# Constrained Detection for Multiple-Input Multiple-Output Channels

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***Abstract*— We develop a family of constrained detectors for multiple-input multiple-output (MIMO) channels by relaxing the maximum likelihood (ML) detection problem. Real constrained linear detectors and decision feedback detectors are proposed for real constellations by forcing the relaxed solution to be real. Generalized minimum mean-square error and constrained least squares detectors are generalized as MIMO detectors for both constant and non-constant modulus constellations. Using our constrained linear detectors, we propose a new ordering scheme to achieve a tradeoff between interference suppression and noise enhancement. Moreover, we introduce a combined constrained linear and decision feedback detector to mitigate the error propagation in decision feedback. Simulation results show that the combined detectors achieve significant performance gain over V-BLAST detection.**

## I. Introduction

Multiple-input multiple-output (MIMO) wireless communication systems employing multiple antennas at both the transmitter and the receiver can potentially achieve remarkably high spectral efficiencies in rich scattering multipath environments. As a result, the design of high date-rate MIMO wireless communications systems has generated significant interest. A prime example is the BLAST (Bell Laboratories layered space time) architecture [1], which has been proposed to exploit the potentially enormous MIMO link capacity

In spatial multiplexing systems such as BLAST, a fundamental receiver function is the detection of transmitted data symbols. The optimal maximum likelihood (ML) detector achieves the minimum error probability for independent and identically distributed (i.i.d.) random data symbols. However, the complexity of the ML detector (MLD) grows exponentially with the number of transmit antennas and the signal constellation size, making it computationally prohibitive in most cases. Therefore, various computationally efficient suboptimal detectors such as the zero-forcing (ZF) detector and the minimum mean-square error (MMSE) detector are developed. In [1], the V-BLAST detector with optimal ordering is proposed. The equivalence between V-BLAST and a zero-forcing (ZF) decision feedback detector (DFD) is demonstrated [2]. If the nulling criterion is MMSE, the resulting MMSE-DFD [3] makes a trade-off between interference suppression and noise enhancement. However, these suboptimal detectors perform much worse than the MLD. In [4], sphere decoding (SD), offering near-optimal performance, is proposed to attain low complexity in high SNR. But its complexity is still high in low SNR or for large systems. The large gap in both performance and complexity between the MLD and suboptimal detectors has motivated the search for alternative detectors.

In this paper, we consider a relaxation approach to the MIMO detection problem. In the relaxation approach, the discrete set of all possible transmit vectors is embedded in a larger multidimensional continuous space and the minimization is performed over this continuous space. The resulting minimum solution is mapped back into the original discrete space. As an example, in code-division multiple-access (CDMA) systems, a generalized MMSE (GMMSE) detector is proposed [5], where the binary phase shift keying (BPSK) vectors are relaxed so that they are inside a hypercube. In [6], a tighter relaxation is used in orthogonal frequency division multiplexing (OFDM) / spatial division multiple access (SDMA) systems employing constant-modulus constellations by restricting the binary vectors on a hypersphere rather than within a hypercube, which is named a constrained least squares (CLS) detector. We extend the idea of constrained detection into uncoded MIMO systems. A class of constrained linear detectors and a class of constrained DFDs are developed. Real constrained linear and decision feedback detectors are proposed for real constellations. We generalize the CLS detector by dividing the signal vector into several subgroups and applying the constant modulus constraints to these subgroups. A new ordering scheme is also proposed using the constrained linear detectors, which maximizes the signal-to-interference and noise ratio (SINR). In V-BLAST, the constrained linear detector and the DFD are combined to improve the quality of the first detected symbol and to mitigate the error propagation inherent in the DFD.

## II. System model

We consider an MIMO system with $n$ transmit antennas and $m$ receive antennas. We focus on spatial multiplexing systems, where the signals are statistically independent. Source data are mapped into a finite constellation $\mathcal{Q}$. The data stream is demultiplexed into $n$ equal length substreams, each of which is simultaneously sent through $n$ antennas over a rich scattering channel. We assume narrowband signals so that the MIMO channel is flat fading. The received signals can be written as

