Moment Analysis of the Equal Gain Combiner Output in Equally Correlated Fading Channels

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***Abstract*—Moments of the multibranch equal gain combiner (EGC) output signal-to-noise ratio (SNR) are only known for independent fading channels or exponentially correlated Nakagami-m fading channels. In this paper, we derive the moments of the EGC output SNR in equally correlated Rayleigh, Rician, and Nakagami-m fading channels. Our moment expressions can be used to evaluate the outage and the average error rate as well as purely moments-based measures such as the average output SNR and the amount of fading as functions of the fading correlation. Numerical results that illustrate the effect of fading correlation on the distribution of the EGC output SNR are also provided.**

***Index Terms*—Amount of fading, diversity, equal gain combining (EGC), equally correlated fading, moments.**

I. Introduction

THE equal gain combining (EGC) scheme offers performance comparable to the optimal maximal ratio combining (MRC) scheme, but with greater simplicity [1]. However, it is notoriously difficult to analyze the performance of EGC. Even for independent fading, a closed-form solution to the probability density function (pdf) of the EGC output signal-to-noise ratio (SNR) is only known for dual-branch EGC in Rayleigh fading [2]. Different methods have, therefore, been developed to analyze the EGC performance. Brennan [3] numerically evaluates the distribution of the EGC output SNR. Beaulieu and Abu-Dayya [2], [4], [5] apply an accurate approximation to the pdf of a sum of independent random variables (RVs). Annamalai *et al.* [6]–[8] use the Parseval theorem to transform the problem to the frequency domain. Zhang [9], [10] provides several closed form solutions to the average bit error rate (BER) using the characteristic function (chf) of the decision variable. While progress has thus been made on analyzing EGC performance in independent fading, little is known about correlated fading.

However, in real-life applications, fading is correlated among diversity branches, and hence characterizing the resultant EGC performance loss is of both theoretical and practical importance. Since the joint pdf of the multiple correlated fading gains is not known, many published results deal with the dual-branch case [11]–[16]. For instance, [13] and [14] derive the average error rate via the chf of the EGC output SNR. An infinite series expression for the symbol error rate (SER) of quadrature amplitude modulation (QAM) in terms of the Appell hypergeometric functions is derived in [15]. Dual predetection EGC systems in correlated Nakagami-m fading channels are analyged. As a result, only several papers address multibranch $(L > 2)$ EGC in correlated fading. Karagiannidis *et al.* [17] derive the moments of the EGC output SNR in exponentially correlated Nakagami-m fading channels. Combining this result with the Pade approximation, [18] approximates the moment generating function (mgf) of the EGC output SNR in correlated Nakagami-m fading channels. Chen and Tellambura [19], [20] develop a new approach for performance analysis of diversity combiners in equally correlated fading channels. They transform a set of equally correlated Rayleigh RVs into a set of conditionally independent Rician RVs using a novel representation of the channel gains. The average error rate and the outage performance of multibranch EGC have been evaluated using this approach [20].

Clearly, it is difficult to derive analytical expressions for the pdf or mgf of the EGC output SNR in correlated fading. In this paper, we thus derive the moments of the EGC output SNR in equally correlated Rayleigh, Rician, and Nakagami-m fading channels. The EGC output moments are important for several reasons. First, purely moment-based measures, such as the average output SNR and the amount of fading are commonly used to characterize the diversity system; for example, higher order moments are derived for certain diversity systems [21]. Second, more widely used performance measures such as the average bit error rate (which is appropriate for digital modulations) and the outage probability, are typically computed using the pdf or the mgf (which are not available in this case). However, these measures can also be evaluated directly via the moments, using the Edgeworth series approach and Gaussian quadrature techniques.

We derive the EGC moments using the approach of [20]. The moments are expressed using the Appell hypergeometric functions, and certain transformation formulas [22] are used to ensure their convergence. We also show how the average error rates and the outage probability can be directly evaluated using the moments without the output mgf. For brevity, we only provide numerical results to show the correlation effect on the EGC output SNR distribution.

This paper is organized as follows. Section II derives the moments of the EGC output SNR in equally correlated Rayleigh, Rician and Nakagami-m fading channels. Section III evaluates the outage and the average error rate performance of EGC using the moments of its output. Some moments-based performance

Manuscript received August 27, 2004; revised February 10, 2005 and June 9, 2005. This work was supported by The National Sciences and Engineering Research Council (NSERC) and the Alberta Informatics Circle of Research Excellence (iCORE), Canada. The review of this paper was coordinated by Dr. C. Tepedelenlioglu.

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Digital Object Identifier 10.1109/TVT.2005.858181

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measures, such as the amount of fading are also derived. Numerical results in Section IV illustrate the effect of fading correlation on the distribution of the EGC output SNR. Section V concludes this paper.

II. Moments of the EGC Output SNR

Consider a diversity combining system with L input branches. The received signal at the kth branch can be written as

$$y_k = g_k s + n_k, \quad k \in \{1, \ldots, L\} \tag{1}$$

where g_k is the random channel gain associated with the kth branch, s is the transmitted signal with energy E_s, and n_k is the additive white Gaussian noise (AWGN) with power spectral density $(N_0/2)$ per dimension. We assume that the noise components are independent of the channel gains, i.e., $E(n_j G_k^*) = 0$ for any $j, k \in \{1, \ldots, L\}$, where x^* and $E(x)$ denote the complex conjugate and the average of x and are uncorrelated with each other, i.e., $E(n_j n_k^*) = N_0 \delta_{jk}$, where δ_{jk} is the Kronecker delta function defined as $\delta_{jj} = 1$, and $\delta_{jk} = 0$ for $j \neq k$. Throughout this paper, all RVs are denoted by uppercase letters, and their realizations are denoted by lowercase letters.

In EGC, the received signals are cophased and equally weighted and then summed to form the resultant output. The instantaneous SNR at the EGC output can be written as [23]

$$\gamma_{\text{egc}} = \frac{(r_1 + r_2 + \cdots + r_L)^2 E_s}{L N_0} \tag{2}$$

where $R_k = |G_k|$ is the amplitude of the channel gain G_k. We assume that the channel gains are equally correlated. That is, the correlation (fading correlation) between any G_k and $G_j (j \neq k)$ is a constant, and all of the channel gains G_k have the same average power; i.e., $E(|G_k|^2) = E(|G_j|^2)$ for any $k, j \in \{1, \ldots, L\}$.

