NEW METHODS TO REDUCE INTERCARRIER INTERFERENCE IN OFDM SYSTEMS

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Abstract

Performance of Orthogonal frequency division multiplexing (OFDM) is heavily dependent on the intercarrier interference (ICI) caused by frequency offset. Recently, this ICI is shown to be mitigated by using the partial transmit sequence (PTS), in which the PTS weights minimizing the peak ICI noise-to-signal power ratio (MinMax search) are selected. The exhaustive search leads to exponential complexity in the number of PTS weights. Therefore, an algorithm using a modified sphere decoder to avoid exhaustive search is proposed. Furthermore, a new algorithm called MinSum is also introduced to minimize the total ICI power. Compared with exhaustive search, the MinMax and MinSum algorithms reduce complexity by 10 and 170 times. Moreover, both algorithms are robust to mismatched optimization.

Keywords*—Orthogonal frequency division multiplexing (OFDM), frequency offset, inter-carrier interference (ICI), partial transmit sequence (PTS), sphere decoder.*

1 Introduction

Orthogonal frequency division multiplexing (OFDM) technique is used widely in broadband communication systems. However, the bit error rate (BER) of OFDM is sensitive to the carrier frequency offset (FO). The partial transmit sequence (PTS) approach, initially proposed to reduce the peak to average power ratio (PAPR) [1], can also be used to mitigate the effect of FO [2] [3] in additive white Gaussian noise (AWGN) channels. The authors in [2] used an exhaustive search to find the best PTS weights to minimize the peak intercarrier interference (ICI) noise-to-signal ratio (PICR). The complexity grows exponentially in the number of the PTS weights. We thus propose a new algorithm by modifying the sphere decoder (SD) [4] to find the best PTS weights. We call this algorithm as MinMax for short, where no notational confusion may arise.

We examine the performance of MinMax algorithm in multipath Rayleigh fading channels. The channel state information (CSI) is assumed to be known exactly at the transmitter and receiver. This assumption is reasonable for time division duplexing (TDD) systems, where the channel is assumed slowly varying an uplink and downlink are assumed reciprocal [5]. With this assumption the MinMax algorithm for AWGN channel can be immediately applied. However, in a fading mobile channel, some subcarriers may experience deep fading and the symbol error rate (SER) is heavily dependent on the SER of those. The PICR of such subcarriers can become large. Minimization of ICI for such deeply faded subcarriers may not improve the overall system SER.

We propose a new method minimizing the total ICI of all the subcarriers. We call this algorithm as MinSum for short. The MinSum algorithm can also be implemented by a SD. From the simulation results, for 16 PTS weights, both MinMax and MinSum algorithms perform nearly identical for SER $> 10^{-5}$. Even without optimizing the parameters of the SD's, MinMax and MinSum algorithms reduce complexity by 10 and 170 times compared with exhaustive search. In practice, the transmitter never knows the exact FO. However, the results show that the two algorithms perform nearly independent of the value of FO that is used for optimization.

The paper is organized as follows. Section II briefly introduces system model, PICR notation and problem formulations. Section III describes the sphere decoding algorithm and its implementations for MinMax and MinSum algorithms. Section IV presents numerical results. Conclusions are summarized in Section V.

Notation: Matrix and vector are denoted by boldface uppercase and lowercase letters, respectively, their entries are denoted by corresponding normal uppercase and lowercase letters with subscripts. Superscripts T and $\dagger$ denote matrix transpose and transpose conjugate operations, respectively. $E[\cdot]$ denotes statistical mean. The diagonal matrix with elements of vector X on the main diagonal is denoted by diag(X).

2 System Model

2.1 OFDM Transmission over Fading Channel

We consider the transmission of OFDM signal over frequency-selective fading channels. The discrete transmited OFDM signal can be represented as

$$\boldsymbol{D} = \boldsymbol{F}\boldsymbol{c} \tag{1}$$

where $\boldsymbol{c}$ is the data vector $\boldsymbol{c} = \begin{bmatrix} c_1 & c_2 & \dots & c_K \end{bmatrix}^T$, $\boldsymbol{F}$ is the inverse fast Fourier transform (IFFT) matrix with the entry $F_{i,k} = \frac{1}{\sqrt{N}} e^{j2\pi(i-1)(k-1)}$, K is the IFFT size, $i, k = 1, 2, \dots, K$, $j = \sqrt{-1}$.

