POLYNOMIAL CONSTRAINED DETECTION FOR MIMO SYSTEMS USING PENALTY FUNCTION

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ABSTRACT

In this paper, we develop a family of approximate maximum likelihood (ML) detectors for multiple-input multiple-output (MIMO) systems by relaxing the ML detection problem. Polynomial constraints are formulated for any signal constellation. The resulting relaxed constrained optimization problem is solved using a penalty function approach. Moreover, to escape from the local minima and to improve the performance of detection, a probabilistic restart algorithm based on noise statistics is proposed. Simulation results show that our polynomial constrained detectors perform better than several existing detectors.

1. INTRODUCTION

The enormous capacity and remarkably high spectral efficiencies promised by multiple-input multiple-output (MIMO) wireless systems in rich scattering multipath environments has motivated much research. High capacity has been the driving force for the development of MIMO systems. Along with the theoretical studies, detection algorithms have been proposed to exploit the capacity provided by MIMO systems. The vertical Bell Laboratories layered space time (V-BLAST) detector was among the first to be developed [1].

The V-BLAST detector, or equivalently the zero-forcing (ZF) decision feedback detector (DFD), is based on nulling and interference with optimal ordering. It is commonly used for performance comparisons of MIMO detection algorithms and provides a reference for our detectors as well. However, the V-BLAST detector performs much worse than the maximum likelihood detector (MLD), which achieves the minimum error probability for independent and identically distributed (i.i.d.) random data symbols. In [2], sphere decoding (SD), which offers near-optimal performance, is proposed to attain low complexity in high SNR. But its complexity is still high in low SNR or for large systems. The large gap in both performance and complexity between the MLD and V-BLAST detector has motivated the search for alternative detectors.

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The relaxation approach has been applied to code-division multiple-access (CDMA) and orthogonal frequency division multiplexing (OFDM) / spatial division multiple access (SDMA) systems. In [3], a generalized MMSE (GMMSE) detector is proposed for CDMA, where the binary phase shift keying (BPSK) vectors are relaxed to lie inside a hypercube. A tighter relaxation for OFDM / SDMA systems is used in [4] by restricting the binary vectors on a hypersphere rather than within a hypercube. In [5], semidefinite relaxation (SDR) has been applied to CDMA systems with BPSK. SDR has been extended to general M-PSK constellations in [6]. However, all these relaxations are loose, which motivates the search for tighter and universal relaxations applicable for any constellations.

In this paper, we propose a polynomial relaxation for MIMO systems detection. The new relaxation can be applied for any constellation. The ML MIMO detection problem is hence reformulated as an equality constrained minimization problem. The constrained optimization problem is solved by using a penalty function with the Newton method. Since the Newton method may be trapped by some local minima, a random restart strategy using the noise statistics is proposed.

Notation: Bold symbols denote matrices or vectors. $(\cdot)^T$, $(\cdot)^H$ and $(\cdot)^*$ denote transpose, conjugate transpose and conjugate, respectively. $(\cdot)^\dagger$ denotes pseudo-inverse. $\mathrm{Re}\{x\}$ and $\mathrm{Im}\{x\}$ denote the real part and imaginary part of x, respectively. $\|(\cdot)\|^2$ is the 2-norm of $(\cdot)$. The set of all complex $K\times 1$ vectors is denoted by $\mathcal{C}^K$. A circularly complex Gaussian variable with mean μ and variance σ^2 is denoted by $z \sim \mathcal{CN}(\mu, \sigma^2)$.

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2. MIMO SYSTEM MODEL

A MIMO system with n transmit antennas and m receive antennas is considered. We focus on spatial multiplexing systems, where the signals are statistically independent rather than jointly encoded. Source data are mapped into complex symbols from a finite constellation $\mathcal{Q}$ of size s, and $\mathcal{Q} = \{q_1, q_2, \ldots, q_s\}$. The data stream is demultiplexed into n equal length substreams, and each of which is simultaneously sent through n antennas over a rich scattering channel. We assume that the MIMO channel is flat fading. Each receive antenna receives signals from all the n transmit antennas. The discrete-time baseband received signals can be written as

