

# A NEW IMPROVED DETECTOR FOR THE V-BLAST SYSTEM

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## ABSTRACT

This paper proposes a new method for symbol detection in the Vertical Bell Labs Layered Space-Time (V-BLAST) system. The performance of V-BLAST is limited by the worst subchannel, i.e. the first iteration, the diversity order of which is  $N_r - N_t + 1$  where  $N_t$  and  $N_r$  are the number of transmit and receive antennas. A diversity order of  $N_r - N_t + 1 + T$  is achievable by delaying decisions by  $T$  stages. Considering the statistical properties of the error patterns, we present a new algorithm to delay the decisions to achieve higher diversity orders with reduced computational complexity. The performance-complexity measure of the proposed algorithm is compared with that of the V-BLAST detector and sphere decoder.

## 1. INTRODUCTION

Multiple-Input Multiple-Output (MIMO) wireless technology promises high data rates that are unachievable using traditional techniques. The optimal MIMO detector is the Maximum Likelihood (ML) detector, but its complexity grows exponentially with the number of transmit antennas, and it may be impractical for many systems. The Vertical Bell Labs Space-Time (V-BLAST) detector [1] is a low-complexity detector, but its bit error rate performance is vastly inferior to that of the ML detector.

A large diversity order, which is the slope of the error rate as a function of signal-to-noise ratio (SNR), ensures that the error rate decreases rapidly. The diversity order in the  $i$ -th iteration of the V-BLAST detector is  $N_r - N_t + i$  [2] for a MIMO system with  $N_t$  and  $N_r$  are the number of transmit and receive antennas. Therefore, due to the error propagation, the V-BLAST performance is limited by the worst iteration, i.e. the first iteration with the diversity order  $N_r - N_t + 1$ . This phenomenon causes a wide performance gap between the V-BLAST detector and the ML detector.

Thus many performance enhancements to the V-BLAST detector have been developed. Reference [3] proposes a

combined ML and decision feedback equalizer (DFE) detector; it performs ML detection of the  $p$  ( $\geq 1$ ) worst subchannels and uses a DFE for the remaining symbols. Another method is proposed by [4] in which the decisions are delayed to improve the detection accuracy. The metric used in [4] considers the distances between equalized samples and the tentative potential candidates. Shen et. al. [5] proposes an iterative approach that uses the decisions in the subchannels with higher diversity order to improve the performance. In [6], the authors propose an iterative method using metrics that consider the post-detection noise.

In this paper we propose a new detector for V-BLAST systems which provides a significantly better performance than the existing V-BLAST detectors. The diversity order of our detector can be chosen to match the detection needs. Other attractive features are its low data dependency, low computational complexity, a roughly deterministic run-time and a parallel structure which makes it a suitable candidate for practical MIMO implementations. The proposed detector can efficiently be processed using a pipelined processing unit [7]. It has significantly improved performance compared to the original V-BLAST detector.

This paper is organized as follows: Section 2 describes the system model and related background. Section 3 explains the new detection method. Simulation results and discussions are presented in Section 4, and finally conclusions are given in Section 5.

*Notation-* Bold letters denote vectors and matrices. Here  $(\cdot)^H$  and  $(\cdot)^T$  denote the Hermitian and transpose respectively and  $\mathbf{I}_n$  denotes the  $n \times n$  identity matrix. The vector norm is defined as  $\|\mathbf{x}\| = (\sum |x_i|^2)^{\frac{1}{2}}$ , and  $(\mathbf{H})_i$  denotes the  $i$ -th column of the matrix  $\mathbf{H}$ .

