

# Transmitter Precoding for ICI Reduction in OFDM Systems

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**Abstract**—A non-linear Tomlinson-Harashima (TH) precoder is proposed for ICI reduction in OFDM. We show the ICI coefficient matrix is approximately unitary and use this property for the precoder design. Our proposed precoder significantly reduces the bit error rate (BER) degradation due to frequency offset. With this proposed design, only partial channel state information (CSI), not including the knowledge of the frequency offset, is needed at the transmitter. Furthermore, the effect of channel mismatch on the precoder scheme is analyzed and demonstrated by simulation results.

## I. INTRODUCTION

Orthogonal frequency division multiplexing (OFDM) is a promising transmission technique for high throughput wireless systems due to its spectral efficiency and ability to mitigate the dispersive effects of wideband channels. It has gained considerable attention in the context of work towards the fourth-generation (4G) cellular communication systems and as an essential component of other wireless access standards such as the IEEE 802.11a and 802.16 [1]. Without transmission channel distortions, OFDM subcarriers are orthogonal and fully separable by the discrete Fourier transform (DFT) operation at the receiver. However, there may be a carrier frequency offset (CFO) due to mismatch of oscillators or the Doppler shift caused by the relative motion between the transmitter and receiver, or movement of other objects around transceivers. OFDM is sensitive to CFO, which distorts the orthogonality between subcarriers and results in intercarrier interference (ICI).

A number of approaches have been proposed to address this problem. An optimum time-domain Nyquist windowing function is derived in [2], which mitigates the joint effect of additive white Gaussian noise (AWGN) and ICI caused by CFO. Selected mapping and partial transmit sequence approaches have been developed in [3] to reduce ICI. In [4] minimum mean square error (MMSE) receivers based on a finite power series expansion for the time-varying frequency response are

proposed. Self-cancellation schemes involving mapping of each input symbol to a group of subcarriers have been investigated in [5] and [6] to reduce ICI between subcarriers, at the price of sacrificing some bandwidth efficiency.

In this paper, we propose transmitter precoding for mitigating ICI in OFDM systems. Note that transmitter precoding can be either linear or non-linear. Linear MMSE or zero-forcing (ZF) precoder, however, requires complete knowledge about channel state information (CSI) at the transmitter [7], including CFO and channel impulse response (CIR). We therefore propose a non-linear Tomlinson-Harashima (TH) precoder, a transmitter-based pre-equalization technique [8], to reduce ICI in OFDM. This TH precoder consists of a feed-forward filter, a scaling matrix and a modulo arithmetic device at the receiver, and a modulo arithmetic feedback structure at the transmitter side (see Fig. 2). We show that the ICI coefficient matrix is approximately unitary and this property is used for the THP design. With this proposed design, only partial CSI, not including the knowledge of the frequency offset, is needed at the transmitter. This information can be obtained from the receivers via feedback channels. Our precoder significantly reduces the bit error rate (BER) degradation due to frequency offset.

## II. SYSTEM MODEL

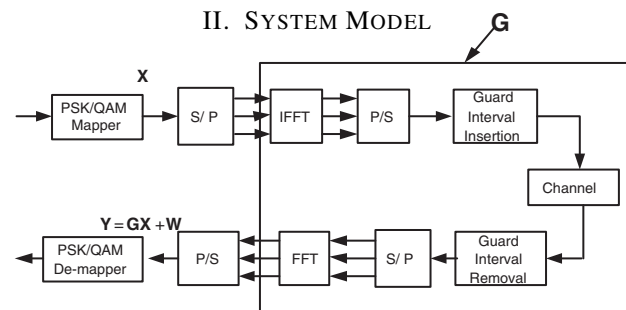


Fig. 1. A simplified block diagram of an OFDM link.

This section will introduce an OFDM system model with the presence of frequency offset.

