NEAR-OPTIMAL LOW-COMPLEXITY DETECTION FOR SPACE-DIVISION MULTIPLE ACCESS WIRELESS SYSTEMS

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ABSTRACT

A symbol detector for wireless systems using space division multiple access (SDMA) and orthogonal frequency division multiplexing (OFDM) is derived. The detector uses a sphere decoder (SD) and provides near optimal symbol detection with computational complexity much less than the naive maximum likelihood (ML) detector. We also show how to detect non-constant modulus signals with constrained least squares (CLS) receiver, which is designed for constant modulus (unitary) signals. The new detector significantly outperforms previous detectors in both uncoded and coded systems.

1. INTRODUCTION

Space division multiple access (SDMA) employs a receiver antenna array and allows different users to share the same spectrum [1], achieving much higher spectral efficiencies. SDMA separates different users based on their spatial signatures, i.e., their unique channel transfer function, by processing the signals received at the receiver antenna array. Orthogonal frequency division multiplexing (OFDM) uses multiple orthogonal subcarriers and offers simple equalization for fading mitigation [2].

The combination of SDMA and OFDM for wireless networks exploits the advantages of both [3–5]. In the uplink channel, the base station handles the SDMA function while the user terminals are plain OFDM modems. This asymmetry keeps the overall cost of this wireless system low. Minimum mean squared error (MMSE) detectors [3] perform worse than the ML detector. The detector designed in [5] is specifically for unitary signals and outperforms the MMSE detector. A minimum bit error rate multiuser detector is designed in [6]. Reference [7] develops two linear demodulators for minimizing the probability of error. However, all these approaches have a significant performance loss compared with the ML detector.

In this paper, we propose a new detector based on a sphere decoder (SD) for OFDM/SDMA based wireless systems. Sphere decoding [8] is investigated for multiple antenna systems in [9]. The average complexity of the sphere decoding used for ML detection in flat fading multiple-antenna systems is polynomial (often sub-cubic) for a wide range of signal-to-noise ratios (SNRs). By employing sphere decoding, we obtain an optimal detector with computational complexity much less than the naive ML detector. Furthermore, we show how the CLS detector of [5] can be used to detect non-unitary signals.

Notation: $(\cdot)^H$ and $(\cdot)^\dagger$ denote conjugate transpose and Moore-Penrose pseudo-inverse, respectively.

2. OFDM/SDMA SYSTEM MODEL

We consider the uplink of an OFDM/SDMA system (Fig. 1) with U users and a basestation (BS). Each user transmit with a single antenna and the BS has $A \geq 1$ receive antennas. Let the number of simultaneous users be no greater than the number of BS antennas. The bit stream of user u is modulated and a block of K modulated symbols is converted to a serial stream of $s_u[1], \cdots, s_u[K]$. This frequency domain series is fed into a K-tap inverse fast Fourier transform (IFFT) to obtain time-domain samples. A cyclic prefix and the samples are transmitted through a multipath fading channel. The received signals at BS antenna a are converted to a parallel stream and the cyclic prefix is removed. A K-tap fast Fourier transform (FFT) operator is used to obtain $y_a[1], \cdots, y_a[K]$, which are processed by SDMA algorithms to separate different users and provide the estimated data $\tilde{s}_u[1], \cdots, \tilde{s}_u[K]$ as for user u.

The cyclic prefix is longer than the channel delay spread, and the k-th subcarrier output is given by

$$\mathbf{y} = \mathbf{H}\mathbf{s} + \mathbf{n}, \tag{1}$$

This work has been supported in part by the Natural Sciences and Engineering Research Council of Canada and Informatics Circle of Research Excellence.

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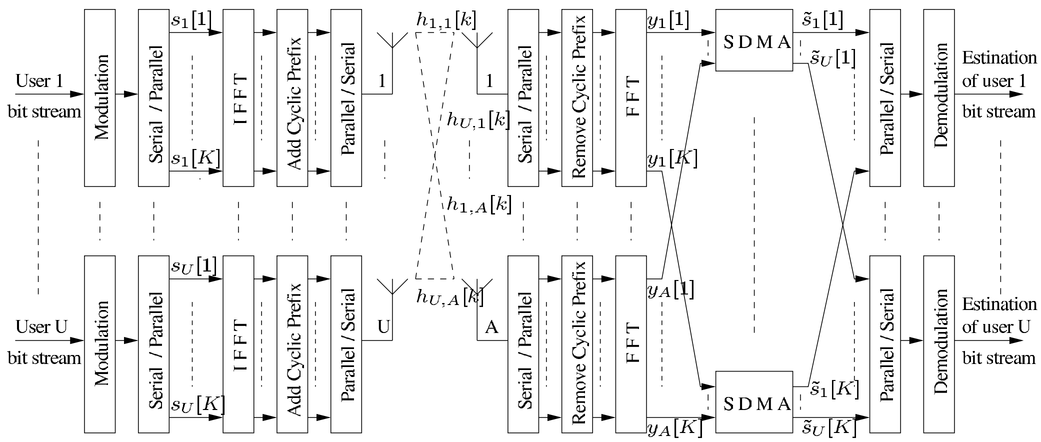