Using the multinomial expansion, we obtain the moments of the EGC output SNR as [18]

$$m_n = E\left(\gamma_{\text{egc}}^n\right) = (2n)\left[\frac{\bar{\gamma}_c}{L E(R_k^2)}\right]^n \times \sum_{\substack{k_1, \ldots, k_L = 0 \\ k_1 + \cdots + k_L = 2n}}^{2n} \frac{1}{\prod_{j=1}^L k_j} E\left(\prod_{j=1}^L R_j^{k_j}\right) \tag{3}$$

where $\bar{\gamma}_c$ is the average branch SNR given by

$$\bar{\gamma}_c = \frac{E_s}{N_0} E(R_k^2). \tag{4}$$

When the channel gain envelopes R_k are independent, (3) can be readily evaluated as

$$m_n = (2n)\left[\frac{\bar{\gamma}_c}{L E(R_k^2)}\right]^n \sum_{\substack{k_1, \ldots, k_L = 0 \\ k_1 + \cdots + k_L = 2n}}^{2n} \prod_{j=1}^L \left[\frac{E(R_j^{k_j})}{k_j}\right]. \tag{5}$$

However, it is an extremely complicated task to compute (3) when the channel gains are correlated and when the corresponding joint pdf is unknown.

In this section, we use the representations for the channel gains developed in [23] to derive the moments of the EGC output SNR in three types of equally correlated fading channels. The equally correlated model presents a worst case among correlated fading channels. It can also be used to describe the correlation among closely placed antennas [24].

A. Rayleigh Fading Channels

The Rayleigh envelopes can be represented by the amplitudes of a set of zero-mean complex Gaussian RVs $\{G_k\}_{k=1}^L$ given by [20]

$$G_k = (\sqrt{1-\rho} X_k + \sqrt{\rho} X_0) + i(\sqrt{1-\rho} Y_k + \sqrt{\rho} Y_0) \tag{6}$$

where $i = \sqrt{-1}, 0 \leq \rho < 1, \{X_k\}_{k=0}^L$ and $\{Y_k\}_{k=0}^L$ are two sets of independent zero-mean Gaussian RVs with variance $E(|X_k|^2) = E(|Y_k|^2) = (1/2)$. Since $\{G_k\}$ is a set of complex Gaussian RVs with zero means, $R_k = |G_k|$ is a set of Rayleigh envelopes with mean-square value

$$E(R_k^2) = 1. \tag{7}$$

The cross-correlation coefficient between any G_k and $G_j (k \neq j)$ equals to ρ [26]. Thus, we can readily represent a set of equally correlated Rayleigh envelopes using.

We consider X_0 and Y_0 to be fixed $(X_0^2 + Y_0^2 = U)$. Then, $\{R_k\}_{k=1}^L$ is a set of independent Rician RVs with the Rice factor and the average power given, respectively, by

$$K_f = \frac{\rho u}{1-\rho} \tag{8a}$$

$$\Omega_f = 1 - \rho + \rho u. \tag{8b}$$

The moments of a Rician RV are given by [25, Eq. (2-1-146)]

$$E(R_j^{k_j}) = (1-\rho)^{\frac{k_j}{2}} \Gamma\left(1 + \frac{k_j}{2}\right) {}_1F_1\left(-\frac{k_j}{2}, 1; -\frac{\rho u}{1-\rho}\right) \tag{9}$$

where $\Gamma(x)$ is the gamma function, and ${}_1F_1(a, c; z)$ is the confluent hypergeometric function defined as [26, Eq. (9.210.1)]

$${}_1F_1(a, c; z) = \sum_{n=0}^{\infty} \frac{(a)_n}{(c)_n n} z^n \tag{10}$$

where $(a)_n = (\Gamma(a+n)/\Gamma(a))$, and $(a)_0 = 1$.

Substituting (7) and (9) into (5), we obtain the moments of the EGC output SNR for a given $x_0^2 + y_0^2 = u$ as

$$m_n(u) = (2n)\left[\frac{\bar{\gamma}_c(1-\rho)}{L}\right]^n \times \sum_{\substack{k_1, \ldots, k_L = 0 \\ k_1 + \cdots + k_L = 2n}}^{2n} \prod_{j=1}^{L} \left] A(k_j) {}_1F_1\left(-\frac{k_j}{2}, 1; -\frac{\rho u}{1-\rho}\right)\right] \tag{11}$$

where $\bar{\gamma}_c = (E_s/N_0)$, and

$$A(k_j) = \frac{\Gamma\left(1 + \frac{k_j}{2}\right)}{k_j}. \tag{12}$$

Notice that $X_0^2+Y_0^2=U$ is chi-square distributed with two degrees of freedom, and its pdf can be found as [25]

$$p_U(u)=e^{-u}, \quad u\geq 0. \tag{13}$$

Averaging the conditional moments (11) over the distribution of U (13), we obtain the moments of the multibranch EGC output SNR in equally correlated Rayleigh fading channels as

$$\begin{aligned} m_n &= \int_0^\infty m_n(u)p_U(u)du \\ &= (2n)\left[\frac{\bar{\gamma}_c(1-\rho)}{L}\right]^n \\ &\quad \times \sum_{\substack{k_1,\dots,k_L=0\\ k_1+\cdots+k_L=2n}}^{2n}\left[I(k_1,\dots,k_L)\prod_{j=1}^{L}A(k_j)\right] \end{aligned} \tag{14}$$

where $I(k_1,\dots,k_L)$ is a single-fold integral which can be derived in terms of the Lth-order Appell hypergeometric function with the aid of [8, Eq. (C.1)]:

$$\begin{aligned} &I(k_1,\dots,k_L) \\ &\quad = \int_0^\infty e^{-u}\prod_{j=1}^{L}{}_1F_1\left(-\frac{k_j}{2},1;-\frac{\rho u}{1-\rho}\right)du \\ &\quad = F_A\left(1;-\frac{k_1}{2},\dots,-\frac{k_L}{2};1,\dots,1;x_1,\dots,x_L\right) \end{aligned} \tag{15}$$