The signal is transmitted over an $L-$path frequency-selective fading channel. The frequency response at the kth subcarrier is

$$H_k = \sum_{l=0}^{L-1} h_l \; e^{-j2\pi(k-1)\Delta f \tau_l} \tag{2}$$

where h_l and τ_l are the gain and delay of path l ($l = 0, 1, \dots, L-1$), respectively. Δf is the subcarrier frequency separation, $\Delta f = 1/T$, where T is the OFDM symbol duration. The path gains are modeled as independent zero-mean complex Gaussian random variables with variance $E\left[|h_l|^2\right] = \delta_l^2$. The channel power is normalized so that $\sum_{l=0}^{L-1} \delta_l^2 = 1$. Define $\boldsymbol{H} = \mathrm{diag}\left[\begin{array}{cccc} H_1 & H_2 & \ldots & H_K \end{array}\right]$.

The received signal are demodulated by a fast Fourier transform (FFT) at the receiver. If there exists a carrier frequency offset (FO) δf, defining the normalized FO by $\varepsilon = \delta f / \Delta f$, the received signal can be represented as [6]

$$\boldsymbol{y} = \boldsymbol{S}\boldsymbol{H}\boldsymbol{c} + \boldsymbol{n} \tag{3}$$

where $\boldsymbol{y}$ is the demodulated vector, $\boldsymbol{n}$ is the noise vector with identically independent distributed (i.i.d) Gaussian noise samples $\boldsymbol{n} = \left[\begin{array}{cccc} n_1 & n_2 & \ldots & n_K \end{array}\right]^T$, the matrix $\boldsymbol{S}$ consists of ICI coefficients

$$S_{ki} = \frac{\sin\left[\pi\left(i-k+\varepsilon\right)\right]}{K\sin\left[\frac{\pi}{K}\left(i-k+\varepsilon\right)\right]} \exp\left[j\pi\left(1-\frac{1}{K}\right)\left(i-k+\varepsilon\right)\right]. \tag{4}$$

Notice that $S_{kk} = S_0$ are independent of the subcarrier index k. In particular, the demodulated signal y_k can be written as

$$y_k = S_0 H_k c_k + I_k + n_k \tag{5}$$

where I_k stands for the ICI from the other subcarriers on subcarrier k and is given by

$$I_k = \sum_{i=0, i\neq k}^{K-1} S_{ki} H_i c_i \,. \tag{6}$$

The parameter Peak Interference-to-Carrier Ratio (PICR) is defined [2] as

$$\mathrm{PICR}(\boldsymbol{c}, \varepsilon) = \max_{1\le k\le K} \left\{ \frac{|I_k|^2}{|S_0 c_k|^2} \right\}. \tag{7}$$

2.2 Reducing ICI by PTS Approach

In AWGN channels, at high signal-to-noise ratio (SNR), ICI power dominates the thermal noise. Thus the data symbol with higher ICI will likely experience higher SER. Therefore, reducing PICR will lead to an improvement of overall system SER. To do this, in [2], the data vector $\boldsymbol{c}$ is partitioned into M equal-sized disjoint blocks $\boldsymbol{c} = [\boldsymbol{c_1}\, \boldsymbol{c_2} \ldots \boldsymbol{c_M}]^T$, each block consists of $K_0 = K/M$ contiguous subcarriers. The symbols of each block are multiplied by weight b_m, where $|b_m| = 1$, for $m = 1, 2, \ldots, M$. Let $\boldsymbol{b} = [b_1\, b_2 \ldots b_M]$. Application of the partial transmit sequence to the data vector $\boldsymbol{c}$ results in vector $\hat{\boldsymbol{c}}$. We can rewrite (3) as

$$\boldsymbol{Y} = \boldsymbol{S}\boldsymbol{H}\hat{\boldsymbol{c}} + \boldsymbol{N} = S_0 \boldsymbol{I}_K \boldsymbol{H}\hat{\boldsymbol{c}} + \hat{\boldsymbol{S}}\boldsymbol{H}\hat{\boldsymbol{c}} + \boldsymbol{N} \tag{8}$$

where $\boldsymbol{I}_K$ is the K-by-K identity matrix and $\hat{\boldsymbol{S}} = \boldsymbol{S} - S_0\boldsymbol{I}_K$.