Fig. 1. System model

where $\mathbf{y} = [y_1[k], \cdots, y_A[k]]^T$ is the received signals, $\mathbf{s} = [s_1[k], \cdots, s_U[k]]^T$ is the transmitted signals from U users, $\mathbf{n} = [n_1[k], \cdots, n_A[k]]^T$ is the additive white Gaussian noise, and

$$\mathbf{H} = \begin{bmatrix} h_{1,1}[k] & \cdots & h_{U,1}[k] \\ \vdots & \ddots & \vdots \\ h_{1,A}[k] & \cdots & h_{U,A}[k] \end{bmatrix} \tag{2}$$

is the frequency domain channel transfer function matrix, where $h_{a,u}[k]$ is the channel between the u-th user and the a-th BS antenna. We assume they are uncorrelated, time-invariant, and complex Gaussian distributed fading processes with zero mean and unit variance. Since SDMA processing is applied on a per-carrier basis, symbol detection can also be solved per subcarrier. For notational simplicity, the index $[k]$ is dropped.

3. NEW DETECTOR

The maximum likelihood (ML) detector is given by

$$\tilde{\mathbf{s}} = \arg\min_{\mathbf{s}\in\mathcal{Q}^U} \|\mathbf{y} - \mathbf{H}\mathbf{s}\|^2, \tag{3}$$

where $\mathbf{y} \in \mathcal{R}^A$, $\mathbf{H} \in \mathcal{R}^{A\times U}$ and $\mathcal{Q} = \{-(2P-1), \cdots, 2P+1\}$ is a Pulse Amplitude Modulation (PAM) constellation of P elements. Since any QAM constellation can be decoupled into real part and image part and transformed into real PAM constellations, this formulation (3) is valid for QAM too. We later consider the case of M-ary phase shift keying (M-PSK). A brute force ML algorithm must search all P^U possible values of $\mathbf{s}$. Hence, its complexity is exponential in U (the number of variables to be estimated).

The original SD by Fincke and Phost [8] restricts the search space to the candidate signals within a hypersphere of an initial radius. Whenever a valid candidate is found, the radius may be updated. Eq. (3) is equivalent to

$$\begin{aligned}\tilde{\mathbf{s}} &= \arg\min_{\mathbf{s}\in\mathcal{Q}^U} (\mathbf{H}^\dagger\mathbf{y} - \mathbf{s})^H\mathbf{R}^H\mathbf{R}(\mathbf{H}^\dagger\mathbf{y} - \mathbf{s}) \\ &= \arg\min_{\mathbf{s}\in\mathcal{Q}^U} (\mathbf{s} - \hat{\mathbf{s}})^H\mathbf{R}^H\mathbf{R}(\mathbf{s} - \hat{\mathbf{s}})\end{aligned} \tag{4}$$

where $\hat{\mathbf{s}} = \mathbf{H}^\dagger\mathbf{y}$ and $\mathbf{R}$ is an upper triangular matrix such that $\mathbf{R}^H\mathbf{R} = \mathbf{H}^H\mathbf{H}$. Let the entries of $\mathbf{R}$ be denoted by r_{ij}, $i \leq j$. Since the number of users U is always less than or equal to the number of BS antennas A, $\mathbf{H}^H\mathbf{H}$ has full rank (positive definite). The diagonal terms of $\mathbf{R}$ are non-zero ($r_{ii} \neq 0$). A candidate is inside a hypersphere of radius r when

$$(\mathbf{s} - \hat{\mathbf{s}})^H\mathbf{R}^H\mathbf{R}(\mathbf{s} - \hat{\mathbf{s}}) = \sum_{i=1}^{U} r_{ii}^2 \|s_i - \rho_i\|^2 \leq r^2 \tag{5}$$

where $\rho_i = \hat{s}_i - \sum_{j=i+1}^{U} \frac{r_{ij}}{r_{ii}}(s_j - \hat{s}_j)$. We assume the initial radius r is large enough so that the hypersphere (5) contains at least one candidate solution. The initial radius may be chosen according to noise variance [9].