## 2. SYSTEM MODEL

We consider a MIMO system with  $N_t$  transmit antennas and  $N_r$  receive antennas. The  $N_r \times 1$  complex baseband received vector  $\mathbf{r}$  is given by

$$\mathbf{r} = \mathbf{H}\mathbf{s} + \mathbf{w} \quad (1)$$

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where  $\mathbf{H} = [h_{ij}] \in \mathbb{C}^{N_r \times N_t}$  is a quasi-static Rayleigh flat fading channel matrix, i.e.  $h_{ij}$  are independent and identically distributed (iid) complex Gaussian with unit variance,  $\mathbf{s} = (s_1, s_2, \dots, s_{N_t})^T$ ,  $s_i \in \mathbb{A}$  is the transmitted signal vector, and the additive noise vector  $\mathbf{w} = (w_1, w_2, \dots, w_{N_r})^T$  is iid complex Gaussian with zero mean and variance  $\sigma_w^2$ . The signal constellation is  $\mathbb{A}$  of size  $|\mathbb{A}|$ . The columns of  $\mathbf{H}$  are linearly independent, so that  $N_r \geq N_t$ . Note that it is customary to assume that all the entries of  $\mathbf{H}$  are independent provided that a rich scattering environment exists between the antennas [8]. We assume that the channel  $\mathbf{H}$  is perfectly known at the receiver (channel estimation via separate training sequences may be necessary). This assumption is also justified by the fact that the channel remains constant over a channel coherence time that is much longer than the transmit signal period. The detection problem is to recover  $\mathbf{s}$  from  $\mathbf{r}$ , given  $\mathbf{H}$ .

In the V-BLAST detector, after determining the detection order, transmitted symbols are detected iteratively with a DFE approach [1]. To simplify the discussion, in this paper it is assumed that  $\mathbf{r}$  is in the desired detection order. In the  $i$ -th iteration,  $\hat{s}_i$  is computed by nulling the effect of the undetected symbols as

$$\tilde{\mathbf{s}}_i = \mathbf{g}_i^T \mathbf{r}_i \quad (2)$$

where  $\mathbf{g}_i^T$  is the  $i$ -th nulling vector, and  $\mathbf{r}_i$  is the received vector after cancellation of the symbols  $\{\hat{s}_1, \hat{s}_2, \dots, \hat{s}_{i-1}\}$  [1]. Either Minimum Mean Square Error (MMSE) or Zero Forcing (ZF) nulling vectors can be utilized in this process. The  $i$ -th transmitted symbol is then estimated by  $\hat{s}_i = Q(\tilde{\mathbf{s}}_i)$  where  $Q(x)$  is a slicer that maps  $x$  to its closest neighbour in  $\mathbb{A}$ .

### 3. THE NEW DETECTION ALGORITHM

Considering the statistical properties of the error patterns in the BLAST detector, we propose a new detector with the minimum required complexity to achieve any performance slope between BLAST and ML detectors.

The proposed detector is a form of tree detection which, unlike the algorithm proposed in [4], in the metric computation, considers the future symbols as well. The idea in this scheme is to use the fact that the diversity order increases as the number of iterations increases, and therefore one can use the accuracy of the higher iterations to improve the performance of the lower ones. In each iteration, a set of the most probable tentative candidates are considered, and the final decision is made based on the metric associated to each one.

The key point here is to keep the number of candidates as low as possible while maintaining the target performance. To describe the new detection method, assume that the slicer provides  $N_e$  most probable symbols  $\mathbf{a} = \{a_1, a_2, \dots, a_{N_e}\} =$

$Q(x)$ . Further, assume that the nulling vectors  $\mathbf{g}_i^T$ ,  $i = 1..N_t$  are computed and stored for each data frame in advance. In the proposed detector, the decision on the  $i$ -th iteration is delayed for  $T - 1$  stages (constraint length) to provide a more accurate decision. In the following subsection, we describe the algorithm for the case  $T = 1$ , and the general case is presented in the next subsection.

