A General Method for Calculating Error Probabilities Over Fading Channels

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***Abstract*—Signal fading is a ubiquitous problem in mobile and wireless communications. In digital systems, fading results in bit errors, and evaluating the average error rate under fairly general fading models and multichannel reception is often required. Predominantly to date, most researchers perform the averaging using the probability density function method or the moment generating function (MGF) method. This paper presents a third method, called the characteristic function (CHF) method, for calculating the average error rates and outage performance of a broad class of coherent, differentially coherent, and noncoherent communication systems, with or without diversity reception, in a myriad of fading environments. Unlike the MGF technique, the proposed CHF method (based on Parseval's theorem) enables us to *unify* the average error-rate analysis of different modulation formats and *all* commonly used predetection diversity techniques (i.e., maximal-ratio combining, equal-gain combining, selection diversity, and switched diversity) within a *single* common framework. The CHF method also lends itself to the averaging of the conditional error probability involving the complementary incomplete Gamma function and the confluent hypergeometric function over fading amplitudes, which heretofore resisted to a simple form. As an aside, we show some previous results as special cases of our unified framework.**

***Index Terms*—Characteristic function (CHF) method, digital communications, fading channels, frequency-domain analysis, wireless communications.**

I. Introduction

IN THE analysis of communications systems over wireless channels, we frequently encounter the task of averaging the conditional error probability (CEP) over either the fading amplitudes or the received signal power. The CEPs for binary and M-ary modulation formats employing coherent, differentially coherent, or noncoherent detection schemes are usually in one of the following forms: exponential function $\exp(.)$; complementary error function $\mathrm{erfc}(.)$ (or Gaussian probability integral $Q(.)$), a combination of $\mathrm{erfc}(.)$ and $\mathrm{erfc}^2(.)$, complementary incomplete Gamma function $\Gamma(.,.)$, confluent hypergeometric series ${}_1F_1(.;.;.)$, Marcum-Q function $Q(.,.)$, or generalized Marcum-Q function $Q_M(.,.)$ [1]–[10]. One of the most commonly used techniques for this task has, in the past, been the probability density function (PDF) method

$$\overline{P_e}(L) = \int_0^\infty P_e(\gamma) p_\gamma(\gamma, L) d\gamma \tag{1}$$

where $P_e(\gamma)$ denotes the symbol (or bit) error probability on an additive white Gaussian noise (AWGN) channel conditioned by the signal-to-noise ratio (SNR) at the combiner output, L is the diversity order, and $p_\gamma(\gamma, L)$ corresponds to the PDF of combiner output SNR in a specified fading environment. This approach has been used by many authors over the past five decades to analyze the performance of various single-channel reception systems over fading channels. In some cases, it is more convenient if the CEP and the PDF are expressed in terms of the combiner output envelope $\vartheta = \sqrt{\gamma}$, viz.,

$$\overline{P_e}(L) = \int_0^\infty P_e(\vartheta) p_\vartheta(\vartheta, L) d\vartheta. \tag{2}$$

Diversity systems (e.g., maximal-ratio or equal-gain combining) and multichannel signaling further complicate the problem at hand by requiring that the PDF of a sum of random variables (RVs) be determined as well. The PDF method has several variations, which can be categorized depending on how the PDF is obtained. In the most traditional form, the evaluation of (1) or (2) will require an L-fold convolution integral. For this case, it is more insightful if we transform the PDF into frequency domain; this way, the evaluation of the PDF will only involve a single integral, namely

$$p_x(x, L) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \Psi_x(\omega, L) e^{-j\omega x} d\omega \tag{3}$$

where $\Psi_x(\omega, L)$ is the characteristic function (CHF) of random variable x at the combiner output and $x \in \{\gamma, \vartheta\}$. This CHF is also related to the moment generating function (MGF) (i.e., Laplace transform of the PDF) via relationship $\Psi_x(\omega, L) = \langle e^{j\omega x} \rangle = \phi_x(-j\omega, L)$ or, alternatively, $\phi_x(s, L) = \langle e^{-sx} \rangle = \Psi_x(js, L)$.