Let $\boldsymbol{P} = \hat{\boldsymbol{S}}\boldsymbol{H}$, we formulate another matrix $\boldsymbol{T}$ such that : $T_{ki} = P_{ki} c_i$, $(i, k = 1, 2, ..., K)$. Then we create a matrix $\boldsymbol{Q}$ by applying the following rule: $Q_{km} = \sum_{n=(m-1)K_0+1}^{mK_0} T_{kn}$, where $k = 1, 2, ..., K, m = 1, 2, ..., M$. One can see that the ICI term I_k becomes $I_k = \boldsymbol{q}_k\boldsymbol{b}$, where $\boldsymbol{q}_k$ is the k-th row of $\boldsymbol{Q}$. The PICR in (7) becomes:

$$\mathrm{PICR}(\boldsymbol{c}, \varepsilon) = \max_{1\le k\le K} \left\{ \frac{|\boldsymbol{q}_k\boldsymbol{b}|^2}{|S_0 c_k|^2} \right\} = \max_{1\le k\le K} \left\{ \boldsymbol{b}^\dagger \hat{\boldsymbol{q}}_k^\dagger \hat{\boldsymbol{q}}_k \boldsymbol{b} \right\}. \tag{9}$$

where $\hat{\boldsymbol{q}}_k = \boldsymbol{q}_k / S_0 c_k$. Let $\hat{\boldsymbol{Q}} = [\hat{\boldsymbol{q}}_1\; \hat{\boldsymbol{q}}_2 \ldots \hat{\boldsymbol{q}}_K]^T$. Assume that the channel state information (CSI) $\boldsymbol{H}$ and FO ε are available at the transmitter, we can restate the problem of minimizing maximum value of PICR as follows: "*Given the matrix $\hat{\boldsymbol{Q}}$, find a vector $\boldsymbol{b}$ of PTS weights so that the maximum value of PICR defined in (9) is minimized.*" The mathematical representation of the problem is:

$$\boldsymbol{b}_{MM} = \arg\min_{\boldsymbol{b}} \max_{1\le k\le K} \left\{ \boldsymbol{b}^\dagger \hat{\boldsymbol{q}}_k^\dagger \hat{\boldsymbol{q}}_k \boldsymbol{b} \right\}. \tag{10}$$

If the total ICI power is minimized, the overall system SER will also be reduced. The total ICI power can be found as follows:

$$P_S = \sum_{k=1}^{K} |I_k|^2 = \boldsymbol{b}^\dagger \boldsymbol{Q}^\dagger \hat{\boldsymbol{Q}} \boldsymbol{b}\,. \tag{11}$$

Thus the problem of minimizing the total ICI power is equivalent to the finding of vector $\boldsymbol{b}$ such that

$$\boldsymbol{b}_{MS} = \arg\min_{\boldsymbol{b}} \boldsymbol{b}^\dagger \boldsymbol{Q}^\dagger \hat{\boldsymbol{Q}} \boldsymbol{b}\,. \tag{12}$$

We refer to searching for $\boldsymbol{b}$ satisfying (10) and (12) as MinMax and MinSum problems, respectively. The algorithms to solve these problems are called MinMax and MinSum algorithms accordingly. Interestingly, both the problems can be solved by a sphere decoder, a decoder which has been proposed to solve several problems in communications such as lattice detection, V-BLAST detection, algebraic space-time code decoder (See [4] and references therein). We next derive sphere decoder based algorithms to solve (10) and (12).

3 MinMax and MinSum Algorithms

Eqs. (10) and (12) have the same quadratic form $\boldsymbol{b}^\dagger \boldsymbol{X}^\dagger \boldsymbol{X}\boldsymbol{b}$. We first present a general method to solve the problem

$$\boldsymbol{b} = \arg\min_{\boldsymbol{b}} \boldsymbol{b}^\dagger \boldsymbol{X}^\dagger \boldsymbol{X}\boldsymbol{b}\,. \tag{13}$$

Assume that the matrix $\boldsymbol{X}$ is real and full rank. We further assume that $\boldsymbol{b}$ consists of binary symbols ($b_k \in \{-1, 1\}$). The sufficient condition for $\boldsymbol{b}$ to minimize the cost metric (13) is the lattice point $\boldsymbol{X}\boldsymbol{b}$ should lie in a hypersphere of a large enough covering radius r.