$$\mathbf{r} = \mathbf{H}\mathbf{x} + \mathbf{n} \tag{1}$$

where $\mathbf{x} = [x_1, \ldots, x_n]^T$, $x_i \in \mathcal{Q}$ is the transmitted signal vector, $\mathbf{r} = [r_1, \ldots, r_m]^T$, $r_i \in \mathcal{C}$ is the received signal vector, $\mathbf{H} = [h_{i,j}] \in \mathcal{C}^{m \times n}$ is the channel matrix, and $\mathbf{n} = [n_1, \ldots, n_m]^T$, $n_i \in \mathcal{C}$ is an additive white Gaussian noise (AWGN) vector. The elements of $\mathbf{H}$ are identically independent distributed (i.i.d.) complex Gaussian, $h_{i,j} \sim \mathcal{CN}(0, 1)$. The components of $\mathbf{n}$ are i.i.d. and $n_i \sim \mathcal{CN}(0, \sigma_n^2)$. We assume that the channel is perfectly known to the receiver and $n \leq m$. If $n > m$, we can readily transform the rank deficient problem into a full rank problem as shown in [7]. Note (1) models any linear, synchronous and flat fading channels, i.e., it can be directly applied to multiuser detection in CDMA. Therefore, all of the detectors proposed in this paper can be readily applied to CDMA systems.

Assuming uncorrelated noise, the MLD is given by

$$\hat{\mathbf{x}} = \arg\min_{\mathbf{x} \in \mathcal{Q}^n} \|\mathbf{r} - \mathbf{H}\mathbf{x}\|^2. \tag{2}$$

Due to the discrete nature of $\mathcal{Q}$, (2) is a NP-hard problem and exhaustive search for $\hat{\mathbf{x}}$ has a complexity exponential in $n$.

## III. Constrained Linear Detectors

### A. Real constrained detectors

If the constellation $\mathcal{Q}$ is real i.e., binary phase shift keying (BPSK) and pulse amplitude modulation (PAM), the ZF and MMSE solutions are usually complex vectors. The imaginary part may cause additional interference since the transmitted vector is real. To impose a real constraint on ZF and MMSE, we rewrite (1) as

$$\tilde{\mathbf{r}} = \begin{bmatrix} \mathrm{Re}\{\mathbf{r}\} \\ \mathrm{Im}\{\mathbf{r}\} \end{bmatrix} = \begin{bmatrix} \mathrm{Re}\{\mathbf{H}\} \\ \mathrm{Im}\{\mathbf{H}\} \end{bmatrix} \mathbf{x} + \begin{bmatrix} \mathrm{Re}\{\mathbf{n}\} \\ \mathrm{Im}\{\mathbf{n}\} \end{bmatrix} = \tilde{\mathbf{H}}\mathbf{x} + \tilde{\mathbf{n}}. \tag{3}$$

Note that the components of $\tilde{\mathbf{n}}$ have zero mean and variance $\sigma_n^2/2$. The ZF and MMSE detectors for (3) can be obtained as

$$\hat{\mathbf{x}}_{\mathrm{R-ZF}} = \left(\tilde{\mathbf{H}}^H \tilde{\mathbf{H}}\right)^{-1} \tilde{\mathbf{H}}^H \tilde{\mathbf{r}} \tag{4}$$

and

$$\hat{\mathbf{x}}_{\mathrm{R-MMSE}} = \left(\tilde{\mathbf{H}}^H \tilde{\mathbf{H}} + \sigma_n^2/2\mathbf{I}_n\right)^{-1} \tilde{\mathbf{H}}^H \tilde{\mathbf{r}} \tag{5}$$

where R-ZF and R-MMSE denote real constrained ZF and MMSE detectors.

### B. Constrained subgroup detectors based on constant-modulus constellations

When $\mathcal{Q}$ is a complex constellation, the constraint on the constellation modulus is exploited. We first consider a constant modulus constellation with unity modulus $|x_i|^2 = 1$. In [6], the CLS relaxes the candidate vectors to be on the hypersphere $\mathbf{x}^H\mathbf{x} = n$. We partition the vector $\mathbf{x}$ into $g$ groups and each group forms a subvector $\mathbf{x}_i$ with size $s_i$, $i = 1, \ldots, g$, where $\sum_{i=1}^{g} s_i = n$. We relax each $\mathbf{x}_i$ on an $s_i$-dimensional hypersphere $\mathbf{x}_i^H\mathbf{x}_i = s_i$. The constrained detector is thus given by