where $x_k=-(\rho/1-\rho)$ for $k\in\{1,\dots,L\}$, and $F_A(\alpha;\beta_1,\dots,\beta_n;\gamma_1,\dots,\gamma_n;x_1,\dots,x_n)$ is the nth-order Appell hypergeometric function defined as [26, (9.180.2)]

$$\begin{aligned} &F_A(\alpha;\beta_1,\dots,\beta_n;\gamma_1,\dots,\gamma_n;x_1,\dots,x_n) \\ &\quad = \sum_{p_1=0}^{\infty}\cdots\sum_{p_n=0}^{\infty}\frac{(\alpha)_{p_1+\cdots+p_n}(\beta_1)_{p_1}\cdots(\beta_n)_{p_n}}{(\gamma_1)_{p_1}\cdots(\gamma_n)_{p_n}p_1\cdots p_n}x_1^{p_1}\cdots x_n^{p_n} \end{aligned} \tag{16}$$

whose convergence region is defined by $|x_1|+\ldots+|x_L|<1$ [22]. If all β_j's are either zeros or negative integers in (16), then the Appell hypergeometric function reduces to a finite series

$$\begin{aligned} &F_A(\alpha;\beta_1,\dots,\beta_n;\gamma_1,\cdots,\gamma_n;x_1,\cdots,x_n) \\ &\quad = \sum_{p_1=0}^{-\beta_1}\cdots\sum_{p_n=0}^{-\beta_n}\frac{(\alpha)_{p_1+\cdots+p_n}(\beta_1)_{p_1}\cdots(\beta_n)_{p_n}}{(\gamma_1)_{p_1}\cdots(\gamma_n)_{p_n}p_1\cdots p_n}x_1^{p_1}\cdots x_n^{p_n}. \end{aligned} \tag{17}$$

Notice that (15) only converges for $-(1/L-1)<\rho<(1/L+1)$. Hence a transformation formula for the higher transcendental function F_A [22] must be used to obtain a converge expression for (15) as

$$\begin{aligned} &I(k_1,\dots,k_L) \\ &\quad = \frac{1-\rho}{1+(L-1)\rho}F_A(1;\beta_1,\dots,\beta_L;1,\dots,1;z_1,\dots,z_L) \end{aligned} \tag{18}$$

where $\beta_j=1+(k_j/2)$, and $z_j=(\rho/1+(L-1))\rho$ for $j\in\{1,\dots,L\}$. Since $0\leq\rho<1, |z_1|+\ldots+|z_L|=(L\rho/L\rho+(1-\rho))<1$, i.e., the Appell hypergeometric function F_A in (18) converges. Therefore, the moments of the EGC output SNR in an equally correlated Rayleigh fading channel can be calculated using (18) and (12) with (14).

Next we present some special cases of (14).

1) Average Output SNR: Using (17) and the relation between the second order Appell hypergeometric function and the Gauss hypergeometric function [8, Eq. (C.4)], we can readily simplify the average output SNR of EGC in equally correlated Rayleigh fading channel as [18]

$$m_1=\bar{\gamma}_c\left[1+\frac{(L-1)\pi}{4}{}_2F_1\left(-\frac{1}{2},-\frac{1}{2};1;\rho^2\right)\right] \tag{19}$$

where ${}_2F_1(a,b;c;z)$ is the Gauss hypergeometric function defined in [26, Eq. (9-100)]

$${}_2F_1(a,b;c;z)=\sum_{n=0}^{\infty}\frac{(a)_n(b)_n}{(c)_n\, n}z^n. \tag{20}$$

When $\rho=0$, (19) reduces to

$$m_1=\bar{\gamma}_c\left[1+\frac{(L-1)\pi}{4}\right] \tag{21}$$

which is the well-known result [27, Eq. (5.2-20)].

2) Second Moment of the Output SNR: We can also show that the second moment of the EGC output SNR is given by (see the Appendix)

$$\begin{aligned} m_2 = \frac{\bar{\gamma}_c^2}{L}\Bigg\{&2+3(L-1)(1+\rho^2) \\ &+\frac{3\pi(L-1)}{2}{}_2F_1\left(-\frac{3}{2},-\frac{1}{2};1;\rho^2\right) \\ &+\frac{3\pi(L-1)(L-2)}{2}\Bigg[(1-\rho){}_2F_1\left(-\frac{1}{2},-\frac{1}{2};1;\rho^2\right) \\ &+\frac{9\rho}{4}{}_2F_1\left(-\frac{1}{2},-\frac{1}{2};2;\rho^2\right) \\ &-\frac{3\rho(1-\rho)}{2}{}_2F_1\left(-\frac{3}{2},\frac{1}{2};2;\rho^2\right) \\ &+\frac{\rho(1-\rho)^2}{4}{}_2F_1\left(\frac{1}{2},\frac{1}{2};2;\rho^2\right)\Bigg] \\ &+\frac{\pi^2(L-1)(L-2)(L-3)(1-\rho)^3}{16(1-\rho+L\rho)} \\ &\times F_A\left(1;\frac{3}{2},\dots,\frac{3}{2};1,\dots,1;z_1,\dots,z_L\right)\Bigg\} \end{aligned} \tag{22}$$

where $z_k=(\rho/1+(L-1)\rho)$ for any $k\in\{1,\dots,L\}$.

For dual-branch and triple-branch EGC, the second moment of the EGC output SNR (22) can be simplified as

$$m_2=\bar{\gamma}_c^2\left[\frac{5}{2}+\frac{3}{2}\rho^2+3\pi 4{}_2F_1\left(-\frac{3}{2},-\frac{1}{2};1;\rho^2\right)\right] \tag{23}$$

and

$$\begin{aligned} m_2 = \bar{\gamma}_c^2 \Bigg\{ \frac{8}{3} + 2\rho^2 + \pi \Bigg[{}_2F_1\left(-\frac{3}{2}, -\frac{1}{2}; 1; \rho^2\right) \\ +(1-\rho){}_2F_1\left(-\frac{1}{2}, -\frac{1}{2}; 1; \rho^2\right)\Bigg] \\ + \pi\rho \Bigg[\frac{9}{4} {}_2F_1\left(-\frac{1}{2}, -\frac{1}{2}; 2; \rho^2\right) \\ - \frac{3(1-\rho)}{2} {}_2F_1\left(-\frac{1}{2}, \frac{1}{2}; 2; \rho^2\right) \\ + \frac{(1-\rho)^2}{4} {}_2F_1\left(\frac{1}{2}, \frac{1}{2}; 2; \rho^2\right)\Bigg]\Bigg\} \end{aligned} \tag{24}$$

respectively. It can be shown that (23) is a special case of [17, Eq.(14)].