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We consider an OFDM system with  $N$  subcarriers as shown in Fig. 1. Let  $X_n$  denote the transmitted M-ary phase-shift keying (PSK) or quadrature amplitude modulation (QAM) symbols on the  $n$ -th subcarrier. The OFDM symbol can be written as  $\mathbf{X}^{(u)} = [X_0 X_1 \dots X_{N-1}]^T$ . At the inverse DFT (IDFT) output, the time-domain transmitted OFDM symbol is  $\mathbf{x} = \mathbf{Q}\mathbf{X}$ , where  $\mathbf{Q}$  is an  $N \times N$  matrix with entries  $Q_{m,n} = \frac{1}{N} e^{j\frac{2\pi}{N}mn}$ . A cyclic prefix is customarily inserted at the beginning of each time-domain OFDM symbol to prevent intersymbol interference (ISI). We assume that the cyclic prefix is longer than the expected maximum excess delay so that there are no overlaps between adjacent OFDM symbols.

We consider a wideband frequency-selective fading channel and the following tapped delay line impulse response model is used:

$$h(t) = \sum_{l=0}^{L-1} \alpha_l e^{j[2\pi f_D(t)t + \theta_l]} \delta(t - \tau_l), \quad (1)$$

where  $f_D(t)$  is Doppler frequency at time  $t$ ,  $\tau_l$  is the propagation delay, and  $\theta_l$  is the initial phase of the  $l$ -th path, respectively. Here,  $\theta_l$  is assumed to be 0 without loss of generality.  $\alpha_l, l = 0, 1, \dots, L-1$  denotes complex path gain on the  $l$ -th tap of the discrete time-domain multipath channel impulse response, and  $L$  is the number of resolvable paths. Each path fades according to a wide-sense stationary uncorrelated scattering (WSSUS) Rayleigh fading model. The usual assumption for precoding is that the multipath channel is sufficiently slowly fading, i.e., it does not change over one block of precoded bits (quasi-static fading). Under this assumption, both  $\alpha_l$  and  $\tau_l$  are constant over an OFDM symbol interval  $T_s$ . Normally,  $f_D(t)$  changes slowly and assumes a constant value  $f_D$  within an OFDM symbol period. The discrete time-domain received signal can therefore be written as

$$y(k) = \sum_{n=0}^{N-1} \alpha_n x(k-n) e^{j\frac{2\pi}{N}\varepsilon k} + w(k), \quad (2)$$

where  $\varepsilon = f_D T_s$  is the normalized CFO. The  $w(k)$  is a discrete-time AWGN sample and  $x(n) \in \mathbf{x}$ . Discarding the cyclic prefix and performing DFT on the signal vector  $\mathbf{y}$ , the received signal for the  $k$ -th subcarrier can be obtained as

$$Y_k = \underbrace{S_0 H_k X_k}_{\text{desired signal}} + \underbrace{\sum_{n=0, n \neq k}^{N-1} S_{n-k} H_n X_n}_{\text{ICI component}} + W_k, \quad (3)$$

for  $k = 0, 1, \dots, N-1$ , where  $W_k$  is an AWGN sample with zero mean and variance  $\sigma_W^2$ ;  $W_k$  for different  $k$  are assumed independent and identically distributed (i.i.d).

$H_k = \sum_{l=0}^{N-1} \alpha_l e^{j\frac{2\pi}{N}lk}$  and  $S_{n-k}$  is an ICI coefficient, which can be given by

$$S_k = \frac{\sin \pi(\varepsilon + k)}{N \sin \frac{\pi}{N}(\varepsilon + k)} e^{j\pi(1-\frac{1}{N})(\varepsilon+k)}. \quad (4)$$

The received signal can therefore be represented in matrix form as

$$\mathbf{Y} = \mathbf{G}\mathbf{X} + \mathbf{W} = \mathbf{S}\mathbf{H}\mathbf{X} + \mathbf{W}, \quad (5)$$

where  $\mathbf{Y} = [Y_0 Y_1 \dots Y_{N-1}]^T$ ,  $\mathbf{H}$  is the frequency response matrix of the  $N$  orthogonal-subcarrier channels, defined as a diagonal matrix with entries of  $H_k$

$$\mathbf{H} = \text{diag} ( H_0 \ H_1 \ \dots \ H_{N-1} ) \quad (6)$$

and the ICI coefficient matrix can be given as

$$\mathbf{S} = \begin{pmatrix} S_0 & S_1 & \dots & S_{N-1} \\ S_{-1} & S_0 & \dots & S_{N-2} \\ \vdots & & \ddots & \vdots \\ S_{-(N-1)} & S_{-(N-2)} & \dots & S_0 \end{pmatrix}. \quad (7)$$

The matrix  $\mathbf{G}$  is the overall channel frequency response matrix, accounting for the ICI due to frequency offset

$$\mathbf{G} = \mathbf{S}\mathbf{H}. \quad (8)$$

### III. PRECODING FOR ICI REDUCTION

This section develops a non-linear TH precoder for ICI reduction in OFDM systems. We show that the ICI coefficient matrix is approximately unitary. With this property, for our TH precoder the knowledge of frequency offset is not needed at the transmitter, i.e., only the knowledge of CIR is needed.