$$\mathbf{r} = \mathbf{H}\mathbf{x} + \mathbf{n} \tag{1}$$

where $\mathbf{x} = [x_1, \ldots, x_n]^T$, $x_i \in \mathcal{Q}$ is the transmitted signal vector, $\mathbf{y} = [y_1, \ldots, y_m]^T$, $y_i \in \mathcal{C}^n$ is the received signal vector, $\mathbf{H} = [h_{i,j}] \in \mathcal{C}^{m \times n}$ is the channel matrix, and $\mathbf{n} = [n_1, \ldots, n_m]^T$, $n_i \in \mathcal{C}^m$ is an additive white Gaussian noise (AWGN) vector. The elements of $\mathbf{H}$ are identically independent distributed (i.i.d.) complex Gaussian, $h_{i,j} \sim \mathcal{CN}(0, 1)$. The components of $\mathbf{n}$ are i.i.d., and $n_i \sim \mathcal{CN}(0, \sigma_n^2)$. We assume that the channel is perfectly known to the receiver, and $n \leq m$. If $n > m$, we can readily transform the rank deficient system into a full rank system using the algorithm in [7]. Note (1) models any linear, synchronous and flat fading channels. It can be directly applied to multiuser detection in CDMA. Therefore, the relaxation proposed in this paper can be readily applied to CDMA systems.

Assuming uncorrelated noise and transmitted signals, the MLD that minimizes the average error probability is given by

$$\hat{\mathbf{x}} = \arg\min_{\mathbf{x} \in \mathcal{Q}^n} \|\mathbf{r} - \mathbf{H}\mathbf{x}\|^2. \tag{2}$$

Eq. (2) is an NP-hard problem, and the complexity of brute-force search is exponential in n.

3. POLYNOMIAL CONSTRAINT AND PENALTY FUNCTION METHOD

Due to the finite alphabet nature of $\mathcal{Q}$, for each $x_i \in \mathcal{Q}$, we have the following equation

$$f(x_i) = \prod_{k=1}^{s} (x_i - q_k) = 0. \tag{3}$$

The ML detection problem (2) can thus be relaxed as

$$\begin{aligned} &\min_{\mathbf{x} \in \mathcal{C}^n} \|\mathbf{r} - \mathbf{H}\mathbf{x}\|^2 \\ &\text{s.t. } f(x_i) = \prod_{k=1}^{s} (x_i - q_k) = 0,\ i = 1, \ldots, n. \end{aligned} \tag{4}$$

Both the objective function and constraints are polynomials in $\mathbf{x}$. For example, for BPSK, $f(x_i) = (x_i - 1)(x_i + 1)$. To avoid the complex operation, Eq. (4) can be transformed into a real problem as

$$\begin{aligned} &\min_{\tilde{\mathbf{x}} \in \mathcal{R}^{2n}} \|\tilde{\mathbf{r}} - \tilde{\mathbf{H}}\tilde{\mathbf{x}}\|^2 \\ &\quad \text{s.t. } g^r(\Re\{x_i\}, \Im\{x_i\}) = 0 \text{ and } g^i(\Re\{x_i\}, \Im\{x_i\}) = 0 \\ &\quad\quad i = 1, \ldots, n \end{aligned} \tag{5}$$

where

$$\tilde{\mathbf{r}} = \begin{bmatrix} \Re\{\mathbf{r}\} \\ \Im\{\mathbf{r}\} \end{bmatrix}, \tilde{\mathbf{x}} = \begin{bmatrix} \Re\{\mathbf{x}\} \\ \Im\{\mathbf{x}\} \end{bmatrix} \tag{6}$$

and

$$\tilde{\mathbf{H}} = \begin{bmatrix} \Re\{\mathbf{H}\} & -\Im\{\mathbf{H}\} \\ \Im\{\mathbf{H}\} & \Re\{\mathbf{H}\} \end{bmatrix}. \tag{7}$$