$$\hat{\mathbf{x}}_{\mathrm{CML}} = \arg\min_{\mathbf{x}_1^H\mathbf{x}_1 = s_1, \ldots, \mathbf{x}_g^H\mathbf{x}_g = s_g} \|\mathbf{r} - \mathbf{H}\mathbf{x}\|^2 \tag{6}$$

where CML denotes constrained ML detector. The minimization problem (6) can be written as

$$\begin{aligned} &\min_{\mathbf{x}} \|\mathbf{r} - \mathbf{H}\mathbf{x}\|^2 \\ &\text{s.t. } \mathbf{x}_1^H\mathbf{x}_1 = s_1, \ldots, \mathbf{x}_g^H\mathbf{x}_g = s_g. \end{aligned} \tag{7}$$

The lagrangian $\mathcal{L}(\mathbf{x}, \lambda_1, \ldots, \lambda_g)$ for this minimization problem is

$$\mathcal{L}(\mathbf{x}, \lambda_1, \ldots, \lambda_g) = \|\mathbf{r} - \mathbf{H}\mathbf{x}\|^2 + \sum_{i=1}^{g} \lambda_i \left(\mathbf{x}_i^H\mathbf{x}_i - s_i\right). \tag{8}$$

Taking partial derivatives with respect to $x_i$'s, the solution for $\mathbf{x}$ can be derived as

$$\hat{\mathbf{x}}(\lambda_1, \ldots, \lambda_g) = \left(\mathbf{H}^H\mathbf{H} + \mathbf{\Lambda}\right)^{-1} \mathbf{H}^H\mathbf{r} \tag{9}$$

where $\mathbf{\Lambda}$ is a diagonal matrix and is given by

$$\mathbf{\Lambda} = \mathrm{diag}\{\underbrace{\lambda_1, \ldots, \lambda_1}_{s_1}, \ldots, \underbrace{\lambda_g, \ldots, \lambda_g}_{s_g}\}. \tag{10}$$

When $g = 1$, there is only one $\lambda_1$. Eq. (10) reduces to the CLS solution in [6]. When $\lambda_1 = \ldots = \lambda_g = \sigma_n^2$, the CML detector becomes the MMSE detector.

In order to obtain the CML solution in (9), $\hat{\mathbf{x}}(\lambda_1, \ldots, \lambda_g)$ is substituted into (7). We need to find the zeros of the set of equations

$$\begin{aligned} F_1(\lambda_1, \ldots, \lambda_g) &= \|\hat{\mathbf{x}}_1(\lambda_1, \ldots, \lambda_g)\|^2 - s_1 = 0 \\ &\vdots \\ F_g(\lambda_1, \ldots, \lambda_g) &= \|\hat{\mathbf{x}}_g(\lambda_1, \ldots, \lambda_g)\|^2 - s_g = 0. \end{aligned} \tag{11}$$

Note that (11) are nonlinear equations, but (9) has the form of LS or MMSE. Therefore, we also consider the CML detector as a linear detector.

The multidimensional Newton-Raphson root finding method can be used to solve (11). It needs to compute the partial derivative of $F_i$ with respect to $\lambda_j$, $\partial F_i/\partial\lambda_j$, $1 \leq i, j \leq g$; the partial derivatives may be computed by finite differences. There are several sets of roots for (11) and an initial estimate is needed to guarantee the convergence to the desired root. Since the MMSE detector provides a good solution, the initial values for $\lambda_i$ are chosen as $\lambda_1 = \ldots = \lambda_g = \sigma_n^2$. If the Newton method does not converge after a specified number of iterations, we simply set $\lambda_1 = \ldots = \lambda_g = \sigma_n^2$ or the CML detector outputs the MMSE solution.