#### A. Property of ICI coefficient matrix

The ICI coefficient matrix  $\mathbf{S}$  (7) is a unitary matrix, i.e.,  $\mathbf{S}^H \mathbf{S} = \mathbf{S} \mathbf{S}^H = \mathbf{I}_N$ . Therefore, the inverse of the ICI coefficient matrix can be easily calculated by taking the conjugate transpose since  $\mathbf{S}^{-1} = \mathbf{S}^H$ . Using (4), this property is proven in Appendix. It is used for non-linear precoding design.

#### B. Tomlinson-Harashima precoding

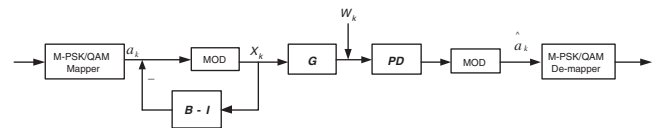


Fig. 2. The Tomlinson-Harashima precoding in an OFDM link.

We propose a non-linear precoder employing Tomlinson-Harashima precoding (THP) [8] to reduce ICI in OFDM. The structure of the scheme is shown in Fig. 2. The receiver side consists of a feedforward filter  $\mathbf{D}$ , a scaling matrix  $\mathbf{P}$  and a modulo arithmetic

device at the receiver. The transmitter side has a modulo arithmetic feedback structure employing the matrix  $\mathbf{B}$  at the transmitter, by which the transmitted symbols  $X_k$  are calculated for the data carrying symbols  $a_k$  drawn from a signal constellation.

Given the channel matrix  $\mathbf{H}$ , we design the matrices  $\mathbf{D}$ ,  $\mathbf{P}$  and  $\mathbf{B}$  using the ZF criterion. QR-type factorization of the channel matrix yields

$$\mathbf{H} = \mathbf{F}^H \mathbf{T}, \quad (9)$$

where  $\mathbf{F}$  is a  $N \times N$  unitary matrix and  $\mathbf{T} = [T_{ij}]$ , a  $N \times N$  upper triangular matrix [9]. The overall channel matrix  $\mathbf{G}$  can thus be given as

$$\mathbf{G} = \mathbf{S}\mathbf{H} = \mathbf{S}\mathbf{F}^H \mathbf{T} = \mathbf{D}^H \mathbf{T}. \quad (10)$$

Since both of the  $\mathbf{S}$  and  $\mathbf{F}$  are  $N \times N$  unitary matrices, the feedforward matrix  $\mathbf{D}^H = \mathbf{S}\mathbf{F}^H$  is also a  $N \times N$  unitary matrix. Therefore  $\mathbf{G} = \mathbf{D}^H \mathbf{T}$  is a QR factorization as well. However, if we directly factorize the overall channel matrix  $\mathbf{G}$ , we need to know both channel impulse response and carrier frequency offset at the transmitter, which may be difficult to achieve in practical implementation. Since the ICI coefficient matrix  $\mathbf{S}$  is unitary, it is sufficient to know only channel impulse response information at the transmitter and include  $\mathbf{S}$  as part of the feedforward matrix  $\mathbf{D}$  at the receiver. Hence, the knowledge of carrier frequency offset at the transmitter is not necessary. To constrain transmitted power, we define a diagonal scaling matrix  $\mathbf{P} = \text{diag}(T_{11}^{-1}, T_{22}^{-1}, \dots, T_{NN}^{-1})$ , and then the transmitter feedback filter becomes

$$\mathbf{B} = [B_{mn}] = \mathbf{P}\mathbf{D}\mathbf{G} = \mathbf{P}\mathbf{T}. \quad (11)$$