3) Independent Fading: In the independent case ($\rho = 0$), (14) reduces to

$$m_n = \left(\frac{\bar{\gamma}_c}{L}\right)^n (2n) \sum_{\substack{k_1,\ldots,k_L=0 \\ k_1+\cdots+k_L=2n}}^{2n} \left[\prod_{j=1}^{L} A(k_j)\right]. \tag{25}$$

4) Dual-Branch EGC: The moments of the dual-branch EGC output SNR can be written in terms of the Gauss hypergeometric function as

$$\begin{aligned} m_n = \left(\frac{\bar{\gamma}_c}{2}\right)(2n) \sum_{k=0}^{2n} \frac{\Gamma\left(1+\frac{k}{2}\right)\Gamma\left(1+n-\frac{k}{2}\right)}{k(2n-k)} \\ \times {}_2F_1\left(-\frac{k}{2}, -\frac{2n-k}{2}; 1; \rho^2\right). \end{aligned} \tag{26}$$

As a comparison, we present the moments of the MRC output SNR in equally correlated Rayleigh fading channels here. The pdf of the MRC output SNR in such channels is given by [24, Eq. (12)]. Hence, the output moments are obtained as

$$\begin{aligned} E\left(\gamma_{\text{mrc}}^n\right) = \bar{\gamma}_c^n \frac{(1-\rho)^{1+n}\Gamma(L+n)}{(1-\rho+L\rho)\Gamma(L)} \\ \times {}_2F_1\left(1, L+n; L; \frac{L\rho}{1-\rho+L\rho}\right). \end{aligned} \tag{27}$$

B. Rician Fading Channels

The equally correlated Rician channel gains can be associated with a set of nonzero mean complex Gaussian RVs $\{G_k\}_{k=1}^L$:

$$\begin{aligned} G_k = (\sqrt{1-\rho}X_k + \sqrt{\rho}X_0 + M_1) \\ + i(\sqrt{1-\rho}Y_k + \sqrt{\rho}Y_0 + M_2) \end{aligned} \tag{28}$$

where $\{X_k\}_{k=0}^L$ and $\{Y_k\}_{k=0}^L$, are two sets of independent zero-mean Gaussian RVs with identical variance $E(|X_k|^2) = E(|Y_k|^2) = (1/2)$, and M_1 and M_2 are the light-of-sight (LOS) components. It has been shown that $R_k = |G_k|$ is Rician distributed with the Rician factor $K = M_1^2 + M_2^2$, and the mean-square value

$$E(R_k^2) = 1 + K. \tag{29}$$

Also the fading correlation between any G_k and $G_j (k \neq j)$ is shown to be equal to the constant ρ.

Fix $(X_0 + (M_1/\sqrt{\rho}))^2 + (Y_0 + (M_2/\sqrt{\rho}))^2 = U$ in (28), then R_k is independent Rician distributed with the Rice factor and the average power given by (8). Then, the Rician moments (9) can be used with (29) and (5) to derive the output moments of EGC for a given u:

$$\begin{aligned} m_n(u) = (2n)\left[\frac{\bar{\gamma}_c(1-\rho)}{L(1+K)}\right]^n \\ \times \sum_{\substack{k_1,\ldots,k_L=0 \\ k_1+\cdots+k_L=2n}}^{2n} \left[\prod_{j=1}^{L} A(k_j) {}_1F_1\left(-\frac{k_j}{2}, 1; -\frac{\rho u}{1-\rho}\right)\right]. \end{aligned} \tag{30}$$

Also, note that U is noncentral chi-square distributed with two degrees of freedom and noncentrality parameter (K/ρ), and its pdf is given by [25]

$$p_U(u) = e^{-\left(u+\frac{K}{\rho}\right)} I_0\left(2\sqrt{\frac{Ku}{\rho}}\right), \quad u \geq 0 \tag{31}$$

where $I_0(x)$ is zeroth-order modified Bessel function of the first kind defined as

$$I_0(x) = \frac{1}{\pi}\int_0^\pi e^{x\cos\theta} d\theta. \tag{32}$$

Thus, averaging the conditional output moments (30) over the distribution of U (31), we obtain the moments of the EGC output SNR in equally correlated Rician fading channels as

$$\begin{aligned} m_n = (2n)\left[\frac{\bar{\gamma}_c(1-\rho)}{(1+K)L}\right]^n \\ \times \sum_{\substack{k_1,\ldots,k_L=0 \\ k_1+\cdots+k_L=2n}}^{2n} \left[J(k_1,\ldots,k_L)\prod_{j=1}^{L} A(k_j)\right] \end{aligned} \tag{33}$$

where $A(k_j)$ is defined as (12), and $J(k_1, \ldots, k_L)$ is a single-fold integral

$$\begin{aligned} J(k_1,\ldots,k_L) = e^{-\frac{K}{\rho}} \int_0^\infty e^{-u} I_0\left(2\sqrt{\frac{Ku}{\rho}}\right) \\ \times \prod_{j=1}^{L} {}_1F_1\left(-\frac{k_j}{2}, 1; -\frac{\rho u}{1-\rho}\right) du. \end{aligned} \tag{34}$$

Using an infinite series expression [28, Eq. (9.6.10)] for $I_0(x)$ and applying the integral identity [8, Eq. (C.1)] and the transformation formula for F_A [22], we may express (34) in terms of the L-th order Appell hypergeometric function as

$$\begin{aligned} J(k_1,\ldots,k_L) \\ = e^{-\frac{K}{\rho}} \sum_{l=0}^\infty \frac{1}{l}\left(\frac{K}{\rho}\right)^l \left(\frac{1-\rho}{1+(L-1)\rho}\right)^{l+1} \\ \times F_A\left(l+1; \beta_1, \cdots, \beta_L; 1, \ldots, 1; z_1, \ldots, z_L\right) \end{aligned} \tag{35}$$

where $\beta_j = 1 + (k_j/2)$, and $z_j = (\rho/1 + (L-1)\rho)$ for $j \in \{1, \ldots, L\}$. The moments of the L-branch EGC output SNR in equally correlated Rician fading channels can thus be computed using (35) and (12) with (33).

