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CYCLIC-PREFIX BASED NOISE VARIANCE AND POWER DELAY PROFILE ESTIMATION FOR OFDM SYSTEMS

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ABSTRACT

In this paper, we derive cyclic-prefix (CP) based parameter estimators for Orthogonal Frequency Division Multiplexing (OFDM) systems. Signal correlation due to the use of the CP is exploited without requiring additional pilot symbols. Noise variance, power delay profile, the number of paths and the signal to noise ratio are estimated. The proposed algorithms can be readily used for applications such as minimum mean-square error (MMSE) equalization.

1. INTRODUCTION

Noise variance, or (equivalently) signal to noise ratio (SNR), is an important measure of channel quality. Their estimation is hence required in many communication applications such as adaptive modulation, turbo coding and others. Several SNR estimation algorithms have been proposed for systems using unitary constellations (i.e., binary phase shift keying (BPSK) and quaternary phase shift keying (QPSK)) over AWGN channels [1, 2]. They can be classified as data-aided (DA), which requires pilot symbols, and non-data aided (NDA) estimators, which do not. In [3], an NDA estimator is extended to systems with non-unitary constellations over Rayleigh fading channels.

In orthogonal frequency division multiplexing (OFDM) systems, noise variance and power delay profile (PDP) are needed for many algorithms such as minimum mean-square error (MMSE) equalization, channel estimation and ML frequency offset estimation. In [4], a noise-variance estimator is proposed by directly using the receiver statistics. A subspace approach is presented in [5] using the sample covariance matrix of the received signal. However, both algorithms are DA estimators, resulting in a bandwidth loss. The estimation of the number of paths and associated multipath time delays is proposed in [6], where pilot symbols are also needed. In [7], a noise-variance and SNR estimator for multiple antenna OFDM systems is developed using training symbols. Except for these contributions, to the best of our knowledge, no other NDA noise variance and PDP estimators for OFDM systems have been published to date.

In this paper, we develop novel noise-variance and PDP estimators for OFDM systems over multipath fading channels; the key is to use the fact that the cyclic prefix contains the repeated samples which will introduce a special correlation structure on the received samples. The noise variance, the number of paths, multipath time delays and powers are jointly estimated without pilots. We first derive the joint maximum likelihood (ML) function for these parameters and approximately solve the ML function to obtain the estimates.

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2. NOISE VARIANCE AND PDP ESTIMATOR

In OFDM, source data are grouped and mapped into $X_k \in \mathcal{Q}$, where $\mathcal{Q}$ is a complex signal constellation, and $E\{|X_k|^2 = 1\}$. Complex data are modulated by the inverse discrete Fourier transform (IDFT) on N parallel subcarriers. The symbol interval and block interval are denoted by T_s and NT_s. The resulting OFDM symbol during the mth block interval that comprises N samples is given by

$$x_n(m) = \frac{1}{N}\sum_{k=0}^{N-1} X_k(m)e^{\jmath(2\pi kn/N)},\ n = 0, \cdots, N-1. \tag{1}$$

The guard interval, inserted to prevent inter-block interference, includes a cyclic prefix that replicates the end of the IDFT output samples. The number of samples in the guard interval N_g is assumed to be larger than the delay spread of the channel. The signal is transmitted over a multipath

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fading channel given by

$$h(t) = \sum_{l=0}^{L-1} h_l \delta(t - \tau_l) \quad (2)$$

where L is the total number of multipaths, $h_l \sim \mathcal{CN}(0, \sigma_l^2)$, and τ_l is the delay of the l-th path. The received signal after sampling is given by

$$y_n(m) = \sum_{l=0}^{L-1} h_l x_{n-d_l}(m) + w_n(m) \quad (3)$$

where $w_n \sim \mathcal{CN}(0, \sigma_n^2)$ is an Additive White Gaussian Noise (AWGN) and $d_l = \lfloor \tau_l / T_s \rceil$ is the delay normalized by T_s. For simplicity, we round d_l to an integer without considering leakage. However, the correlation approach in this paper may also be extended to fractional d_l. We assume perfect synchronization, and that the channel is invariant within each OFDM block. If there exists a synchronization error, a decision directed algorithm may be applied using our proposed parameter estimators and the joint ML time and frequency offset estimator in [8].