Given the data symbols  $a_k$  drawn from an M-ary PSK or QAM square signal constellation

$$\mathcal{A} = \left\{ a_I + ja_Q \mid a_I, a_Q \in \pm 1, \pm 3, \dots, \pm(\sqrt{M}-1) \right\}, \quad (12)$$

the transmitted symbols are successively calculated as

$$\begin{aligned} X_k &= \text{MOD}_{2\sqrt{M}} \left( a_k - \sum_{q=k+1}^{N_B} B_{k,q} X_q \right) \\ &= a_k + c_k - \sum_{q=k+1}^{N_B} B_{k,q} X_q, \end{aligned} \quad (13)$$

where  $N_B$  is the number of columns of feedback matrix  $\mathbf{B}$ . The precoding symbols  $c_k \in \left\{ 2\sqrt{M}(c_I + jc_Q) \mid c_I, c_Q \in \mathbb{Z} \right\}$  are chosen implicitly by a modulo reduction of the real and imaginary parts of  $X_k$  into the half-open interval  $-\sqrt{M}, \sqrt{M}$ ;  $\mathbb{Z}$  indicates the set of integers. Hence, the initial signal constellation  $\mathcal{A}$  is periodically extended by the feedback structure at the transmitter. All points in real and imaginary parts spaced by integer multiples of  $2\sqrt{M}$  are congruent and represent the same data. We select  $X_k$  such that

$\text{Re}\{X_k\} \in [-\sqrt{M}, \sqrt{M}]$  and  $\text{Im}\{X_k\} \in [-\sqrt{M}, \sqrt{M}]$ , i.e.,  $X_k$  lies in the boundary region of  $\mathcal{A}$ , resulting in a transmit signal with smaller power. The modulo operation employed at the transmitter is non-linear, and a slicer at the receiver uses the same modulo operation before deciding the points of the initial constellation  $\mathcal{A}$ .

Since the linear pre-distortion via  $\mathbf{B}^{-1}$  equalizes the cascade  $\mathbf{G}\mathbf{P}\mathbf{D}$ , after pre-filter  $\mathbf{D}$  and scaling  $\mathbf{P}$  at the receiver, the transmitted symbols  $X_k$  are corrupted by an additive noise as  $Y_k = X_k + W'_k$ . Note that  $W'_k$  is the  $k$ -th entry of filtered noise vector  $\mathbf{W}' = \mathbf{P}\mathbf{D}\mathbf{W}$  with individual variance  $\sigma_{W'_k}^2 = \sigma_W^2 T_{k,k}^{-2}$ , since  $\mathbf{D}$  is unitary. As the modulo arithmetic device at the receiver applies the same modulo operation as that at the transmitter, unique estimates of the data symbols are generated. Consequently, after discarding the modulo congruence, the proposed precoding cancels the fading channel distortion and ICI.

The proposed THP for OFDM systems, which does not require the knowledge of carrier frequency offset at the transmitter, reduces information load in the feedback channel and makes it useful for practical implementation. With perfect channel information at the transmitter and knowledge of frequency offset at the receiver, THP performs better than linear precoding schemes, (similar observation has also been made for multiple-antenna systems as in [10]). Furthermore, because the feedback part is moved to the transmitter in THP algorithm, the error propagation can be voided. Therefore, lower BER can be expected for THP.

#### IV. EFFECT OF PRECODER MISMATCH ON PERFORMANCE

We consider two cases of precoder mismatch, when the channel assumed by the THP transmitter differs from the actual multipath channel at the time of transmission. In the first case, the transmitter and the receiver share the same imperfect channel estimate. In the second case, the receiver has perfect channel information, while the transmitter has incorrect channel information due to feedback errors or delay.