The average output SNR of EGC in equally correlated Rician fading channels can be simplified in terms of the hypergeometric function of two variables, i.e., the second-order Appell hypergeometric function, as

$$m_1 = \bar{\gamma}_c \left[1 + e^{-\frac{K}{\rho}} \frac{(1-\rho)^2(L-1)\pi}{4(1+\rho)(1+K)} \sum_{j=0}^{\infty} \frac{1}{j} \left(\frac{K(1-\rho)}{\rho(1+\rho)} \right)^j \times F_A\left(j+1; \frac{3}{2}, \frac{3}{2}; 1, 1; \frac{\rho}{1+\rho}, \frac{\rho}{1+\rho} \right) \right]. \quad (36)$$

Notice that the second-order Appell hypergeometric function is available in common mathematical software such as Maple and Mathematica.

C. Nakagami-m Fading Channels

When m is an integer, the equally correlated Nakagami-m envelopes can also be represented using a set of complex Gaussian RVs

$$G_{kj} = (\sqrt{1-\rho}X_{kj} + \sqrt{\rho}X_{0j}) + i(\sqrt{1-\rho}Y_{kj} + \sqrt{\rho}Y_{0j}) \quad (37)$$

for $k \in \{1, \ldots, L\}$ and $j \in \{1, \ldots, m\}$, where $\{X_{kj}\}$ and $\{Y_{kj}\}, k \in \{0, 1, \ldots, L\}$ and $j \in \{1, \ldots, m\}$ are two sets of independent zero-mean Gaussian RVs with identical variances of $(1/2)$. It has been shown that $R_k = \sqrt{\sum_{j=1}^{m} |G_{kj}|^2}$ is equally correlated Nakagami-m distributed with mean-square value

$$E(R_k^2) = m. \quad (38)$$

The correlation between any R_k^2 and $R_j^2 (k \neq j)$ is equal to ρ^2.

Now fix $X_{0j} = x_{0j}$ and $Y_{0j} = y_{0j}, (j \in \{1, \ldots, m\})$ and let $u = \sum_{j=1}^{m}(x_{0j}^2 + y_{0j}^2)$, then R_k^2 is noncentral chi-square distributed with $2m$ degrees of freedom and noncentrality parameter ρu. The moments of R_k can be found as [25, Eq. (2-1-146)]

$$E\left(R_i^{k_i}\right) = (1-\rho)^{\frac{k_i}{2}} \frac{\Gamma\left(m + \frac{k_i}{2}\right)}{\Gamma(m)} {}_1F_1\left(\frac{-k_i}{2}, m; \frac{-\rho u}{1-\rho} \right). \quad (39)$$

Substituting (38) and (39) into (5), we obtain the output moments of EGC conditioned on u as

$$m_n(u) = (2n) \left[\frac{\bar{\gamma}_c(1-\rho)}{mL} \right]^n \times \sum_{\substack{k_1, \ldots, k_L = 0 \\ k_1 + \cdots + k_L = 2n}}^{2n} \left[\prod_{j=1}^{L} B(k_j) {}_1F_1\left(-\frac{k_j}{2}, m; -\frac{\rho u}{1-\rho} \right) \right] \quad (40)$$

where

$$B(k_j) = \frac{\Gamma\left(m + \frac{k_j}{2}\right)}{(m-1)k_j}. \quad (41)$$

Also notice that U is chi-square distributed with $2m$ degrees of freedom, and its pdf is given by [25]

$$p_U(u) = \frac{1}{\Gamma(m)} u^{m-1} e^{-u}, \quad u \geq 0. \quad (42)$$

Averaging the conditional output moments (40) over the distribution of U (42), we can obtain the moments of the EGC output SNR in equally correlated Nakagami-m fading channels as

$$m_n = (2n) \left[\frac{\bar{\gamma}_c(1-\rho)}{mL} \right]^n \times \sum_{\substack{k_1, \ldots, k_L = 0 \\ k_1 + \cdots + k_L = 2n}}^{2n} \left[K(k_1, \ldots, k_L) \prod_{i=1}^{L} B(k_j) \right] \quad (43)$$

where $K(k_1, \ldots, k_L)$ can also be expressed in terms of the Lth order Appell hypergeometric function as

$$\begin{aligned} K(k_1, \ldots, k_L) &= \frac{1}{(m-1)} \int_0^\infty u^{m-1} e^{-u} \prod_{j=1}^{L} {}_1F_1\left(-\frac{k_j}{2}, m; -\frac{\rho u}{1-\rho} \right) du \\ &= \frac{1}{(m-1)} \left[\frac{1-\rho}{1+(L-1)\rho} \right]^m \times F_A(m; \beta_1, \ldots, \beta_L; m, \ldots, m; z_1, \ldots, z_L) \end{aligned} \quad (44)$$

where $\beta_j = m + (k_j/2)$, and $z_j = (\rho/1 + (L-1)\rho)$ for $j \in \{1, \ldots, L\}$. Combining (44) with (41) and (43), we can readily compute the moments of the L-branch EGC output SNR in equally correlated Nakagami-m fading channels. For the dual-branch case, (43) reduces to the previously-derived average output SNR of EGC [18]

$$m_1 = \bar{\gamma}_c \left[1 + \frac{(L-1)\Gamma^2\left(m + \frac{1}{2}\right)}{m[(m-1)]^2} {}_2F_1\left(-\frac{1}{2}, -\frac{1}{2}; m; \rho^2 \right) \right]. \quad (45)$$

III. Performance Measures

The moments of the EGC output SNR derived in Section II can be further used to evaluate the average error rate and the outage performance of multibranch EGC in equally correlated fading channels. In this section, we derive the output cdf and the BER of a wide class of digital modulations with EGC in terms of the output moments. The cdf or the outage probability can therefore be expressed as an Edgeworth series related to the moments, and the average error rate performance of EGC can be evaluated using a Gaussian quadrature approximation. Such moment-based methods circumvent the need for pdf or mgf expressions, which are not available for the EGC problem. Moment-based statistical measures, such as skewness, kurtosis and Pearson's coefficient of variation are also evaluated.