For a non-constant modulus constellation such as quadrature amplitude modulation (QAM), we assume $\rho$ to be the largest modulus of the constellation. Similarly, we also partition the vector $\mathbf{x}$ into $g$ groups. We thus relax each $\mathbf{x}_i$ within an $s_i$-dimensional hypercube $\mathbf{x}_i^H\mathbf{x}_i \leq \rho^2 s_i$. The CML detector is modified as

$$\hat{\mathbf{x}}_{\mathrm{CML}} = \arg\min_{\mathbf{x}_1^H\mathbf{x}_1 \leq \rho^2 s_1, \ldots, \mathbf{x}_g^H\mathbf{x}_g \leq \rho^2 s_g} \|\mathbf{r} - \mathbf{H}\mathbf{x}\|^2 \tag{12}$$

The minimization problem (12) can be written as

$$\begin{aligned} &\min_{\mathbf{x}} \|\mathbf{r} - \mathbf{H}\mathbf{x}\|^2 \\ &\text{s.t. } \mathbf{x}_1^H\mathbf{x}_1 \leq \rho^2 s_1, \ldots, \mathbf{x}_g^H\mathbf{x}_g \leq \rho^2 s_g. \end{aligned} \tag{13}$$

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Using the convex duality theorem, the Lagrangian dual function for (13) can be expressed as

$$\mathcal{L}(\mathbf{x}, \lambda_1, \ldots, \lambda_g) = \|\mathbf{r}-\mathbf{H}\mathbf{x}\|^2 + \sum_{i=1}^{g} \lambda_i \left(\mathbf{x}_i^H \mathbf{x}_i - \rho^2 s_i\right). \quad (14)$$

Solving (14) for $\mathbf{x}$, the solution is the same as (9). Substituting it back into (14), we obtain

$$\max_{\lambda_1 \geq 0, \ldots, \lambda_g \geq 0} -\mathbf{r}^H \mathbf{H} \left(\mathbf{H}^H \mathbf{H} + \mathbf{\Lambda}\right)^{-1} \mathbf{H}^H \mathbf{r} - \rho^2 \sum_{i=1}^{g} \lambda_i s_i \quad (15)$$

where (15) is a $g$-dimensional optimization problem. The simple unconstrained multidimensional gradient descent algorithm can be used to solve (15). The partial derivatives are computed by finite differences. If $g = 1$, the CML detector (12) reduces to the GMMSE in [5].

The constrained detectors using constellation modulus can be combined with the real constraint. We denote the combined receiver as R-CML.

### C. Iterative improvement

An iterative detector can be used to improve the performance of our constrained linear detectors by correcting unreliable decisions of the detector. As before, we partition $\mathbf{x}$ into $g$ groups each with $s_i$ symbols. In each iteration, for $i = 1, \ldots, g$, we fix the other $g-1$ groups and solve

$$\hat{\mathbf{x}} = \arg\min_{\mathbf{x}_i \in \mathcal{Q}^{s_j}} \|\mathbf{r} - \mathbf{H}\mathbf{x}\|^2 \quad (16)$$

where $\mathbf{x}_i$ is the $i$-th group in $\mathbf{x}$. $\mathbf{x}_i$ is updated to $\hat{\mathbf{x}}_i$. The iteration starts with any solution from a constrained detector and terminates when $\mathbf{x}$ ceases to change during an iteration. Typically, we choose $g = n$ and $s_i = 1$. It resembles the chase decoder [8] for soft decoding of linear block codes.

## IV. Constrained Decision Feedback Detectors

### A. V-BLAST detection

The V-BLAST detection algorithm [1] uses nulling and interference cancellation and consists of $n$ iterations. In the $k$-th iteration, the signal with maximum post-detection SNR among the remaining $n-k+1$ symbols is detected. The whole algorithm is described as follows

- Initialization:

$$\begin{aligned} \mathbf{r}_1 &= \mathbf{r} && (17a)\\ \mathbf{G}_1 &= \mathbf{H}^\dagger && (17b)\\ k_1 &= \arg\min_j \|(\mathbf{G}_1)_j\|^2 && (17c) \end{aligned}$$

- Recursion: for $i = 1$ to $n$

$$\begin{aligned} \mathbf{w}_{k_i} &= (\mathbf{G}_i)_{k_i} && (17d)\\ \hat{x}_{k_i} &= \arg\min_{x \in \mathcal{Q}} \left|x - \mathbf{w}_{k_i}^H \mathbf{r}_i\right|^2 && (17e)\\ \mathbf{r}_{i+1} &= \mathbf{r}_i - \hat{x}_{k_i} (\mathbf{H})_{k_i} && (17f)\\ \mathbf{G}_{i+1} &= \mathbf{H}_{\bar{k}_i}^\dagger && (17g)\\ k_{i+1} &= \arg\min_{j \notin \{k_1, \ldots, k_i\}} \|(\mathbf{G}_{i+1})_j\|^2 && (17h) \end{aligned}$$

where $(\mathbf{A})_i$ is the $i$-th column of matrix $\mathbf{A}$ and $\mathbf{H}_{\bar{k}_i}$ is obtained by zeroing the $k_1, \ldots, k_i$-th columns of $\mathbf{H}$.