$g^r(\Re\{x_i\}, \Im\{x_i\}) = \Re\{f(x_i)\}$, and $g^i(\Re\{x_i\}, \Im\{x_i\}) = \Im\{f(x_i)\}$, which are also polynomial in $\Re\{x_i\}$ and $\Im\{x_i\}$. Specifically for decouplable constellations, i.e., QAM, (5) can be simplified as

$$\begin{aligned} &\min_{\tilde{\mathbf{x}} \in \mathcal{R}^{2n}} \|\tilde{\mathbf{r}} - \tilde{\mathbf{H}}\tilde{\mathbf{x}}\|^2 \\ &\quad \text{s.t. } g(\tilde{x}_i) = 0,\ i = 1, \ldots, 2n. \end{aligned} \tag{8}$$

For example, for 16-QAM, $g(\tilde{x}_i) = (\tilde{x}_i + 3)(\tilde{x}_i - 3)(\tilde{x}_i + 1)(\tilde{x}_i - 1)$.

We next show how to apply the penalty function method to (8). Eq. (5) can be solved similarly. The most common penalty function is the one that associates a penalty that is proportional to the square of the constraints. We replace (8) by

$$\min_{\tilde{\mathbf{x}} \in \mathcal{R}^{2n}} \|\tilde{\mathbf{r}} - \tilde{\mathbf{H}}\tilde{\mathbf{x}}\|^2 + \frac{1}{2} c \sum_{i=1}^{2n} |g(\tilde{x}_i)|^d \tag{9}$$

where the positive scalar c controls the magnitude of the penalty, and d controls the acceleration of the penalty. If $d = 2$, it reduces to the usual penalty function in [8]. In the following, we choose $d = 2$. Since (9) is a polynomial in $\tilde{\mathbf{x}}$, the Hessian matrix of (9) can be computed in a closed form. The well-known Newton or Quasi-Newton method can be used to solve (9). The initial point for the Newton method can be the least squares (LS) or minimum mean square error (MMSE) solution.

It seems logical to choose a large c to ensure that no constraint is violated. However, a large c may lead to numerical difficulties or ill-condition, and the search will be trapped by the local minima around the LS or MMSE initial solution. The minimization is started with a relatively small c, and c is increased gradually. A typical value for $c^{(k+1)}/c^{(k)}$ is 5 [8]. If $\tilde{\mathbf{x}}$ is the true solution, $\|\tilde{\mathbf{r}} - \tilde{\mathbf{H}}\tilde{\mathbf{x}}\|^2$ is a chi-square random variable. The initial c can be set to $2\sigma_n^2$.

To overcome the ill-conditioning, the penalty function method can be combined with the Lagrange multipliers. The so-called augmented Lagrangian function [8] is defined as

$$L(\tilde{\mathbf{x}}, \lambda_1, \ldots, \lambda_{2n}) = \|\tilde{\mathbf{r}} - \tilde{\mathbf{H}}\tilde{\mathbf{x}}\|^2 + \sum_{i=1}^{2n} \lambda_i g(\tilde{x}_i) + \frac{1}{2} c \sum_{i=1}^{2n} |g(\tilde{x}_i)|^2. \quad (10)$$

The initial λ_i can be set to $\lambda_i^{(0)} = 0$. After minimizing (10), [8] suggests updating λ_i as

$$\lambda_i^{(k+1)} = \lambda_i^{(k)} - c^{(k)} g(\tilde{x}_i^{(k)}) \quad (11)$$

where $\tilde{x}_i^{(k)}$ is the estimate of $\tilde{x}_i$ in the k-th iteration.

To avoid the trap of local minima, we propose a probabilistic restart to improve the performance of the Newton method. If LS solution is the initial point, it can be written as

$$\hat{\mathbf{x}} = \mathbf{x} + \mathbf{H}^{-1}\mathbf{n} \quad (12)$$

If $\mathbf{n}$ is Gaussian, we generate T samples of $\mathbf{n}_t$ according to its probability density function and use the T samples to obtain T initial vectors as $\hat{\mathbf{x}} - \mathbf{H}^{-1}\mathbf{n}_t$. For each initial vector, we apply the penalty function method to (8). The best of the T resulting solutions with the minimum cost $\|\mathbf{r} - \mathbf{H}\mathbf{x}\|^2$ is output as the solution.

To find the global minimum of (9), the stochastic mechanics algorithm in [9] and simulated annealing in [10] can be applied, which will give near ML performance.

