codes [9], [15], the maximum mutual information of SAST codes is higher than that of LTAST codes. Note that for $M = 2$, the SAST code reverts to the Alamouti code. The numerical results show that for $M = 4$, SAST codes attain more than 95% (and up to 98%) of channel capacity. QSTBC (with $M = 4$ only) also attain the same capacity [18] because SAST codes and QSTBC for $M = 4$ are equivalent [19]. For a specific SNR, the channel capacity actually does not increase when the number of Tx antennas keeps growing but the number of Rx antennas is fixed [17]. Fig. 1 also shows that the capacity increment of a MISO channel is negligible when the number of Tx antennas increases from 8 to 16.

IV. SIMULATION RESULTS

We present simulations results for SAST codes and DST and LTAST codes, all with rate-one. Fig. 2 plots the BER of SAST and LTAST codes with spectral efficiencies 2, 4 and 6 bits pcu. The SNR gain of SAST codes over LTAST codes is quite large. For example, for a (4, 1) system, the SNR gain is about 1.3, 2, and 2.5 dB for 2, 4, and 6 bits pcu, respectively. The gain increases with spectral efficiency. For $M = 8$, similar results can be obtained but are omitted for brevity.

Fig. 3 illustrates the performance of SAST codes and space-time linear constellation precoding (ST-LCP) codes [7] with the same 2 bits pcu. ST-LCP codes in fact are equivalent to the DAST codes proposed in [5], and by using the Fast fourier transform (FFT), one can convert LTAST codes to the DAST codes (see [9] and Property 4 of circulant matrices in Section II-C). The slopes of the BER curves of SAST and ST-LCP codes are almost parallel. This indicates that SAST codes achieve full diversity. Furthermore, notable gains of 1 and 1.5 dB over ST-LCP codes are obtained for $M = 3$ and $M = 5$,

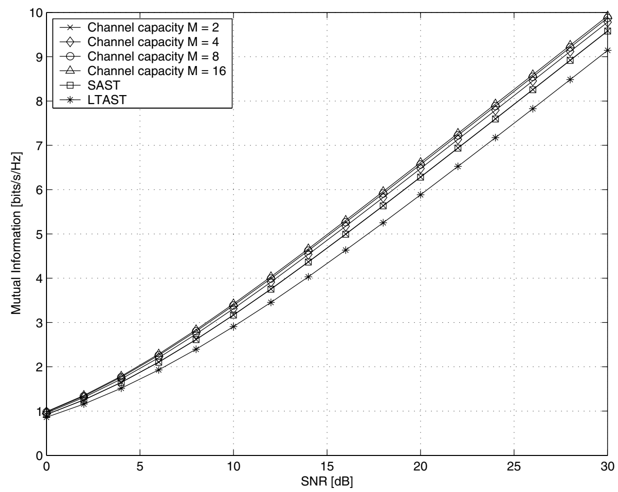

Fig. 1. Channel capacity and maximum mutual information of SAST and LTAST codes over MISO channels.

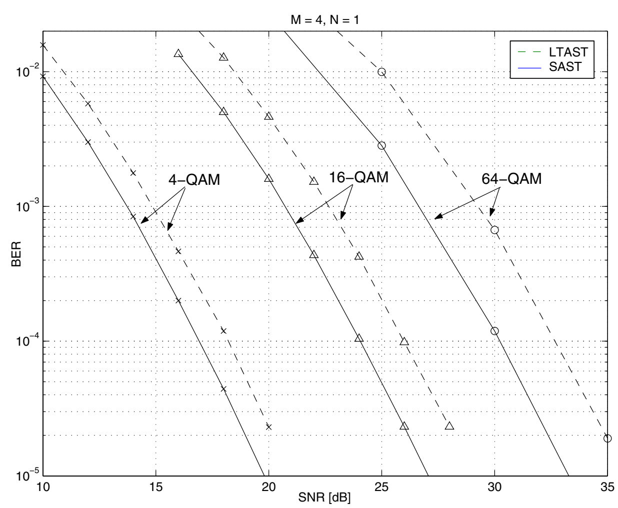

Fig. 2. BER of SAST and LTAST codes with $M = 4$ and $N = 1$.

respectively. Thus SAST codes perform better compared to LTAST codes for any number of Tx antennas.

V. Conclusion

We presented a new class of space-time block codes called SAST codes. They are of rate one symbol pcu and are derived from rate-one LTAST codes. Compared with rate-one LTAST codes, SAST codes reduce the decoding complexity significantly and achieve SNR gains up to 3 dBs. In terms of maximum mutual capacity, the use of SAST codes is nearly optimal for MISO channels. These improvements come from the orthogonality embedded into the codeword structure, where the left-half columns are orthogonal with the right-half columns.

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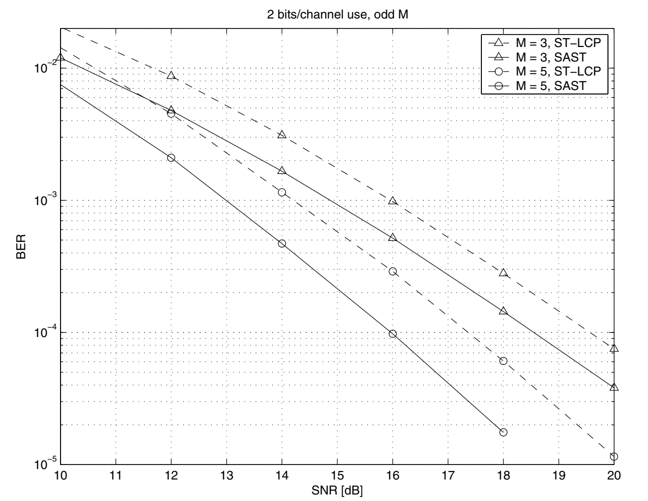

Fig. 3. BER of SAST and ST-LCP codes with 4-QAM, $M = 3, 5$, $N = 1$.

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