At the border between two OFDM blocks ($-N_g \le n < 0$), the received signal samples can be written as

$$\begin{aligned} y_n(m) = & \sum_{l=0}^{L-1} h_l x_{n-d_l}(m) U(n - d_l) \\ & + \sum_{l=0}^{L-1} h_l x_{N+n-d_l}(m-1) U(d_l - n) + w_n(m) \end{aligned} \quad (4)$$

where $U(\cdot)$ is the step function. The correlation between each received signal sample over the CP interval and its corresponding sample at the end of the OFDM block can thus be given by

$$E\{y_{-k}(m) y_{N-k}^*(m)\} = \begin{cases} \sigma_y^2 + \sigma_n^2 & m = 0 \\ \sum_{l=0}^{L-1} \sigma_l^2 U(N_g - k - d_l) & m = N \\ 0 & \text{otherwise} \end{cases} \quad (5)$$

where $\sigma_y^2 = E\{|y_n(m)|^2\} = \sum_{l=0}^{L-1} \sigma_l^2$ and $k = 1, \ldots, N_g$. The expectation in (5) is taken with respect to both h_l and $x_n(m)$.

We may approximately model $y_n(m)$ as complex Gaussian using the central limited theorem, and the probability density function (pdf) is given by

$$f(y_n(m)) = \frac{\exp\left(-\frac{|y_n(m)|^2}{\sigma_y^2 + \sigma_n^2}\right)}{\pi(\sigma_y^2 + \sigma_n^2)}. \quad (6)$$

Samples $y_{-k}(m)$ and $y_{N-k}(m)$ are jointly Gaussian with pdf

$$\begin{aligned} & f(y_{-k}(m), y_{N-k}(m)) \\ & = \frac{\exp\left(-\frac{|y_{-k}(m)|^2 + |y_{N-k}(m)|^2 - 2\rho_k \Re\{y_{-k}(m) y_{N-k}^*(m)\}}{(\sigma_y^2 + \sigma_n^2)(1 - \rho_k^2)}\right)}{\pi^2(\sigma_y^2 + \sigma_n^2)(1 - \rho_k^2)} \end{aligned} \quad (7)$$

where

$$\begin{aligned} \rho_k = & \left| \frac{E\{y_{-k}(m) y_{N-k}^*(m)\}}{\sqrt{E\{|y_{-k}(m)|^2\} E\{|y_{N-k}(m)|^2\}}} \right| \\ = & \frac{\sum_{l=0}^{L-1} \sigma_l^2 U(N_g - k - d_l)}{\sum_{l=0}^{L-1} \sigma_l^2 + \sigma_n^2}. \end{aligned} \quad (8)$$

We use M OFDM blocks to estimate those parameters and assume that they remain unchanged during the M blocks. Define $\mathbf{p} = [\sigma_0^2, \ldots, \sigma_{L-1}^2]$, $\mathbf{d} = [d_0, \ldots, d_{L-1}]$ and $\mathbf{y} = [y_{-N_g}(1), y_{-N_g+1}(1), \ldots, y_{N-1}(M)]$. Using (6) and (7), the joint log-likelihood function can be written as

$$\begin{aligned} & \Lambda(\sigma_n^2, \mathbf{p}, \mathbf{d}) \\ & = \sum_{m=1}^{M} \log \left(\prod_{k=1}^{N_g} f(y_{-k}(m), y_{N-k}(m)) \prod_{k=0}^{N-1} f(y_k(m)) \right) \\ & = -M \left(\sum_{k=1}^{N_g} \frac{a_k - \rho_k b_k}{c(1 - \rho_k^2)} + \log(c(1 - \rho_k^2)) + \sum_{k=1}^{N} \frac{g_k}{c} + \log(c) \right) \end{aligned} \quad (9)$$

where

$$\begin{aligned} a_k = & \frac{\sum_{m=1}^{M} |y_{-k}(m)|^2 + |y_{N-k}(m)|^2}{M} \\ b_k = & \frac{\sum_{m=1}^{M} \Re\{y_{-k}(m) y_{N-k}^*(m)\}}{M} \\ g_k = & \frac{\sum_{m=1}^{M} |y_k(m)|^2}{M}, \ c = \sigma_y^2 + \sigma_n^2. \end{aligned} \quad (10)$$