Using Cholesky factorization, one can find a square upper triangle matrix $\boldsymbol{R}$ such that $\boldsymbol{R}^\dagger\boldsymbol{R} = \boldsymbol{X}^\dagger\boldsymbol{X}$. Then

$$r^2 \ge \boldsymbol{b}^\dagger \boldsymbol{X}^\dagger \boldsymbol{X}\boldsymbol{b} = \boldsymbol{b}^\dagger \boldsymbol{R}^\dagger \boldsymbol{R}\boldsymbol{b} = \sum_{i=1}^{M}\sum_{k=i}^{M} (R_{i,k} b_k)^2 \tag{14}$$

or

$$r^2 \ge (R_{MM} b_M)^2 + (R_{M-1,M} b_M + R_{M-1,M-1} b_{M-1})^2 + \ldots \tag{15}$$

The following necessary conditions can be drawn from (15):

$$\begin{aligned} &r^2 \ge (R_{MM} b_M)^2 \\ &r^2 \ge (R_{M-1,M} b_M + R_{M-1,M-1} b_{M-1})^2 \\ &\ldots \end{aligned}$$

This set of inequalities can be used to find admissible b_k ($k = 1, 2, \ldots, M$) as follows. We first look for possible values of b_M. Since b_M is an integer, the admissible values of b_M should satisfy

$$-\left\lceil \frac{r}{R_{MM}} \right\rceil \leq b_M \leq \left\lfloor \frac{r}{R_{MM}} \right\rfloor \tag{16}$$

where $\lceil \cdot \rceil$ and $\lfloor \cdot \rfloor$ denote ceiling and floor functions, respectively. Define $L_M = -\left\lceil \frac{r}{R_{MM}} \right\rceil$ and $U_M = \left\lfloor \frac{r}{R_{MM}} \right\rfloor$ as the lower and upper bounds of b_M. Hence

$$L_M \leq b_M \leq U_M \,. \tag{17}$$

For each value of b_M, define $a_{M-1} = R_{M-1,M} b_M$, $r_{M-1}^2 = r^2 - R_{M,M} b_M$, the admissible values of b_{M-1} are

$$\left\lceil \frac{-r_{M-1} - a_{M-1}}{R_{M-1,M-1}} \right\rceil \leq b_{M-1} \leq \left\lfloor \frac{r_{M-1} - a_{M-1}}{R_{M-1,M-1}} \right\rfloor. \tag{18}$$

or $L_{M-1} \leq b_{M-1} \leq U_{M-1}$.

Similarly, all admissible values of b_{M-2} to b_1 can be found. We obtain the set of candidate vectors $\boldsymbol{b}$.

In our problem, we deal with complex matrices $\boldsymbol{X}$. If vector $\boldsymbol{b}$ is real, the following transformation can be applied.

$$\bar{\boldsymbol{X}} = \begin{bmatrix} Re(\boldsymbol{X}) \\ Im(\boldsymbol{X}) \end{bmatrix} \tag{19}$$

where $Re(\boldsymbol{X})$ and $Im(\boldsymbol{X})$ denote the real and imagine parts of $\boldsymbol{X}$. Then $\bar{\boldsymbol{X}}^\dagger \bar{\boldsymbol{X}}$ can be Cholesky factorized.

We can readily apply the sphere decoder described above for the MinSum problem. However, the sphere decoder for MinMax problem needs following modifications.