4. SIMULATION RESULTS

The error rates of our proposed constrained detector are simulated for a MIMO system with 8 transmit and 8 receive antennas over a flat Rayleigh fading channel. We assume the receiver has perfect channel state information (CSI) and noise variance. In our constrained detector, $T = 5$. Our polynomial constrained detection is denoted as PCD. The V-BLAST detection and SD are used as benchmark detectors.

Fig. 1 shows the BER performance of different detectors in a BPSK modulated system. Our PCD has significantly better performance than that of V-BLAST and MMSE. At BER $= 10^{-4}$, the performance loss over SD is only 1.3 dB.

The BER of different constrained linear detectors for 16QAM is shown in Fig. 2. Our PCD still performs better than V-BLAST. However, the performance improvement is reduced compared to that of the BPSK case. At BER $= 4 \times 10^{-3}$, our PCD has a 3.4-dB gain over V-BLAST. When SNR is less than 20 dB, PCD has identical performance to SD. In high SNR, the performance gap between PCD and SD is large. This suggests using the global optimization algorithm in [9] and [10].

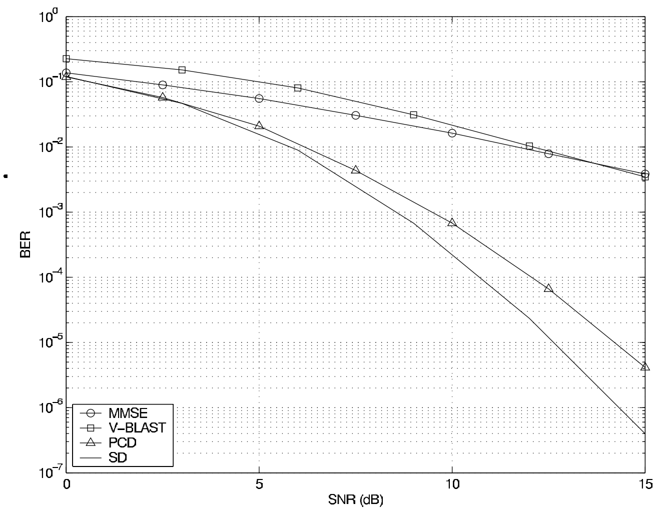

Fig. 1. Performance comparison of different detectors in an 8×8 MIMO system with BPSK.

5. CONCLUSION

In this paper, we have proposed a relaxation approach to near maximum likelihood detection for MIMO systems. A polynomial relaxation is proposed for any constellation. The ML MIMO detection problem was reformulated as an equality constrained optimization problem. A generalized penalty function method is proposed to solve the constrained optimization problem using the Newton method. Since the Newton method may be trapped by some local minima, a random restart strategy using the noise statistics is proposed. Simulation results show that our proposed relaxation detector outperforms the V-BLAST detector, but the complexity is significantly less than that of ML brute-force search.

6. REFERENCES

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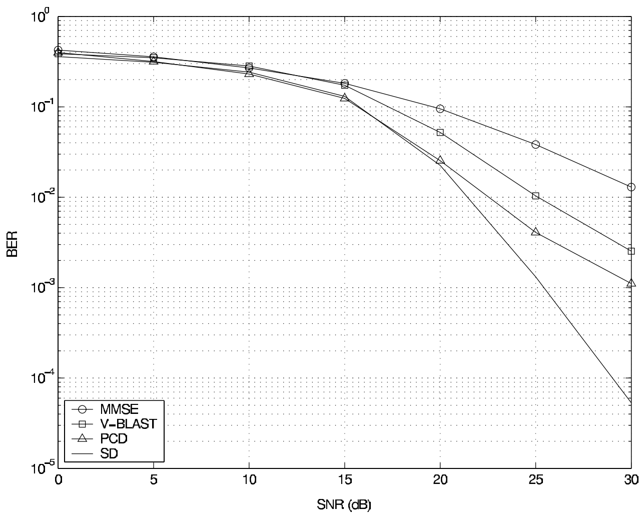

Fig. 2. Performance comparison of different detectors in an 8×8 MIMO system with 16QAM.

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