As suggested in [2], given an optimum order $k_1, \ldots, k_n$, V-BLAST detection is equivalent to the ZF-DFD. Assuming $\mathbf{\Pi}$ is the column permutation matrix obtained from the optimum order, we apply $\mathbf{\Pi}$ to $\mathbf{H}$. Let the QR factorization of $\mathbf{G} = \mathbf{H}\mathbf{\Pi}$ be

$$\mathbf{G} = [\mathbf{Q}_1, \mathbf{Q}_2] \begin{bmatrix} \mathbf{R} \\ \mathbf{0} \end{bmatrix} \quad (18)$$

where $\mathbf{R}$ is an $n \times n$ upper-triangular matrix, $\mathbf{0}$ is an $(m-n) \times n$ zero matrix, $\mathbf{Q}_1$ is an $m \times n$ unitary matrix and $\mathbf{Q}_2$ is an $m \times (m-n)$ unitary matrix. Eq. (1) is equivalent to

$$\mathbf{y} = \mathbf{R}\mathbf{x} + \mathbf{v} \quad (19)$$

where $\mathbf{y} = \mathbf{Q}_1^H \mathbf{r}$ and $\mathbf{v} = \mathbf{Q}_1^H \mathbf{n}$ is also an i.i.d. complex Gaussian vector with mean zero and variance $\sigma_n^2$. The second description of the V-BLAST algorithm is given by

- for $i = n$ to 1

$$\begin{aligned} \hat{x}_i &= \arg\min_{x \in \mathcal{Q}} |y_i - R_{i,i} x|^2 && (20a)\\ \mathbf{y} &= \mathbf{y} - (\mathbf{R})_i \hat{x}_i && (20b) \end{aligned}$$

where $R_{i,i}$ is the $(i,i)$-th entry of $\mathbf{R}$.

### B. Real decision feedback detectors

For real valued constellations, using the same arguments in Section III.B, V-BLAST detection of (3) performs better than that of (1) directly, and we denote V-BLAST for (3) as R-V-BLAST. If $n = m$ and no permutations are used, the squared-norm of the entries of $\mathbf{R}$ are known to be $\chi^2$ distributed. Specifically, $|R_{i,i}|^2 \sim \chi^2(2i)$, for $i = 1, \ldots, n$ and $|R_{i,j}|^2 \sim \chi^2(2)$, for $j > i$, where $\chi^2(k)$ denotes the chi-squared distribution with $k$ degrees of freedom. Since the performance of V-BLAST is limited by the first detected symbol [9], the diversity order of V-BLAST detection is only one. However, if QR decomposition is performed on the real matrix $\tilde{\mathbf{H}}$ in (3), the squared-norm of the entries of $\tilde{\mathbf{R}}$ are also $\chi^2$ distributed, but $|R_{i,i}|^2 \sim \chi^2(i+n)$, for $i = 1, \ldots, n$ and $|R_{i,j}|^2 \sim \chi^2(1)$, for $j > i$. Therefore, it can be readily verified that the diversity order of R-V-BLAST increases to $(n+1)/2$.

For decoupleable complex constellations, i.e., QAM, (1) can be rewritten as

$$\begin{bmatrix} \mathrm{Re}\{\mathbf{r}\} \\ \mathrm{Im}\{\mathbf{r}\} \end{bmatrix} = \begin{bmatrix} \mathrm{Re}\{\mathbf{H}\} & -\mathrm{Im}\{\mathbf{H}\} \\ \mathrm{Im}\{\mathbf{H}\} & \mathrm{Re}\{\mathbf{H}\} \end{bmatrix} \begin{bmatrix} \mathrm{Re}\{\mathbf{x}\} \\ \mathrm{Im}\{\mathbf{x}\} \end{bmatrix} + \begin{bmatrix} \mathrm{Re}\{\mathbf{n}\} \\ \mathrm{Im}\{\mathbf{n}\} \end{bmatrix} \quad (21)$$

or

$$\tilde{\mathbf{r}} = \tilde{\mathbf{H}}\tilde{\mathbf{x}} + \tilde{\mathbf{n}}. \quad (22)$$