#### 3.1. Case $T = 1$

Here,  $T = 1$  implies that the decision on the  $k$ -th symbol is made at the same stage (i.e. without delay). The proposed detection algorithm for the case  $T = 1$  is shown in Algorithm 1. For detection of  $\hat{s}_k$  in the  $k$ -th iteration, first  $\tilde{\mathbf{s}}_k$  is

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#### Algorithm 1 Proposed detection ( $T=1$ )

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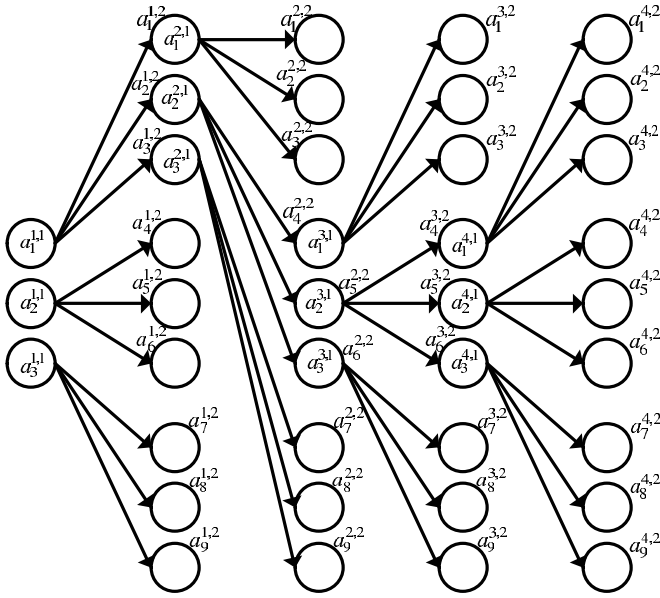
1:	$\mathbf{r}_1 = \mathbf{r}$	{Initialization}
2:	<b>for</b> $k = 1 : N_t - 1$ <b>do</b>	
3:	$\tilde{\mathbf{s}}_k = \mathbf{g}_k^T \mathbf{r}_k$ ;	{Nulling}
4:	$\{a_1, a_2, \dots, a_{N_e}\} = Q(\tilde{\mathbf{s}}_k)$ ;	{Slicing}
5:	<b>for</b> $l = 1 : N_e$ <b>do</b>	
6:	$\hat{s}_k = a_l$	{Tentative candidate}
7:	$\hat{\mathbf{r}} = \mathbf{r}_k - a_l(\mathbf{H})_k$	{Tentative cancellation}
8:	<b>for</b> $j = k + 1 : N_t$ <b>do</b>	
9:	$\hat{s}_j = Q(\mathbf{g}_j^T \hat{\mathbf{r}})$ ;	
10:	$\hat{\mathbf{r}} = \hat{\mathbf{r}} - \hat{s}_j(\mathbf{H})_j$	{Future Symbols}
11:	<b>end for</b>	
12:	$m_l = \ \mathbf{H}\hat{\mathbf{s}} - \mathbf{r}\ ^2$	{Metric computation}
13:	<b>end for</b>	
14:	$j = \arg \min_i \{m_i\}$	{Finding the best one}
15:	$\hat{s}_k = a_j$ ; $\hat{s}_k = \hat{s}_k$	{Selecting the best one}
16:	$\mathbf{r}_{k+1} = \mathbf{r}_k - \hat{s}_k(\mathbf{H})_k$	{Cancellation}
17:	<b>end for</b>	
18:	$\hat{s}_{N_t} = Q(\mathbf{g}_{N_t}^T \mathbf{r}_{N_t})$ ;	{Last symbol}

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computed using the  $k$ -th nulling vector  $\mathbf{g}_k^T$  and then a set of  $N_e$  tentative candidates for  $\hat{s}_k$ , say  $\mathbf{a} = \{a_1, a_2, \dots, a_{N_e}\}$ , is generated by slicing  $\tilde{\mathbf{s}}_k$  as shown in the line 4. Assuming  $a_l$  to be the true transmitted sample for the  $k$ -th symbol (line 6), in the next step the interference from the  $k$ -th symbol is cancelled from the remainder of the received signal (line 7) and the rest of the symbols are temporarily estimated as  $\hat{s}_j$ ,  $j = k + 1..N_t$  (lines 8-11). Using the previously detected symbols  $\hat{s}_j$ ,  $j = 1..k - 1$ , the tentative candidate  $a_l$  and  $\hat{s}_j$ ,  $j = k + 1..N_t$  the metric for  $a_l$  is computed (line 12). The tentative candidate  $a_l$  resulting in the minimum metric will be chosen as the final estimation for  $\hat{s}_k$ . This procedure is performed for  $k = 1..N_t - 1$ . As shown in line 18, the last symbol can be simply computed from the remainder of the received signal.