Since (9) involves many variables, we first estimate c by maximizing only the last sum in (9), and c is obtained as

$$\hat{c} = \frac{\sum_{k=1}^{N} g_k}{N} = \frac{\sum_{k=1}^{N} \sum_{m=1}^{M} |y_k(m)|^2}{NM}. \quad (11)$$

Substituting $\hat{c}$ back into the sum of (9), and maximizing the k-th summand individually, we get an estimate for ρ_k as the real root of the equation

$$2\hat{c}\rho^3 - b\rho^2 - 2(\hat{c} - a_k)\rho - b = 0. \quad (12)$$

Note that there is an explicit expression for this root, but it is omitted for brevity. We compute the value s_k as

$$s_k = \begin{cases} \rho_{N_g} \hat{c} & k = 1 \\ (\rho_{N_g-k+1} - \rho_{N_g-k+2})\hat{c} & k = 2, \ldots, N_g \end{cases}. \quad (13)$$

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A threshold value is set as $\alpha\hat{c}$, where α is a constant less than 1. If $s_k > \alpha\hat{c}$, it is identified as a path, s_k is the estimate of path power and k is estimate of delay time. The number of paths is estimated as the number of s_k with $s_k > \alpha\hat{c}$. We denote the maximum delay time as $d_{\max}$. The noise variance can thus be estimated as

$$\hat{\sigma_n}^2 = \hat{c}\left(1 - \frac{\sum_{k=1}^{N_g - d_{\max}} \hat{\rho}_k}{N_g - d_{\max}}\right). \quad (14)$$

Due to the repetition of samples in the CP region in each OFDM block, in the absence of noise, $y_{-k}(m) = y_{N-k}(m)$ for $k = 1, \ldots, N_g - d_{\max}$. The noise variance can thus be obtained alternatively as

$$\hat{\sigma_n}^2 = \frac{\sum_{m=1}^{M}\sum_{k=1}^{N_g - d_{\max}} |y_{N-k}(m) - y_{-k}(m)|^2}{2M(N_g - d_{\max})}. \quad (15)$$

Using the results of (11) and (14) or (15), SNR can be estimated by

$$\widehat{\mathrm{SNR}} = \frac{\hat{c}}{\hat{\sigma_n}^2}. \quad (16)$$

3. SIMULATION RESULTS

We now investigate how our proposed estimators perform for a QPSK-OFDM system with $N = 64$ subcarriers and CP length $N_g = 16$. A channel with six taps ($L = 6$ and a power profile $\mathbf{p} = [0.189, 0.379, 0.239, 0.095, 0.061, 0.037]$) is used. The delay profile after sampling is $\mathbf{d} = [0, 1, 2, 4, 6, 8]$. Each path is an independent, zero-mean complex Gaussian random process.

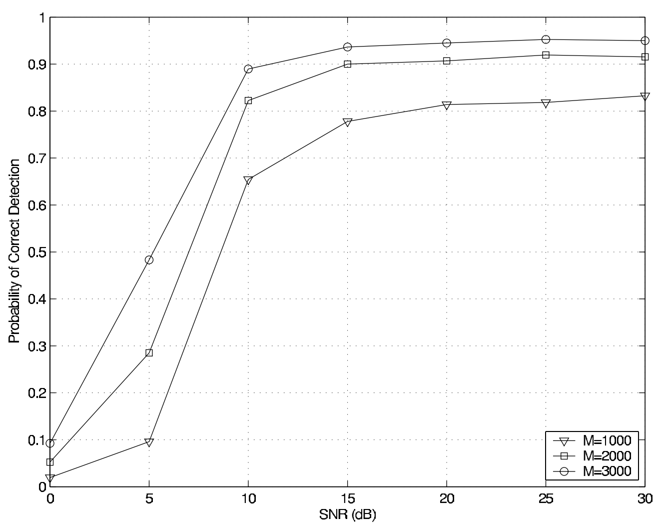

Fig. 1. The probability of correct detection of the number of paths.

Fig. 1 shows the probability of correct detection of the number of paths using our proposed algorithm with different M. The threshold parameter is set to $\alpha = 0.01$. In low SNR, the paths with smaller power are dominated by the noise, and there may be many paths larger than the threshold. Therefore, the number of paths may be overestimated. The probability of correct detection increases in high SNR. With increasing M, the probability of detection error decreases.