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For some special cases (e.g., the sum of exponential RVs), the inverse Fourier transform (FT)[1] (3) can be evaluated in a closed form, and thus a closed-form solution for the PDF is available for these situations. However, it is difficult (if not impossible) to get closed-form PDFs for all common fading environments, especially for the diversity systems. In this case, one may resort to an *approximate* PDF, which are easily determined using a Fourier series technique [11], [12]

$$p_x(x, L) \approx \frac{2}{T} \sum_{n=1, n \text{ odd}}^{\infty} \left[\Psi_x(n\omega_0, L) e^{-jn\omega_0 x} + \Psi_x(-n\omega_0, L) e^{jn\omega_0 x} \right] \quad (4)$$

where $\omega_0 = 2\pi/T$ and T is selected such that $Pr(x > T) \leq \varepsilon$, and ε can be set to a very small value. This variation of the PDF method has been widely used in the analysis of both coherent equal-gain combining (EGC) diversity receivers and the co-channel interference [13]. In [10] and [14]–[16], the authors have derived the PDF of the composite phase of the fading signal and noise, and subsequently determining the average symbol error rate (ASER) for M-ary signals from its cumulative distribution function (CDF).

More recently, some authors have suggested using the MGF technique for analyzing the performance of a broad class of modulation formats in different fading environments [1]–[9]. The key idea here is to express the CEP in a desirable exponential form (e.g., [1], [12], [17]–[20]) so that the averaging can be easily performed, once knowledge of the Laplace or Fourier transform of the PDF is obtained. However, there are several limitations to this approach: 1) it fails to work in the analysis of coherent EGC diversity receivers because of the cross-product terms in the CEP except for cases involving only $\text{erfc}(.)$ or $Q(.)$ functions;[2] 2) it fails to work if CEP cannot be expressed in a desirable exponential form (e.g., trigonometric integral for the complementary incomplete Gamma function given in [21] is not in a "desired" form); 3) it fails to yield an exact analytical expression for binary DPSK or binary orthogonal signaling with post-detection EGC combining (square-law combining) due to the limitation of trigonometric integral representation for the generalized Marcum-Q function; and 4) it fails to analyze the performance of MFSK with post-detection EGC.

Recognizing that the product integral in (1) can be easily transformed into the frequency domain, with the aid of Parseval's theorem, and that the Fourier transform (FT) of the PDF is the CHF, we immediately obtain a simple expression for computing (1) in the frequency domain. However, in this case, we also need the knowledge of FT of the CEP. Fortunately, this turns out to be easily computed. Therefore, it is evident that we have reformulated the task of finding a desirable exponential form for the CEP (required for the MGF method) to simply computing its FT. This is obviously a much simpler task and generally works for all forms of the CEPs! Note the implications of our results: 1) first of all, we have, *within a single common framework, developed a general method for calculating the average error probability performance of single and multiple channel reception* (MGF method cannot facilitate the analysis of coherent EGC within a common framework); (b) more importantly, *the CHF method overcomes all the limitations of the MGF method* highlighted above; and (c) if the CEP can be expressed in a desirable exponential form, then it can be shown, using Cauchy's integral formula, that the solution from the CHF method reduces to the familiar expression obtained via MGF method. Given this, we can safely state that *the results obtained from the CHF method encapsulates all those obtained by the MGF method.* Several new solutions are also derived.

Notice also that the CHF method described above differs from that in [22] and [23] by considering the FT of the CEP instead of the FT of the conditional decision variable. This distinction is important because it may not be trivial to extend the latter framework to consider different modulation and/or diversity-combining techniques. The utility of our approach to derive easy-to-evaluate formulas for a broad range of modulation/detection/diversity-combining schemes are further highlighted by numerous examples in Section III.

The outline of this paper is as follows. In Section II, generic expressions for calculating both the average bit error rate (ABER) or average symbol error rate (ASER) and the outage performance, with/without diversity reception, are derived using Parseval's theorem and the Fourier inversion formula. Subsequently, six selected applications of our new performance analysis technique are presented in Section III. Some previous results are shown as special cases of our framework. Finally, the main points are summarized in Section IV.

II. Frequency-Domain Analysis

By transforming (1) and (2) into the frequency domain [i.e., using (A4)], we find that

$$\overline{P_e}(L) = \frac{1}{2\pi} \int_{-\infty}^{\infty} G_x(\omega) \Psi_x(\omega, L) d\omega \quad (5)$$

where $x \in \{\gamma, \vartheta\}$, and $G_x(\omega) = \text{FT}[P_e(x)] = \int_0^{\infty} P_e(x) e^{-j\omega x} dx$.