Without the proposed TH precoding, the signal-to-noise ratio (SNR) of an OFDM system in the presence of frequency offset can be given as [11]

$$\text{SNR} = \frac{\text{sinc}^2(\varepsilon) \sigma_H^2 E_s}{[1 - \text{sinc}^2(\varepsilon)] \sigma_H^2 E_s + \sigma_W^2}, \quad (14)$$

where  $\sigma_H^2 = E[|H_k|^2]$ . When  $\varepsilon = 0$ , i.e., there is no frequency offset, the SNR converts to  $\text{SNR}_0 = \frac{\sigma_H^2 E_s}{\sigma_W^2}$ . With the TH precoder, if the channel frequency response matrix is perfectly known at both transmitter and receiver, the SNR for the  $k$ -th subcarrier can be given as  $\text{SNR}_{0'_k} = \frac{\sigma_H^2 E_s}{\sigma_{W'_k}^2}$ . The THP with mismatch has residual ICI. Under mismatch,  $\hat{\mathbf{P}}\hat{\mathbf{D}}\mathbf{G} - \hat{\mathbf{B}}$  has a term  $g_{0,k}$

at the zero-lag tap for the  $k$ -th subcarrier. We can write the output SNR as

$$\text{SNR}_k = \frac{g_{0k}^2 E_s}{\sigma_{\text{ICI}_k}^2 + \sigma_{W'_k}^2}. \quad (15)$$

The residual ICI limits the output SNR and degrades the system performance.

In the first case, transmitter and receiver share the same incorrect channel estimate. Let  $\hat{\mathbf{B}}$  and  $\hat{\mathbf{P}}\hat{\mathbf{D}}$  correspond to the feedback and feedforward filters of the THP designed for  $\hat{\mathbf{G}} \neq \mathbf{G}$ . The received symbols therefore is

$$Y_k = \hat{\mathbf{P}}\hat{\mathbf{D}}\hat{\mathbf{G}}^{-1}X_k + W'_k = \hat{\mathbf{P}}\hat{\mathbf{D}}\hat{\mathbf{D}}^H\hat{\mathbf{T}}\hat{\mathbf{T}}^{-1}\hat{\mathbf{P}}^{-1}X_k + W'_k. \quad (16)$$

Since  $\hat{\mathbf{D}}\hat{\mathbf{D}}^H \neq \mathbf{I}_N$  and  $\hat{\mathbf{T}}\hat{\mathbf{T}}^{-1} \neq \mathbf{I}_N$ , residual ICI is introduced. In the second case, the receiver has perfect knowledge of the channel  $\mathbf{H}$ , but the transmitter still has an incorrect estimate  $\hat{\mathbf{H}}$ , because of errors or delay in feedback channels. The receiver can compute the feedforward matrix as  $\hat{\mathbf{D}} = \mathbf{T}\mathbf{G}^{-1}$ . The received symbols  $Y_k$  become

$$Y_k = \mathbf{P}\mathbf{T}\mathbf{G}^{-1}\hat{\mathbf{G}}\hat{\mathbf{B}}^{-1}X_k + W'_k = \mathbf{B}\hat{\mathbf{B}}^{-1}X_k + W'_k. \quad (17)$$

Since  $\mathbf{B} \neq \hat{\mathbf{B}}^{-1}$ , the received symbols are corrupted by the non-identity matrix  $\mathbf{B}\hat{\mathbf{B}}^{-1}$ .

## V. SIMULATION RESULTS

In this section, simulation results show how the TH precoder suppresses ICI in OFDM. A QPSK-OFDM system with 64 subcarriers on a 6-tap Rayleigh fading channel is considered. The channel taps are zero-mean complex Gaussian random processes with variances 0.189, 0.379, 0.239, 0.095, 0.061, and 0.037. The CFO is estimated at the receiver.

We assume that the channel frequency response matrix  $\mathbf{H}$  changes independently from one OFDM symbol to next. Fig. 3 gives the BER as a function of SNR for different values of the normalized CFO in OFDM with perfect CIR. The performance of OFDM without precoding is shown as a reference. The THP reduces ICI significantly. When  $\varepsilon = 0.1$ , THP-OFDM has almost the same BER as an OFDM system in the absence of CFO, i.e., the ICI has been cancelled completely.