### 3.2. Generic Algorithm

For the case when  $T > 1$ , the detection of the  $k$ -th symbol is delayed by  $T - 1$  stages, and combinations of tentative candidates are examined for the best estimation of  $\hat{s}_k$ . The progress of the proposed algorithm can be shown on a tree diagram. Fig. 1 shows an example tree with  $N_e = 3$  and  $T = 2$  for a  $N_t = 5 \times N_r$  MIMO system. For estimating  $\hat{s}_k$ , a set of tentative candidates  $\mathbf{a}^{k,1}$  are considered. Then for each of the tentative candidates  $a_l^{k,1}$ ,  $l = 1..N_1 \leq N_e$ , a new set of tentative candidates for the symbol  $\hat{s}_{k+1}$  are



**Fig. 1.** Example tree for a  $N_t = 5$  and  $N_r \geq N_t$  MIMO system. The tree is designed for  $N_e = 3$  and  $T = 2$ .

considered  $a_l^{k,2}$ ,  $l = 1..N_2 \leq N_e^2$ , and so on. Then the rest of the symbols are temporarily estimated, and the metrics are computed in a way similar to the case  $T = 1$ .

## 4. DISCUSSION AND RESULTS

Denote the error event in the  $i$ -th iteration of detection by  $E_i$ . Also denote the event of at least one error in detection of future symbols  $\hat{s}_j$ ,  $j = i + 1..N_t$  by  $E_i^f$ . It is assumed that the BLAST algorithm is used for detecting the future symbols. Because in BLAST detection the bottleneck is detection of the first symbol [2], we focus on the error rate of the first iteration. In the first iteration, an error may happen due to either a lack of the proper sample in  $\mathbf{a}$  or error in detection of the future symbols  $\hat{s}_j$ ,  $j = i + T, \dots, N_t$ . For the case  $T = 1$ , the error rate of the first symbol can be

bounded to

$$\Pr\{E_1\} \leq \Pr\{o_1\} + (1 - \Pr\{o_1\}) \times \Pr\{E_1^f\} \quad (3)$$

where  $o_1$  is the event of not considering the transmitted symbol in  $\mathbf{a}$  (i.e.  $s_1 \notin \{a_1, a_2, \dots, a_{N_e}\}$ ) in the first step. With a similar argument as in [2], it can be shown that  $\Pr\{E_1^f\}$  results in an error rate associated with a diversity order of  $N_r - N_t + 2$ . On the other hand,  $\Pr\{o_1\}$  is a function of  $N_e$ , the modulation scheme, the number of antennas, the type of used nulling vectors and ordering discipline. As (3) suggests, as long as  $\Pr\{o_1\}$  is small enough comparing to  $\Pr\{E_1^f\}$ ,  $\Pr\{E_1\}$  follows the trend (and consequently slope) of  $\Pr\{E_1^f\}$ . Therefore, to design a tree with minimum computational complexity one should choose the number of slicer outputs,  $N_e$ , such that the algorithm provides the target error rate slope. For given system parameters,  $N_e$  can be chosen properly to have a target error rate performance slope for a given range of SNR.

For the case  $T > 1$ , the probability of missing the transmitted symbols in higher stages should be considered as well. For the special case  $T = 2$ , the error rate of the proposed algorithm can be expressed as follows:

$$\Pr\{E_1\} \leq \Pr\{o_1\} + (1 - \Pr\{o_1\}) \times (\Pr\{o_2\} + (1 - \Pr\{o_2\}) \times \Pr\{E_2^f\}) \quad (4)$$

where  $o_2$  is the event of not considering the transmitted symbol in  $\mathbf{a}$  given that the previous candidate was the transmitted symbol. Fig. 2 shows the  $\Pr\{o_1\}$  and  $\Pr\{o_2\}$  curves versus SNR for a  $8 \times 8$  MIMO system with 64-QAM modulation of symbols. In this figure,  $N_e = 5$  and  $N_e = 9$  are used, and nulling vectors are computed with the MMSE criterion. Since  $\Pr\{o_2\}$  is much smaller than  $\Pr\{o_1\}$  for the medium-to-high range of SNR,  $\Pr\{o_1\}$  is more significant than the rest of the error events. This can be used for further complexity reduction of the proposed algorithm.