In [10], it has been shown that applying V-BLAST to the equivalent real system (21) yields an additional performance gain. We can quantify the improvement. In the original system, the diversity order for $x_n$ is 1. In (21), after QR decomposition on $\tilde{\mathbf{H}}$, we find $|R_{i,i}|^2 \sim \chi^2(i)$, for $i = 1, \ldots, 2n$. Therefore, the diversity order for $\mathrm{Re}\{x_n\}$ is $(n+1)/2$ and for $\mathrm{Im}\{x_n\}$ is $1/2$, which may improve the total performance on $x_n$.

### C. Constrained ordering decision feedback detectors

The ZF nulling vector $\mathbf{w}_{k_i}$ (17d) in V-BLAST completely removes the interferences from the other antennas but also amplifies the additive noise. To get a better tradeoff between noise enhancement and interference suppression, we use our proposed constrained linear detectors in Section III instead of the ZF detector in V-BLAST. We replace (17b) and (17g) with

$$\mathbf{G}_1 = \left(\mathbf{H}^H\mathbf{H} + \mathbf{\Lambda}\right)^{-1}\mathbf{H}^H \tag{23}$$

and

$$\mathbf{G}_{i+1} = \left(\mathbf{H}_{\bar{k}_i}^H\mathbf{H}_{\bar{k}_i} + \mathbf{\Lambda}_i\right)^{-1}\mathbf{H}_{\bar{k}_i}^H \tag{24}$$

where $\mathbf{\Lambda}$ and $\mathbf{\Lambda}_i$ can be calculated using (11) and (15) for constant modulus and non-constant modulus constellations, respectively.

When nulling is performed using CML, interference cannot be removed completely. We thus propose to determine the detection order at each iteration by maximizing the SINR defined as

$$\mathrm{SINR}_j = \frac{|(\mathbf{G}_{i+1}\mathbf{H}_{\bar{k}_i})_{j,j}|^2E\{|x_j|^2\}}{\sum_{k=1,k\neq j}^{n}|(\mathbf{G}_{i+1}\mathbf{H}_{\bar{k}_i})_{j,k}|^2E\{|x_k|^2\} + \sigma_n^2\|(\mathbf{G}_{i+1})_j\|^2} \tag{25}$$

where $(\mathbf{A})_{i,j}$ is the $(i,j)$-th entry of matrix $\mathbf{A}$ and $(\mathbf{G}_{i+1})_k$ denotes the $k$-th row of matrix $\mathbf{G}_{i+1}$. In V-BLAST, (17h) is replaced by

$$k_{i+1} = \arg\min_{j\notin\{k_1,\ldots,k_i\}} \mathrm{SINR}_j. \tag{26}$$

This modified V-BLAST detection is denoted as constrained DFD (CDFD). Note that if $\mathbf{\Lambda} = \sigma_n^2\mathbf{I}_n$, CDFD reduces to MMSE-DFD in [3].

### D. Combined constrained linear and decision feedback detectors

The performance of ZF-DFD is limited by the error propagation of decision feedback. The diversity order of V-BLAST detection is only one since the V-BLAST detection is a greedy algorithm. It makes a hard decision only on the "local" metric (20a) without taking into account its effect on the detection for subsequent symbols. We thus propose to combine the constrained linear detectors in Section III and ZF-DFD to let the detector make hard decisions less greedily. At each iteration, a "global" metric is used to make the decision on each symbol, which is obtained by the constrained linear detectors.

In the $i$-th iteration, we define $\mathbf{R}_i = \mathbf{R}(1:i-1, 1:i-1)$, $\mathbf{r}_i = \mathbf{R}(1:i-1, i)$ and $\mathbf{y}_i = \mathbf{y}(1:i-1)$. For detecting the $i$-th symbols, after cancelling $x \in \mathcal{Q}$ from $\mathbf{y}$, the soft decisions for the remaining $n-i$ symbols can be obtained using the constrained linear detectors as

$$\hat{\mathbf{x}}_i = \left(\mathbf{R}_i^H\mathbf{R}_i + \mathbf{\Lambda}_i\right)^{-1}\mathbf{R}_i^H(\mathbf{y}_i - \mathbf{r}_i x) \tag{27}$$

where $\mathbf{x}_i = [x_1, \ldots, x_{i-1}]^T$ and $\mathbf{\Lambda}_i$ can be obtained similarly to (11) and (15). Since the solutions to (7) or (13) give a low bound on $\|\mathbf{r} - \mathbf{H}\mathbf{x}\|^2$, the effect of $x$ on the decision