Fig. 4 and 5 show the BER of THP-OFDM with channel mismatch. The channel is estimated using pilot symbols as in [12], where pilot symbols are multiplexed with the OFDM blocks in the time domain to enable channel estimation. BER of OFDM with perfect CIR and  $\varepsilon = 0$  and BER of OFDM without THP at  $\varepsilon = 0.1$  are shown as references. Fig. 4 presents BER when the receiver has perfect knowledge of CIR, while the transmitter has an estimated channel  $\hat{\mathbf{H}}$ . In order to guarantee reasonable performance of the channel estimator, every OFDM symbol is followed by a pilot block of length  $2N_{CP}$ , where  $N_{CP}$  is the length of cyclic prefix. The throughput loss incurred due to the pilot blocks is

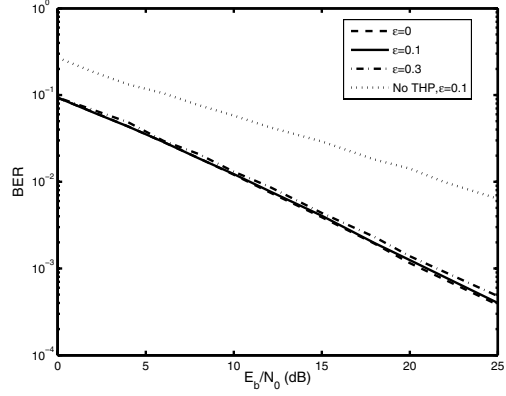


Fig. 3. BER with THP as a function of the SNR for different values of the normalized CFO and QPSK-OFDM ( $N = 64$ ), with perfect CIR at both the transmitter and the receiver.

$2N_{CP}/(N + N_{CP})$ . For a given data rate, it is possible that  $N \gg N_{CP}$  if the number of subcarriers is large. In this case, the throughput loss will be small. In Fig. 5, we assume that the transmitter and the receiver share the estimated channel matrix  $\hat{\mathbf{H}}$ . It can be seen that the channel mismatch only leads to slight BER degradation.

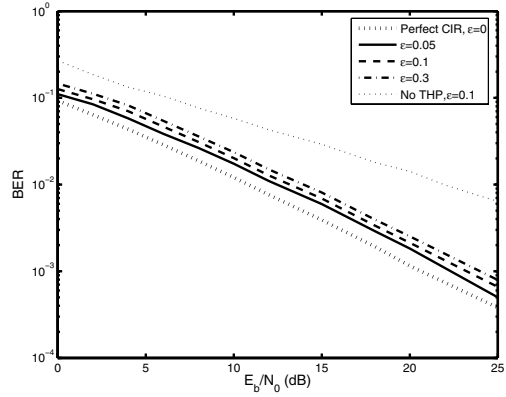


Fig. 4. BER with THP as a function of the SNR for different values of the normalized CFO for QPSK-OFDM ( $N = 64$ ) with perfect CIR at the receiver, and inaccurate CIR at the transmitter.

## VI. CONCLUSION

We have derived a non-linear TH precoder for ICI reduction in OFDM systems. We showed that the ICI coefficient matrix is approximately unitary and used this property to design the precoder for ICI reduction. The degradation due to CFO can be significantly reduced by our proposed non-linear TH precoding in OFDM. With the proposed precoder design, only partial CSI, not including the knowledge of the CFO, is needed at the transmitter. Two cases of channel mismatch are also analyzed for OFDM.

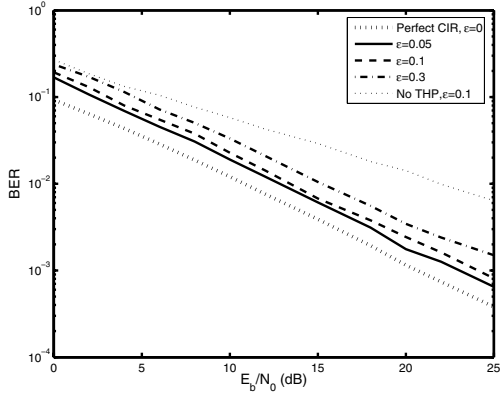


Fig. 5. BER with THP as a function of the SNR for different values of the normalized CFO for QPSK-OFDM ( $N = 64$ ) with inaccurate CIR at both transmitter and receiver.

## VII. APPENDIX

### Proof of unitary property of $\mathbf{S}$

We have two steps to prove that  $\mathbf{S}$  is unitary. Firstly, we prove that the diagonal entries of  $\mathbf{Z} = \mathbf{S}^H \mathbf{S}$ ,  $Z_{m,m}$ , approach unity. Secondly, we prove that off-diagonal terms vanish, i.e.,  $\frac{|Z_{m,n}|^2}{|Z_{m,m}|^2} \rightarrow \infty$  as  $\varepsilon \rightarrow 0$ .