- *Rank deficiency*: Since $\bar{\boldsymbol{q}}_k^\dagger \bar{\boldsymbol{q}}_k$ is of rank two, Cholesky factorization does not work [note that $\bar{\boldsymbol{q}}_k$ is obtained from $\hat{\boldsymbol{q}}_k$ by the transformation (19)]. However, we can use the generalized sphere decoder proposed in [7]. Since b_k's are all unit modulus, $\mu \boldsymbol{b}^\dagger \boldsymbol{b} = \mu M$; thus $|\bar{\boldsymbol{q}}_k \boldsymbol{b}|^2 + \mu M = \boldsymbol{b}^\dagger (\bar{\boldsymbol{q}}_k^\dagger \bar{\boldsymbol{q}}_k + \mu \boldsymbol{I}_M) \boldsymbol{b}$. We now can apply Cholesky factorization for the new full rank matrix $\bar{\boldsymbol{X}} = \bar{\boldsymbol{q}}_k^\dagger \bar{\boldsymbol{q}}_k + \mu \boldsymbol{I}_M$
- *Multiple constraints*: In the MinMax problem, we need to find the vector $\boldsymbol{b}$ to minimize the PICR of K subcarriers or equivalently K sphere decoders must be run at the same time. This can be performed easily by only one sphere decoder. At each step of searching admissible values of b_m ($m = 1, 2, \ldots, M$), the compound lower and upper bounds are calculated as

$$L_m = \max(L_m^1, L_m^2, \ldots, L_m^K),$$
$$U_m = \min(U_m^1, U_m^2, \ldots, U_m^K).$$

where L_m^k and U_m^k ($k = 1, 2, \ldots, K$) are the lower and upper bounds of symbol b_m derived from corresponding equation of subcarrier k.

If vector $\boldsymbol{b}$ is complex, each symbol b_k may belong to phase shift keying (PSK) constellations. One can apply the complex sphere decoder [8], with some modifications described above.

4 Simulation Results

The complexity and SER will be the two performance criteria for comparing the efficiency of our proposed algorithms. We simulated both real (binary) and complex PTS weights (4-PSK). The results show that $2M$ real weights perform much better than M complex weights with similar complexity. Therefore, we present the results for binary PTS weights only. The number of subcarriers is 64 with 4-QAM. For binary weights, the value of b_M can be set to 1 without loss of generality.

4.1 Complexity

The total number of multiplications, divisions and squares of the proposed algorithms for FO 10% are reported in Table I. Comparing MinMax algorithm and exhaustive search (ES), the computation saving is marginal for $M = 8$, but for $M = 16$, the MinMax algorithm reduces about 10 times number of operations. The suboptimum MinMax algorithm saves 1/3 number of operations. Comparing the MinSum and ES algorithms, for $M = 16$, the MinSum algorithm reduces the number of operations by 171 times.

4.2 Symbol Error Rate

1. Ideal Optimization:

Fig. 1 presents the SER of MinMax and MinSum algorithms for $M = 8$ when the transmitter knows exactly the value of FO. If normalized FO is 10%, at low SNR, the two algorithms perform identically. At high SNR, the SER curve of MinSum reaches an error floor. When FO is 8%, the two algorithms perform similarly.

2. Mismatched Optimization:

Let the FO known at the transmitter is ε_T and the true value of FO at the receiver is ε_R. We consider two cases: (a) Case 1: $\varepsilon_T > \varepsilon_R$ and (b) Case 2: $\varepsilon_T < \varepsilon_R$.

Figs. 2 and 3 plot the SER curves for MinMax and MinSum algorithms, respectively. The performance under mismatched optimization is almost identical with that of ideal optimization. For example, in Fig. 2, the SER curve of mismatched optimization done for $\varepsilon_T = 0.1 > \varepsilon_R = 0.08$ is almost overlap the SER curve of the ideal optimization, where $\varepsilon_T = \varepsilon_R = 0.08$. This is true for MinSum algorithm and also holds for the cases $\varepsilon_T = 0.08 < \varepsilon_R = 0.1$ as well. Therefore, both MinMax and MinSum are robust to the mismatched optimization.