A. Cdf of the EGC Output SNR

It can be shown that the cumulants κ_ks of the EGC output SNR $\gamma_{\rm egc}$ can be evaluated in terms of the output moments:

$$\kappa_k = m_k - \sum_{t=1}^{k-1} \binom{k-1}{t-1} \kappa_t m_{k-t}. \tag{46}$$

Using (46) and applying the Edgeworth series [29], we express the cdf of the EGC output SNR in

$$\begin{aligned} F_{\rm egc}(x) = 1 &- Q(u) - c_3\phi^{(2)}(u) \\ &- \left[c_4\phi^{(3)}(u) + c_6\phi^{(5)}(u)\right] \\ &- \sum_{k=2}^{\infty} \Big[c_{3k-1}\phi^{(3k-2)}(u) \\ &+ c_{3k+1}\phi^{(3k)}(u) + c_{3k+3}\phi^{(3k+2)}(u)\Big] \end{aligned} \tag{47}$$

where $Q(x)$ is the upper-tail probability of a unit Gaussian pdf, and $u = (x - m_1)/\sigma, \sigma^2 = E[(\gamma_{out} - m_1)^2]$ is the variance of the EGC output SNR. $\phi^{(k)}(x)$ is the kth-order derivative of the error function $\phi^{(0)}(x)$ defined as

$$\phi^{(0)}(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}. \tag{48}$$

The coefficients c_k are given by

$$c_k = \frac{1}{\sigma^k} \sum_{\substack{t=3, r=1 \\ tr=k}}^{\infty} \frac{\kappa_t^r}{tr}, \quad k = 3, \ldots. \tag{49}$$

Equation (47) only requires the output moments and does not require the chf or the pdf of the output SNR. Notice that the coefficients c_k (49) generally decrease with increasing k, while $\phi^{(k)}(x)$ begins to increase rapidly after a certain value of k. Hence, (47) behaves like an asymptotic series, which converges up to a certain point and then diverges.

B. Average Error Rate Analysis

The average error rate of a certain digital modulation with EGC is often computed by averaging the corresponding conditional error probability (CEP) $g(\gamma)$ over the pdf of the EGC output SNR:

$$\bar{P}_e = E[g(\gamma)] = \int_0^{\infty} g(\gamma) p_{\rm egc}(\gamma) d\gamma \tag{50}$$

where $p_{\rm egc}(\gamma)$ is the pdf of the EGC output SNR. Since the output pdf of multibranch EGC ($L > 2$) is unknown for correlated fading channels, we shall evaluate the average error rates using the moments of the EGC output SNR.

By applying the Taylor's series expansion of $g(\gamma)$, we may express the average error rate as

$$\bar{P}_e = \sum_{k=0}^{\infty} E[(\gamma - \gamma_0)^k] \frac{g^{(k)}(\gamma_0)}{k} \tag{51}$$

where $g(\gamma)$ is analytic at $\gamma = \gamma_0$, and $E[(\gamma - \gamma_0)^k] = \sum_{t=0}^{k} \binom{k}{t} m_k (-\gamma_0)^{k-t}$ can be computed using the output moments m_k. For example, the CEP for binary differential phase-shift-keying (BDPSK) is given by

$$g(\gamma) = \frac{1}{2} e^{-\gamma}. \tag{52}$$

Hence, the average BER of BDPSK can be obtained as

$$\bar{P}_e = \frac{1}{2} \sum_{k=0}^{\infty} m_k \frac{(-1)^k}{k}. \tag{53}$$

In practice, due to roundoff errors, it is impossible to add up as many terms in (51) as are required to achieve a desired accuracy [30].

We thus use Gaussian quadrature approximation to compute the average error rates of a wide class of digital modulations with EGC in equally correlated fading channels. The average error rate can be approximated as a linear combination of values of the CEP $g(\gamma)$:

$$\bar{P}_e \approx \sum_{i=1}^{N} \omega_i g(x_i) \tag{54}$$

where the abscissas x_i and the weights ω_i can be computed by solving the following nonlinear equations as [30], [31]

$$m_k = \sum_{i=1}^{N} \omega_i x_i^k, \quad k = 0, 1, \ldots, 2N - 1. \tag{55}$$

C. Other Moment-Based Measures

The central moments, skewness, kurtosis, and the Karl Pearson's coefficient of variation are useful statistical measures, which are often used to characterize the output SNR of a diversity system [17].

The central moments of EGC output SNR can be obtained as

$$\eta_k = E[(\gamma_{\rm egc} - m_1)^k] = \sum_{n=0}^{k} \binom{k}{n} m_n (-m_1)^{k-n}. \tag{56}$$

Skewness is a measure of the asymmetry of the data around the sample mean, which is defined as $\nu = (\eta_3/\eta_2^{3/2})$ [32]. For symmetric distributions, $\nu = 0$. If $\nu > 0$, the data are spread out more to the right. If $\nu < 0$, the data are spread out more to the left.

Kurtosis is a measure of the "tail weight" of distribution, which is defined as $\kappa = (\eta_4/\eta_2^2)$ [32]. For normal distribution, $\kappa = 3$. The larger κ is, the greater the relative probability in one or both tails will be. Since the tail of the pdf mainly determines the BER and outage probability [17], the kurtosis can be used as an alternative performance measure.