Since each term in (5) is nonnegative, the partial sums must also be less than r^2. This fact can be used to derive a set of admissible values for s_u ($u = U, U-1, \ldots, 1$) as

$$\mathcal{L}_u = \Big\{t \,\Big|\, \lceil \rho_u - r_u/r_{uu} \rceil \leq t \leq \lfloor \rho_u + r_u/r_{uu} \rfloor, t \in \mathcal{Q}\Big\} \tag{6}$$

where $r_u^2 = r^2 - \sum_{i=u+1}^{U} r_{ii}^2 \|s_i - \rho_i\|^2$, $s_i \in \mathcal{L}_i$, $\lceil x \rceil$ is the smallest integer greater than or equal to x and $\lfloor x \rfloor$ is the largest integer less than or equal to x. In (6), the signal constellation $\mathcal{Q}$ can be either PAM or QAM (expressed as two real PAM constellations). Eq. (6) generates all signal vectors satisfying (5), the number of which is much smaller than $|\mathcal{Q}|^U$.

If $\mathcal{Q}$ is M-PSK, (6) is not directly applicable ($\mathcal{Q}$ is generally not decomposable into two real PAM constellations).

We thus form the set of lists for $u = U, U-1, \ldots, 1$ as

$$\mathcal{L}_u = \left\{ t \,|\, r_{uu}^2 \left\| t - \rho_i \right\|^2 \le r_u^2, t \in \mathcal{Q} \right\}. \quad (7)$$

Once all the candidate signal vectors are generated via (6) or (7), the ML estimate is given by the candidate vector which minimizes (4). Putting all these ideas together, we get the following new detector algorithm:

1 Input ($\mathbf{y}$, $\mathbf{H}$, r, $\mathcal{Q}$);

2 $[\mathbf{Q}, \mathbf{R}] = qr(\mathbf{H})$; $\hat{\mathbf{s}} = \mathbf{H}^\dagger \mathbf{y}$; $i = U$; $l = 1$; $\rho_U = \hat{s}_U$; $d^2 = r^2$;$r_U^2 = r^2$; $\mathcal{L}_U = \emptyset$; found=0;

3 Generate $\mathcal{L}_i$ using (6) or (7);

4 If $\mathcal{L}_i \ne \emptyset$ and $i = 1$, goto 8;

5 If $\mathcal{L}_i \ne \emptyset$, $\rho_{i-1} = \hat{s}_{i-1} - \sum_{j=i}^{U} \frac{r_{i-1,j}}{r_{i-1,i-1}}(s_j - \hat{s}_j)$, $r_{i-1}^2 = r_i^2 - r_{ii}^2 \|s_i - \rho_i\|^2$, $i = i-1$, $l = 1$, goto 3;

6 If $i = U$, if found=1, go to 9, else increase r, $l = 1$, goto 3;

7 $i = i+1$, $l = l+1$, if $l > M$, goto 4, else goto 3;

8 found=1, $\mathbf{s}^* = \mathbf{s}$, $\tilde{d}^2 = r_1^2 - r_{11}^2 \left\| s_1 - \rho_1 \right\|^2$, $l = 1$, goto 3;

9 If $\tilde{d}^2 < d^2$, $d = \tilde{d}$, goto 7, else return $\mathbf{s}^*$.

4. NON-UNITARY SIGNALS

An M-QAM vector can be represented as a weighted sum of $n/2$ QPSK vectors when $M = 2^n$ and n is an even number. For example, 16QAM transmit vector $\mathbf{x}$ can be written as $\mathbf{s} = \sqrt{2}\mathbf{s}_1 + \frac{\sqrt{2}}{2}\mathbf{s}_2$ where $\mathbf{s}_1$ and $\mathbf{s}_2$ are QPSK vectors. Thus (1) becomes

$$\mathbf{y} = \begin{pmatrix} \sqrt{2}\mathbf{H} & \frac{\sqrt{2}}{2}\mathbf{H} \end{pmatrix} \begin{pmatrix} \mathbf{s}_1 \\ \mathbf{s}_2 \end{pmatrix} + \mathbf{n}. \quad (8)$$

A 16QAM system with N transmit and M receive antennas is equivalent to a QPSK system with $2N$ transmit antenns and M receive antennas. Similarly, any other M-QAM system can also be expressed as an equivalent QPSK system. Therefore the CLS detector in [5] can be used with M-QAM.

5. RESULTS AND CONCLUSION

A current wireless local-area network (LAN) standard at 5GHz is ETSI BRAN HIPERLAN/2 [10]. We consider such a wireless LAN with four simultaneously transmitting users and a BS with four antennas. The simulation parameters are summarized in Table I.