TABLE I

NUMBER OF OPERATIONS OF THREE ALGORITHMS

Algorithm	M = 8	M = 16
Exhaustive search	147.4×10^3	71.3×10^6
MinMax	115.6×10^3	7.34×10^6
MinSum	38.8×10^3	0.417×10^6

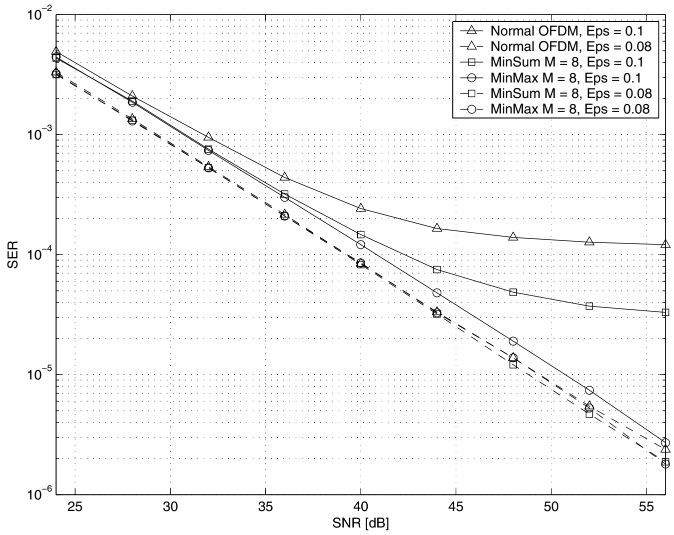

Figure 1. Performance of MinMax and MinSum algorithms, ideal optimization

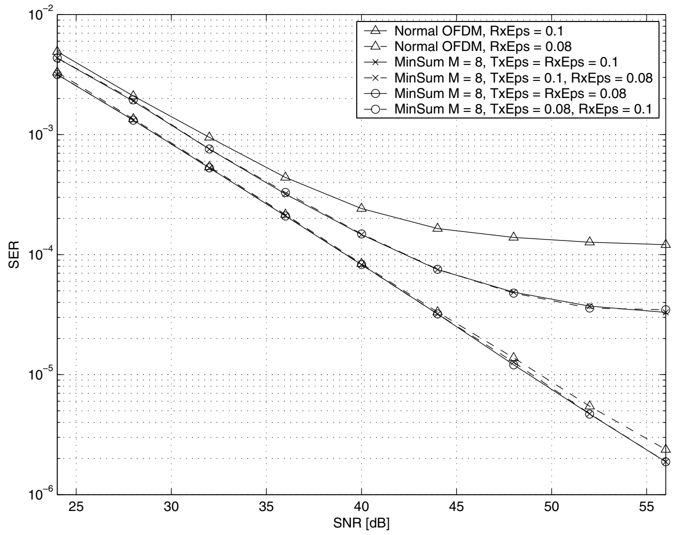

Figure 3: Performance of MinSum algorithm, mismatched optimization.

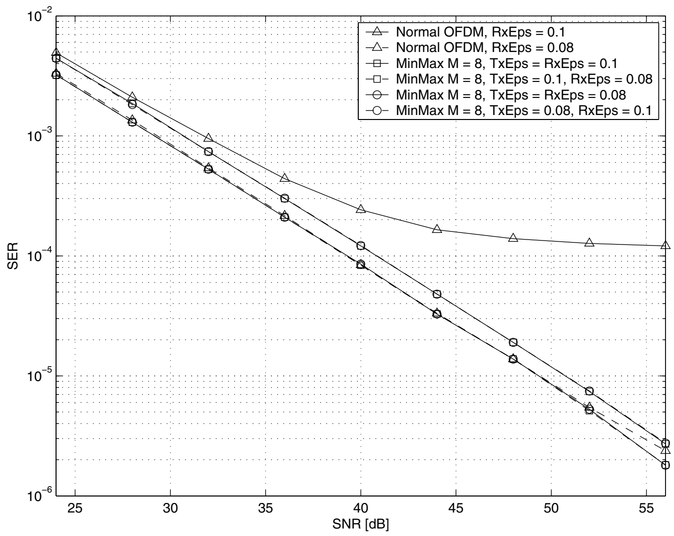

Figure 2: Performance of MinMax algorithm, mismatched optimization.

5 Conclusion

We presented two algorithms (MinMax and MinSum) to search for optimum PTS weights to reduce ICI effect in OFDM transmission over fading channels. The two algorithms are robust to the mismatched FO estimation at the transmitter. When FO is larger (10%) and $M = 16$, MinMax yields better performance compared with MinSum for $SER > 10^{-5}$. For small FO or a large number of PTS weights, the two algorithms perform nearly identical. However, the complexity of MinSum is just a fraction of MinMax.

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