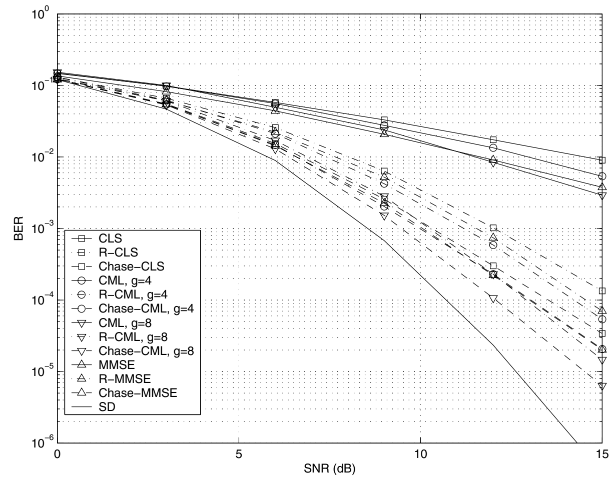


Fig. 1. Performance comparison of constrained linear detectors in an $8 \times 8$ MIMO system with BPSK.

metric for the remaining $n-i$ symbols can be measured using $\|\mathbf{y}_i - \mathbf{r}_i x - \mathbf{R}_i\hat{\mathbf{x}}_i\|^2$. The global metric for $x$ is defined as

$$\begin{aligned} M_i(x) = &\left\|(\mathbf{I}_{n-i} - \mathbf{R}_i(\mathbf{R}_i^H\mathbf{R}_i + \mathbf{\Lambda}_i)^{-1}\mathbf{R}_i^H)(\mathbf{y}_i - \mathbf{r}_i x)\right\|^2 \\ &+ |y_i - R_{i,i}x|^2 \end{aligned}. \tag{28}$$

In ZF-DFD, (20a) is simply replaced by

$$\hat{x}_i = \arg\min_{x\in\mathcal{Q}} M_i(x). \tag{29}$$

The resulting detector is denoted by CL-DFD. Though it does not give a low bound on $\|\mathbf{y}_i - \mathbf{r}_i x - \mathbf{R}_i\hat{\mathbf{x}}_i\|^2$, the metric (28) also measures the effect of $x$ on the overall metric. Combined MMSE and DFD (CMMSE-DFD) also improves the performance.

## V. Simulation results

Our proposed constrained detectors are simulated for a MIMO system with 8 transmit and 8 receive antennas. We assume the receiver has perfect channel state information (CSI) and noise variance. We use the notation Chase-X to denote the combination of the detector X and the iterative correction in Section III.D.

Fig. 1 shows the BER performance of different constrained linear detectors in a BPSK modulated system. We compare our detectors with sphere decoding [4] and the CLS detector [6]. When all of the linear detectors are applied to the complex system (1), the CLS and CML perform close to MMSE. In high SNR, CML with $g = 8$ performs better than MMSE. But the performance of all of them is inferior to the ML performance achieved by SD. When the detectors are applied to the real system (3), the performance of all the detectors improves. At BER=$10^{-3}$, R-MMSE has a 0.5-dB gain over R-CLS. Both R-CML with $g = 4$ and $g = 8$ perform better than R-MMSE. They have a 0.3 dB and 2 dB gain over R-MMSE, respectively. When employing the iterative improvement with all of the detectors, we find that R-MMSE, R-CLS and R-CML with $g = 4$ have 2 dB, 1.8 dB and 1.5 dB gains at

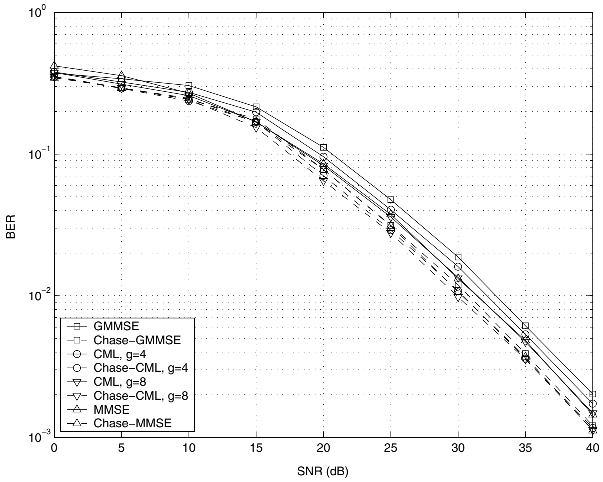


Fig. 2. Performance comparison of constrained linear detectors in an $8 \times 8$ MIMO system with 16QAM.