Since the real part of $G_x(.)$ and $\Psi_x(.,.)$ is an even function while their imaginary part is an odd function, we can simplify (5) as

$$\overline{P_e}(L) = \frac{1}{\pi} \int_0^{\infty} \Re \left\{ G_x(\omega) \Psi_x(\omega, L) \right\} d\omega \quad (6)$$

using the fact that $\int_{-z}^{z} f(t)dt = 2\int_0^z f(t)dt$ if $f(t)$ is an even function or $\int_{-z}^{z} f(t)dt = 0$ if $f(t)$ is an odd function, and the notation $\Re\{\cdot\}$ in (6) denotes the real part of its argument.

Next, using variable substitution $\omega = \tan\theta$ in (6), we obtain an *exact* finite-range integral, which is suitable for numerical integration (i.e., the integrand is well behaved at $\theta = 0$)

$$\overline{P_e}(L) = \frac{1}{\pi} \int_0^{\frac{\pi}{2}} \Re \left\{ G_x(\tan\theta) \Psi_x(\tan\theta, L) \right\} \sec^2\theta \, d\theta. \quad (7)$$

[1]Although (3) differs from the usual definition of an inverse FT by a negative sign in the exponential, we still refer to the integral as an inverse Fourier transform. This is consistent with the definition of CHF (i.e., FT of PDF), $\Psi_x(\omega, L) = \int_{-\infty}^{\infty} p_x(x, L) e^{j\omega x} dx$, which differs likewise from the usual definition of FT [10].

[2]A desirable exponential form for $Q(.)$ that is suitable for EGC receiver analysis is given in [12].