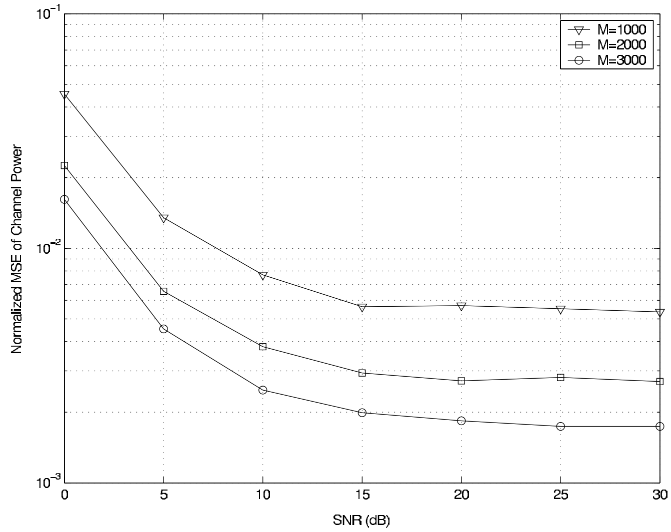

Fig. 2. The NMSE of the channel power estimation for the 3rd path.

Fig. 2 presents the normalized mean square error (NMSE) of the channel power estimation for the 3rd path (arbitrarily chosen), where the NMSE is defined as NMSE $= E\{(\hat{\sigma}_3^2 - \sigma_3^2)^2\}/\sigma_3^4$. The channel power is overwhelmed by the noise in low SNR. In high SNR, the NMSE is constant, and it naturally decreases when the number of OFDM blocks available for estimation, M, increases.

Fig. 3 shows the NMSE of the noise variance estimation using different estimators, where the NMSE is defined as NMSE $= E\{(\hat{\sigma}_n^2 - \sigma_n^2)^2\}/\sigma_n^4$. The estimator using (14) is denoted as ML, and that using (15) is denoted as direct estimator or (DI). In low SNR, DI performs better than ML since the probability of $d_{\max}$ estimation is high. In high SNR, DI and ML both perform identically. When more OFDM blocks are available for estimation, both the estimators perform better.

Note that these estimators require a large number of blocks for their operation. This is rather typical of blind approaches where no pilots are used and is indeed the price paid for not using pilots. We have also observed that the estimated mean of a small set (several hundred) of Gaussian random variables generated by MATLAB is non-zero. We suspect that this discrepancy might impact the convergence

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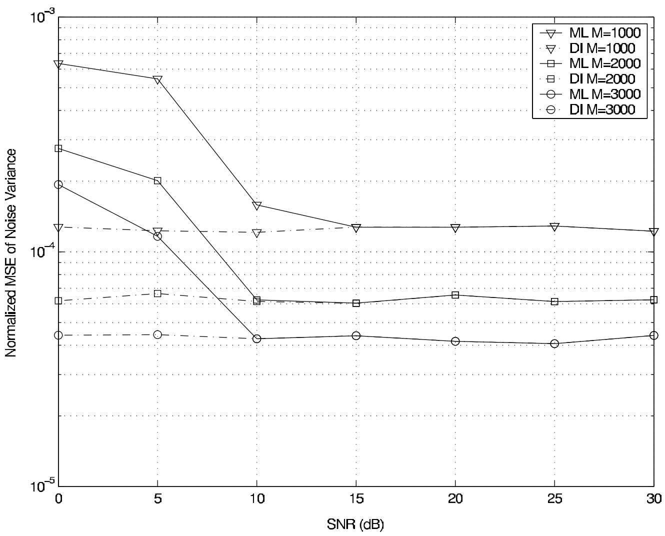

Fig. 3. The NMSE of the noise variance estimation using different estimators.

of the estimators.

4. CONCLUSION

This paper has derived estimators for noise-variance, power-delay-profile, the number of paths and the SNR using the CP, avoiding the use of pilot symbols. This derivation has exploited the correlation structure due to the use of CP. The results show that our proposed estimators effectively estimate the channel parameters and hence may be used for applications such as MMSE equalization and channel estimation in OFDM.

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