From (4), we can immediately get  $S_{N-k} = S_{-k}$ . The  $\{m, n\}$ th entry of  $\mathbf{Z}$  is hence given by

$$Z_{m,n} = \sum_{k=-m}^{N-1-m} S_k S_{k+m-n}^* = \sum_{k=0}^{N-1} S_k S_{k+m-n}^* \quad (18)$$

As  $N \rightarrow \infty$ , for  $1 \leq k < \frac{N}{2}$  and  $k \gg \varepsilon$ , we have

$$\begin{aligned} S_k &\approx \frac{\sin \pi(\varepsilon + k)}{\pi(\varepsilon + k)} e^{j\pi(\varepsilon+k)} \\ &\approx \frac{\sin \pi\varepsilon}{\pi(\varepsilon + k)} e^{j\pi\varepsilon} \approx \frac{\sin \pi\varepsilon}{\pi k} e^{j\pi\varepsilon}. \end{aligned} \quad (19)$$

Note that when  $k = 0$ ,  $S_0 \approx \frac{\sin \pi\varepsilon}{\pi\varepsilon} e^{j\pi\varepsilon}$ .

Let us first consider diagonal entries of  $\mathbf{Z}$ . When  $m = n$ ,  $Z_{m,m}$  are the diagonal entries

$$Z_{m,m} = S_0 S_0^* + \sum_{k=1}^{N-1} S_k S_k^* = \frac{\sin^2 \pi\varepsilon}{(\pi\varepsilon)^2} + \frac{\sin^2 \pi\varepsilon}{\pi^2} \sum_{k=1}^{N-1} \frac{1}{k^2}, \quad (20)$$

where the second term in (20) is the Riemann Zeta function [13]; when  $N \rightarrow \infty$ ,  $\sum_{k=1}^{N-1} \frac{1}{k^2} \approx \frac{\pi^2}{6}$ . Therefore (20) can be approximated as

$$Z_{m,m} \approx \sin^2(\pi\varepsilon) \left[ \frac{1}{(\pi\varepsilon)^2} + \frac{1}{6} \right], \quad (21)$$

and as  $\varepsilon \rightarrow 0$ ,  $Z_{m,m} \rightarrow 1$ . We next consider the off-diagonal terms. When  $m \neq n$ , since  $\mathbf{Z}$  is a Hermitian matrix, i.e.,  $Z_{m,n} = Z_{n,m}^*$ , it is sufficient to consider the

case of  $m > n$ :

$$\begin{aligned} Z_{m,n} &= S_0 S_{m-n}^* + \sum_{k=1}^{N-1} S_k S_{k+m-n}^* \\ &= \frac{\sin^2 \pi\varepsilon}{\pi^2} \frac{1}{\varepsilon(m-n)} + \frac{\sin^2 \pi\varepsilon}{\pi^2} \sum_{k=1}^{N-1} \frac{1}{k(k+m-n)} \\ &= \frac{\sin^2 \pi\varepsilon}{\pi^2(m-n)} \frac{1}{\varepsilon} + \sum_{k=1}^{N-1} \frac{1}{k} - \sum_{k=1}^{N-1} \frac{1}{k+m-n}. \end{aligned} \quad (22)$$

Obviously, only when  $m - n = 1$ ,  $Z_{m,n}$  can reach the maximum value  $Z_{m,n, \max} = \frac{\sin^2 \pi\varepsilon}{\pi^2} \left( \frac{1}{\varepsilon} + 1 - \frac{1}{N} \right)$ . The power ratio of the diagonal entries to the non-diagonal entries can be given by

$$K = \frac{Z_{m,m}^2}{Z_{m,n}^2} > \frac{1 + \frac{\pi^2 \varepsilon^2}{6}}{\varepsilon^2 + \varepsilon}. \quad (23)$$

As  $\varepsilon \rightarrow 0$ ,  $K \rightarrow \frac{1}{\varepsilon^2}$ . For instance, when  $\varepsilon = 0.3$ ,  $K > 11.1$ , which means over 90% energy is concentrated on the main diagonal.  $K$  rapidly increases as  $\varepsilon$  decreases. Therefore, the ICI coefficient matrix  $\mathbf{S}$  can be approximated as a unitary matrix.

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