To simplify the tree structure, one can decrease  $N_e$  as the number of iterations increases while maintaining  $\Pr\{o_i\}$  much smaller than the target Symbol Error Rate (SER). Further, for a given performance, the tree search can be stopped in one of the middle iterations to reduce the computational complexity, as long as this truncation doesn't affect the overall performance.

Fig. 2 shows the SER performance of a  $8 \times 8$  MIMO system with 64-QAM. MMSE nulling vectors are used for the proposed detector with  $T = 1$  or  $T = 2$ . For the former,  $N_e = \{9, 9, 5, 5\}$  is chosen for the first to fourth iterations. The detection is stopped after the fourth iteration, and the best estimates for the symbols  $s_5..s_8$  are selected as the rest of transmitted symbols. For the case  $T = 2$ ,  $N_e = \{21, 13, 9, 9\}$  are used for iterations one to four, and

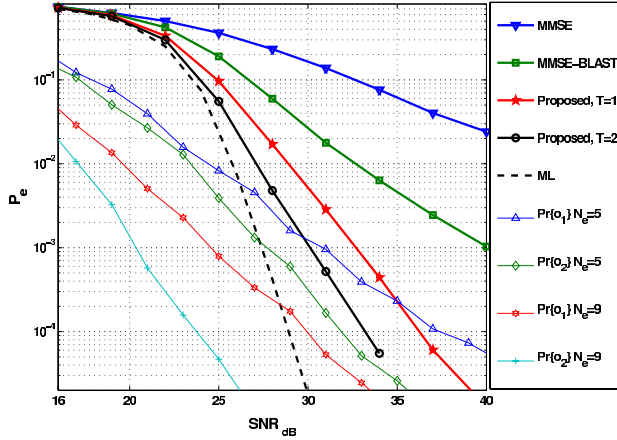


Fig. 2. Performance of different schemes.

again the detection process is stopped at the fourth iteration. With proper selection of detection parameters, the proposed detector provides the target error rate slope for the SNR range. Therefore, for any range of SNR, one can design a detection tree to achieve a compromise between detection complexity and the required SNR (i.e. receiver complexity versus the transmitted power)

Table 1 shows the complexity of different detectors for achieving a target SER of 0.001 in a  $8 \times 8$  MIMO system with 64-QAM. The channel is assumed to be fixed over a frame of 1000 symbols so that the initial overhead is averaged over the entire frame. Here the transmitter complexity is represented as the required SNR, and the receiver complexity is shown in terms of the average number of flops. The Schnorr-Euchner sphere decoder (SD) [9] is also simulated for comparison. For the proposed algorithm,  $T = 1$

	MMSE	BLAST	Proposed	SD
Flops	$5.5e2$	$1.2e3$	$2.1e4$	$1.9e5$
SNR	53.0 dB	40.0 dB	32.6 dB	27.4 dB

Table 1. Complexity of different receivers

and  $N_e = \{9, 5\}$  are selected to have a slope of two, up to an error rate of 0.001. As Table 1 shows, the computational complexity of the proposed receiver is an order of magnitude less than that of SD. Also the proposed method shows a 7.4-dB SNR advantage when compared to the MMSE-BLAST receiver.

## 5. CONCLUSION

We proposed a new enhanced detector which overcomes the performance limitation of the conventional V-BLAST detection. In each iteration, a detection decision is made using

previously detected symbols and tentative estimation of future symbols. The proposed algorithm can be parallelized, has low data dependency and is deterministic. With proper selection of the estimation parameters, the diversity order of the proposed method can be increased.

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