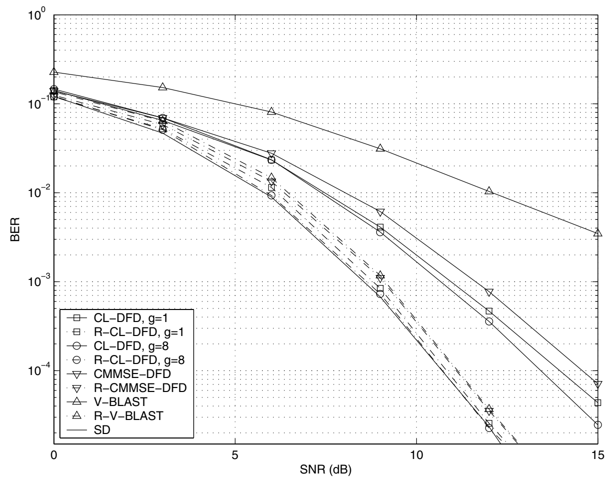


Fig. 3. Performance comparison of combined linear and decision feedback detectors in an $8 \times 8$ BPSK MIMO system.

BER=$10^{-3}$. The detector R-CML with $g = 8$ improves by 1 dB at BER=$10^{-4}$.

The BER of GMMSE [5] and different constrained linear detectors for 16QAM is shown in Fig. 2. GMMSE performs the worst among all the detectors. CML with $g = 4$ has a 0.8 dB loss over MMSE at BER=$10^{-3}$. In low SNR, CML with $g = 8$ performs better than MMSE, but they perform identically in high SNR. With the Chase iterative improvement, R-MMSE, R-CLS, R-CML with $g = 4$ and R-CML with $g = 8$ have 2 dB, 1.8 dB 1 dB and 1.2 dB gains at BER=$10^{-2}$, respectively. Since the group-wise hypercube constraint (13) is loose, the resulting performance improvement is marginal. Tighter constraints are needed for high order QAM constellations.

Finally, we present the results for combined linear and decision feedback detectors in a BPSK system (Fig. 3). Clearly, our proposed CL-DFDs significantly improve the performance, indicating their ability to mitigate error propagation. At BER=$10^{-2}$, the CL-DFD with $g = 1$ has a 4-dB gain over V-BLAST. The CL-DFD with $g = 1$ performs worse than the CMMSE-DFD. But the CL-DFD with $g = 8$ has better performance than the CMMSE-DFD. At BER=$10^{-4}$, the CL-DFD with $g = 8$ performs 0.5 dB better than the CMMSE-DFD. When the real constraint is applied, all the detectors perform close to SD at high SNR. R-CL-DFD with $g = 8$ can almost achieve the ML performance. However, the performance gain using our R-CL-DFDs decreases compared to the complex case. For non-constant modulus constellations, the performance gain by using CL-DFD and MMSE-DFD is also significant. Results of constrained ordering schemes are omitted for brevity.

## VI. Conclusion

In this paper, we have proposed a class of constrained linear detectors and a class of constrained decision feedback detectors. For real constellations, real constrained detectors are proposed to exploit the real-valued property of the constellations. The diversity order increases to $(n + 1)/2$ in a real system. The previous CLS detector for OFDM/SDMA and the GMMSE detector for CDMA were generalized as MIMO detectors for both constant and non-constant modulus constellations. A chase-principle based iterative technique has also been proposed to improve the performance of linear detectors. A constrained ordering scheme for DFDs has been derived to alleviate noise enhancement and to improve interference suppression. We also proposed combined linear and decision feedback detectors, where a global metric is defined to mitigate the error propagation. The complexity of these combined detectors is reasonably low.

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