Pearson's coefficient of variation of a RV X is frequently used to describe the severity of the channel fading. Charash [33] introduced the square value of the Pearson's coefficient as a unified measure of the amount of fading (AF)

$$\mathrm{AF} = \frac{\eta_2}{m_1^2} = \frac{m_2}{m_1^2} - 1. \tag{57}$$

Using (19) and (23), we may obtain the AF of EGC in equally correlated Rayleigh fading channels. For example, the AF of the

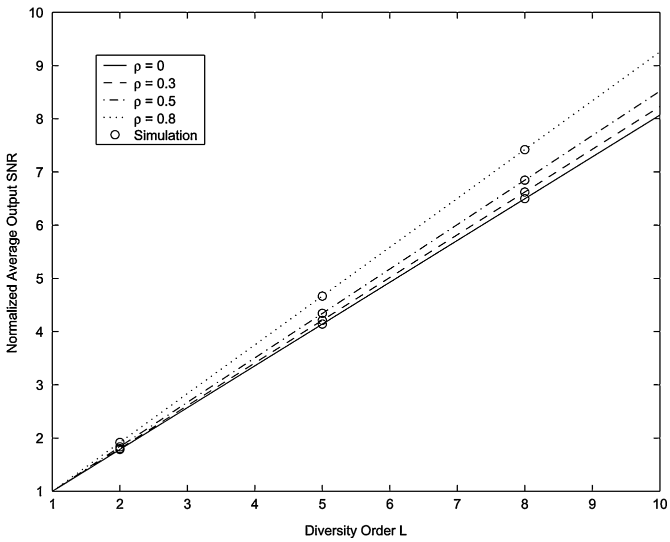

Fig. 1. Average output SNR of four-branch EGC in equally correlated Nakagami-m fading channels.

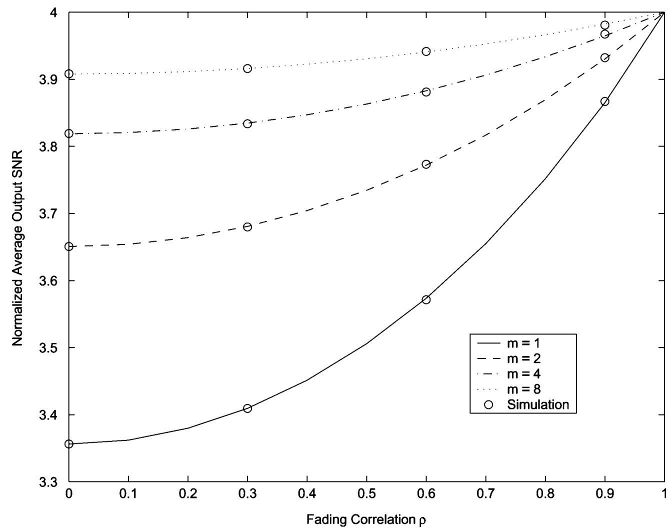

Fig. 2. Average output SNR of four-branch EGC in equally correlated Nakagami-m fading channels.

dual-branch EGC output is obtained as

$$\mathrm{AF} = \frac{\frac{5}{2} + \frac{3}{2}\rho^2 + \frac{3\pi}{4}{}_2F_1\left(-\frac{3}{2}, -\frac{1}{2}; 1; \rho^2\right)}{\left[1 + \frac{\pi}{4}{}_2F_1\left(-\frac{1}{2}, -\frac{1}{2}; 1; \rho^2\right)\right]^2} - 1. \tag{58}$$

IV. Numerical Results

We now present numerical results to show the correlation effect on the distribution of the EGC output SNR. Semi-analytical simulations are provided as an independent check of our analytical results.

Fig. 1 shows the normalized average output SNR $\hat{\gamma}_{\mathrm{egc}} = m_1/\bar{\gamma}_c$ of EGC versus the diversity order L in equally correlated Rayleigh fading channels. As the diversity order L increases, the average output SNR of EGC increases. Fig. 2 plots the normalized average output SNR of four-branch EGC versus the fading correlation ρ in equally correlated Nakagami-m fading channels. As the fading figure m increases, i.e., the channel fading condition improves, the average output SNR increases, and the fading correlation has less effect on the EGC output. This observation agrees with [17]. In both figures, we find that the average output SNR also increases as the fading correlation ρ increases. This has never happened in the MRC and SC performance [3]. This observation departs from our expectations. However, it can be explained by (3). Since $E(R_jR_k) > E(R_j)E(R_k), (j \neq k)$, for $\rho > 0$, the average output SNR of EGC in correlated fading channels is higher than that in independent fading channels [3]. In fact, as ρ approaches 1, the average output SNR of EGC approaches that of MRC as

$$\bar{\gamma}_{\mathrm{egc}} = \bar{\gamma}_{\mathrm{mrc}} = L\bar{\gamma}_c. \tag{59}$$

It should be noted that common EGC performance measures, such as BER, cannot be solely characterized by the average output SNR; they also depend on the higher moments. The average output SNR by itself is not a comprehensive metric for the EGC performance in correlated fading channels. Caution must be exercised when using the average output SNR as a performance measure. Higher order moment-based performance measures of the combiner output SNR are required.

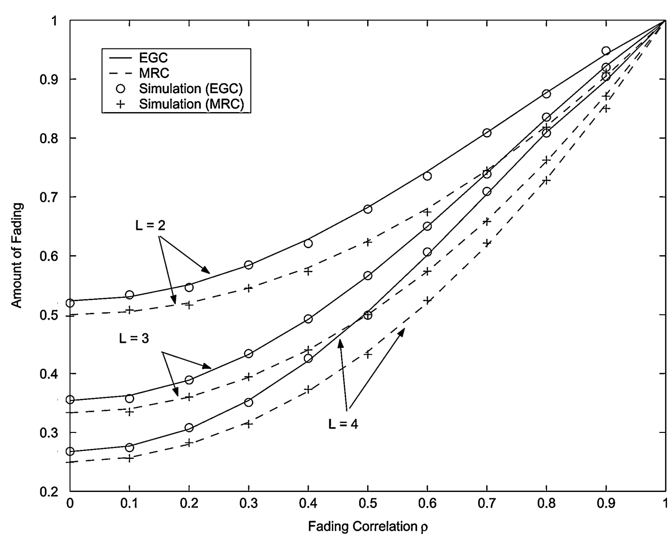

Fig. 3. Amount of fading of EGC in equally correlated Rayleigh fading channels.

Fig. 3 compares the AF of EGC with that of MRC in equally correlated Rayleigh fading channels. As expected, the AF increases as the fading correlation ρ increases and the diversity order L decreases. The AF of EGC is larger than that of MRC. Compared to the average output SNR, AF is a more reliable performance measure because it considers the second moment of the output SNR.