Table 1. Simulation Parameters

Parameter	Value
Number of simultaneous users	4
Number of basestation antennas	4
Bandwidth for each user (MHz)	20
Number of subcarriers	64
Number of data subcarriers	48
Cyclic Prefix Length	16

Fig. 2 and Fig. 3 compare the error performance of the various detectors for the system shown in Table 1. We consider QPSK in Fig. 2 and 16QAM in Fig. 3. For low SNR, MMSE detector gains 2 dB over the V-BLAST detector, while the latter outperforms the former by more than 2 dB in high SNR. The MMSE detector performs slightly better than the CLS detector for both QPSK and 16QAM. Regardless of SNR, the ZF detector has the worst performance and our new detector (labelled as SD) performs significantly better than the other four detectors. It gains about 7 dB over the V-BLAST detector at a BER equal to 10^{-2}. This gain increases as the SNR increases. The diversity order (the slope of the error rate curve) is higher for our new detector than that for V-BLAST.

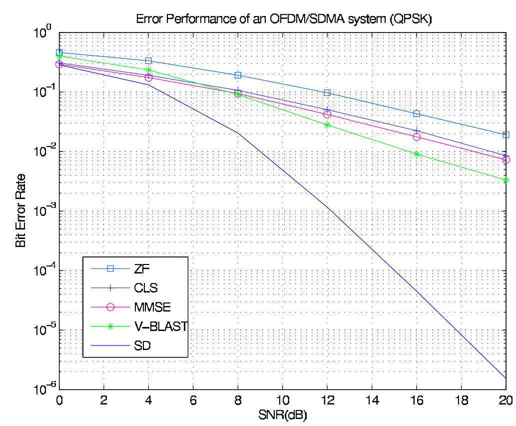

Fig. 2. Bit error rate of the detectors for QPSK.

Fig. 4 shows the performance of the five detectors when channel coding is included. We use the industry-standard generator polynomials $g_1 = 133_{OCT}$ and $g_2 = 171_{OCT}$ of rate 1/2 convolutional code with constraint length 7. We consider binary PSK (BPSK). Again, our new detector performs significantly better than the other four detectors.

Fig. 5 shows the computational complexity of the detectors for QPSK. For low SNR, the new detector has a higher computational complexity, but its complexity decreases significantly as SNR increases. For high SNR, the complexity of our detector approaches that of V-BLAST detector, which has a higher complexity than other detectors.

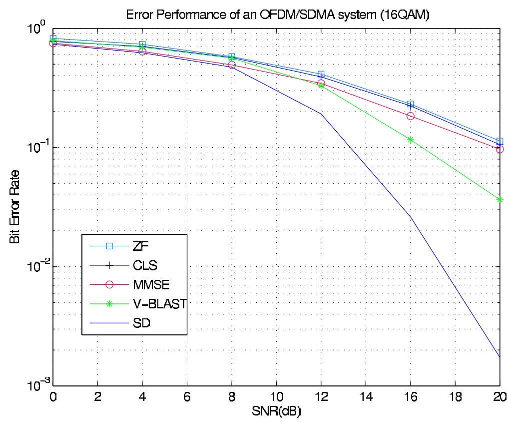

Fig. 3. Bit error rate of the detectors for 16QAM.

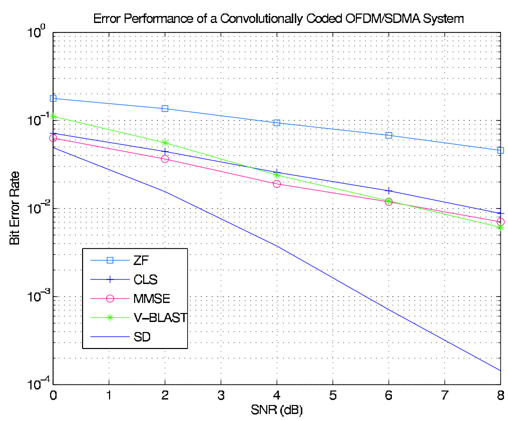

Fig. 4. Bit error rate with convolutional coding.

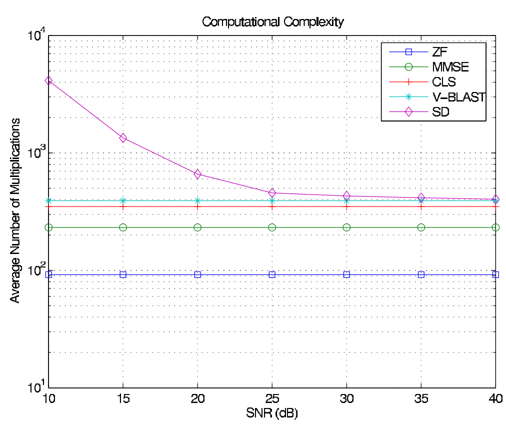

Fig. 5. Computational complexity of the detectors.

We proposed a near-optimal detector based on the sphere decoder for OFDM/SDMA based wireless systems. We also showed how to detect non-unitary signals with a constrained least squares (CLS) receiver. The results show that our new detector significantly outperforms ZF, CLS, MMSE, and V-BLAST detectors in both uncoded and coded systems.

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