V. Conclusion

In this paper, we have derived the moments of the EGC output SNR in equally correlated Rayleigh, Rician, and Nakagami-m fading channels. Moment-based analysis techniques can serve as an alternative to the more common pdf and mgf approaches. Application of our newly derived results to obtaining practical performance measures, such as error probability or outage probability, has been discussed in Section III. The outage probability can be expressed as an Edgeworth series related to the moments, and the average error rate performance of EGC can be evaluated using a Gaussian quadrature approximation. Such moment-based methods circumvent the need for pdf or mgf expressions, which are not available for the EGC problem. Several purely moment-based statistical performance measures such as skewness, kurtosis, and amount of fading have also been evaluated. Numerical and simulation results show that the fading correlation has a negative effect on the EGC performance. The average output SNR is not a complete performance measure. Higher moments are required to study the EGC performance.

APPENDIX

The second moments of EGC output can be obtained as (14)

$$\begin{aligned} m_2 = \left[\frac{\bar{\gamma}_c(1-\rho)}{L}\right]^2 4 \Bigg[& \binom{L}{1}\frac{\Gamma(3)}{\Gamma(5)} F_A\left(1;-2;1;-\frac{\rho}{1-\rho}\right) \\ & + 2\binom{L}{2}\frac{\Gamma(2.5)\Gamma(1.5)}{\Gamma(4)\Gamma(2)} \\ & \times F_A\left(1;-\frac{3}{2},-\frac{1}{2};1,1;-\frac{\rho}{1-\rho},-\frac{\rho}{1-\rho}\right) \\ & + \binom{L}{2}\frac{\Gamma^2(2)}{\Gamma^2(3)} F_A\left(1;-1,-1;1,1;-\frac{\rho}{1-\rho},-\frac{\rho}{1-\rho}\right) \\ & + 3\binom{L}{3}\frac{\Gamma(2)\Gamma^2(1.5)}{\Gamma(3)\Gamma^2(2)} F_A\left(1;-1,-\frac{1}{2},-\frac{1}{2};1,\ldots\right. \\ & \left. 1;\frac{-\rho}{1-\rho},\ldots,\frac{-\rho}{1-\rho}\right) \\ & + \binom{L}{4}\frac{\Gamma^4(1.5)}{\Gamma^4(2)} F_A\left(1;-\frac{1}{2},\ldots,-\frac{1}{2};1,\ldots\right. \\ & \left.\left. 1;\frac{-\rho}{1-\rho},\ldots,\frac{-\rho}{1-\rho}\right)\right] \end{aligned} \tag{60}$$

where

$$F_A\left(1;-2;1;-\frac{\rho}{1-\rho}\right) = \frac{1}{(1-\rho)^2} \tag{61}$$

and

$$F_A\left(1;-1,-1;1,1;-\frac{\rho}{1-\rho},-\frac{\rho}{1-\rho}\right) = \frac{1+\rho^2}{(1-\rho)^2} \tag{62}$$

can be readily derived from (17). Using the relation between the second kind Appell hypergeometric function and the Gauss hypergeometric function [8, Eq. (C.4)], we obtain

$$\begin{aligned} & F_A\left(1;-\frac{3}{2},-\frac{1}{2};1,1;-\frac{\rho}{1-\rho},-\frac{\rho}{1-\rho}\right) \\ & = \frac{1}{(1-\rho)^2}\,{}_2F_1\left(-\frac{3}{2},-\frac{1}{2};1;\rho^2\right). \end{aligned} \tag{63}$$

Notice that the confluent hypergeometric function ${}_1F_1(a,c;z)$ reduces to a finite series when the first parameter a is either zero or a negative integer

$${}_1F_1(a,c;z) = \sum_{m=0}^{-a}\frac{(a)_m}{(c)_m\, m}z^m. \tag{64}$$

Using (64) and the integral identity of the Appell hypergeometric function [8, Eq. (C.1)], we obtain

$$\begin{aligned} & F_A\left(1;-1,-\frac{1}{2},-\frac{1}{2};1,1,1;-\frac{\rho}{1-\rho},\ldots,-\frac{\rho}{1-\rho}\right) \\ & = \frac{1}{1-\rho}\,{}_2F_1\left(-\frac{1}{2},-\frac{1}{2};1;\rho^2\right) \\ & \quad + \frac{\rho}{1-\rho}\int_0^\infty u e^{-u}\,{}_1F_1^2\left(-\frac{1}{2},1;-\frac{\rho}{1-\rho}\right)du. \end{aligned} \tag{65}$$

Using Gauss's contiguous relation [26, Eq. (9.212.3)]

$${}_1F_1(a,c;z) = \frac{c-a}{c}\,{}_1F_1(a,c+1;z) + \frac{a}{c}\,{}_1F_1(a+1,c+1;z) \tag{66}$$

to expand ${}_1F_1(\cdot,\cdot;\cdot)$ in (65), we finally obtain

$$\begin{aligned} & F_A\left(1;-1,-\frac{1}{2},-\frac{1}{2};1,1,1;-\frac{\rho}{1-\rho},\ldots,-\frac{\rho}{1-\rho}\right) \\ & = \frac{\rho}{1-\rho}\,{}_2F_1\left(-\frac{1}{2},-\frac{1}{2};1;\rho^2\right) \\ & \quad + \frac{9\rho}{4(1-\rho)^2}\,{}_2F_1\left(-\frac{1}{2},-\frac{1}{2};2;\rho^2\right) \\ & \quad - \frac{3\rho}{2(1-\rho)^2}\,{}_2F_1\left(-\frac{1}{2},\frac{1}{2};2;\rho^2\right) \\ & \quad + \frac{\rho}{4}\,{}_2F_1\left(\frac{1}{2},\frac{1}{2};2;\rho^2\right). \end{aligned} \tag{67}$$

Thus, we may write the second moment of EGC output SNR as (22) using (61)–(63) and (67) with the transformation formula for F_A [22].

ACKNOWLEDGMENT

The authors would like to thank the anonymous reviewers for their critical comments that greatly improved this paper.

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