TABLE I
CHF OF FADING ENVELOPE (AMPLITUDE) AND SNR FOR SEVERAL FADING CHANNEL MODELS

Channel Model	PDF and CHF of fading amplitude α and SNR $\gamma = \alpha^2$
Rayleigh	PDF: $f_\alpha(\alpha) = \frac{2\alpha}{\bar{\gamma}} \exp\left(\frac{-\alpha^2}{\bar{\gamma}}\right)$, $\alpha \geq 0$ where $\bar{\gamma} = E[\alpha^2]$ = average SNR per symbol $f_\gamma(\gamma) = \frac{1}{\bar{\gamma}} \exp\left(\frac{-\gamma}{\bar{\gamma}}\right)$, $\gamma \geq 0$ CHF: $\psi_\alpha(\omega) = {}_1F_1\left(1; \frac{1}{2}; \frac{-\bar{\gamma}\omega^2}{4}\right) + j\omega\sqrt{\frac{\pi\bar{\gamma}}{4}} \exp\left(\frac{-\bar{\gamma}\omega^2}{4}\right)$ $\psi_\gamma(\omega) = \frac{1}{1-j\omega\bar{\gamma}}$
Rician $(K \geq 0)$	PDF: $f_\alpha(\alpha) = \frac{2(K+1)\alpha}{\bar{\gamma}} \exp\left(-K - \frac{(1+K)\alpha^2}{\bar{\gamma}}\right) I_0\left(2\alpha\sqrt{\frac{K(K+1)}{\bar{\gamma}}}\right)$, $\alpha \geq 0$ $f_\gamma(\gamma) = \frac{1+K}{\bar{\gamma}} \exp\left(-K - \frac{(1+K)\gamma}{\bar{\gamma}}\right) I_0\left(2\sqrt{\frac{K(1+K)\gamma}{\bar{\gamma}}}\right)$, $\gamma \geq 0$ CHF: $\psi_\alpha(\omega) = \exp(-K) \sum_{i=0}^{\infty} \frac{K^i}{i!} \, {}_1F_1\left(i+1; \frac{1}{2}; \frac{-\bar{\gamma}\omega^2}{4(1+K)}\right)$ $+j\omega\sqrt{\frac{\bar{\gamma}}{1+K}} \exp(-K) \sum_{i=0}^{\infty} \frac{\Gamma(i+3/2)K^i}{(i!)^2} \, {}_1F_1\left(i+\frac{3}{2}; \frac{3}{2}; \frac{-\bar{\gamma}\omega^2}{4(1+K)}\right)$ $\psi_\gamma(\omega) = \frac{1+K}{1+K-j\omega\bar{\gamma}} \exp\left(\frac{j\omega K\bar{\gamma}}{1+K-j\omega\bar{\gamma}}\right)$
Nakagami-q $(-1 \leq b \leq 1)$ where $b = \frac{1-q^2}{1+q^2}$ and $0 \leq q \leq \infty$	PDF: $f_\alpha(\alpha) = \frac{2\alpha}{\bar{\gamma}\sqrt{1-b^2}} \exp\left(\frac{-\alpha^2}{(1-b^2)\bar{\gamma}}\right) I_0\left(\frac{b\alpha^2}{(1-b^2)\bar{\gamma}}\right)$, $\alpha \geq 0$ $f_\gamma(\gamma) = \frac{1}{\bar{\gamma}\sqrt{1-b^2}} \exp\left(\frac{-\gamma}{(1-b^2)\bar{\gamma}}\right) I_0\left(\frac{b\gamma}{(1-b^2)\bar{\gamma}}\right)$, $\gamma \geq 0$ CHF: $\psi_\alpha(\omega) = \sqrt{1-b^2} \sum_{i=0}^{\infty} \frac{(b/2)^{2i}\Gamma(2i+1)}{(i!)^2} \, {}_1F_1\left(2i+1; \frac{1}{2}; \frac{-\bar{\gamma}(1-b^2)\omega^2}{4}\right)$ $+j\omega\sqrt{\bar{\gamma}}(1-b^2) \sum_{i=0}^{\infty} \frac{(b/2)^{2i}\Gamma(2i+3/2)}{(i!)^2} \, {}_1F_1\left(2i+\frac{3}{2}; \frac{3}{2}; \frac{-\bar{\gamma}(1-b^2)\omega^2}{4}\right)$ $\psi_\gamma(\omega) = \frac{1}{\sqrt{[1-j\omega\bar{\gamma}(1+b)][1-j\omega\bar{\gamma}(1-b)]}}$
Nakagami-m $(m \geq 0.5)$	PDF: $f_\alpha(\alpha) = \frac{2}{\Gamma(m)} \left(\frac{m}{\bar{\gamma}}\right)^m \alpha^{2m-1} \exp\left(\frac{-m\alpha^2}{\bar{\gamma}}\right)$, $\alpha \geq 0$ $f_\gamma(\gamma) = \frac{1}{\Gamma(m)} \left(\frac{m}{\bar{\gamma}}\right)^m \gamma^{m-1} \exp\left(\frac{-m\gamma}{\bar{\gamma}}\right)$, $\gamma \geq 0$ CHF: $\psi_\alpha(\omega) = {}_1F_1\left(m; \frac{1}{2}; \frac{-\bar{\gamma}\omega^2}{4m}\right) + j\omega\frac{\Gamma(m+1/2)}{\Gamma(m)}\sqrt{\frac{\bar{\gamma}}{m}} \, {}_1F_1\left(m+\frac{1}{2}; \frac{3}{2}; \frac{-\bar{\gamma}\omega^2}{4m}\right)$ $\psi_\gamma(\omega) = \left(\frac{m}{m-j\omega\bar{\gamma}}\right)^m$

Equally, one may arrive at (5) by substituting (3) into (1) and then rearranging the order of integration. Similarly, by substituting (4) into (1), we get an *approximate* solution for $\overline{P_e}(L)$

$$\overline{P_e}(L) \approx \frac{2\omega_0}{\pi} \sum_{n=1, n \text{ odd}}^{\infty} \Re\left\{G_x(n\omega_0)\Psi_x(n\omega_0, L)\right\}. \quad (8)$$

Interestingly, this turns out to be a trapezoidal rule approximation of (6).

Since both the MGF and the CHF of the fading statistics are readily available for both single and multichannel reception, the outage probability can be easily calculated by invoking the Fourier inversion formula (also known as Gil–Pelaez inversion theorem [24]). This formula gives the relationship between the CDF and the CHF (or the MGF)

$$F_x(x^*, L) = \frac{1}{2} - \frac{2}{\pi} \int_0^{\frac{\pi}{2}} \frac{1}{\sin 2\theta} \Im\left[\phi_x(-j\tan\theta, L) \times \exp(-jx^*\tan\theta)\right] d\theta \quad (9)$$

where $\Im\{.\}$ denotes the imaginary part of its argument. Note that, using trapezoidal rule approximation, (9) can be restated as

$$F_x(x^*, L) \approx \frac{1}{2} - \frac{2}{\pi} \sum_{n=1, n \text{ odd}}^{\infty} \frac{1}{n} \Im\{\Psi_x(n\omega_0, L) \exp(-jn\omega_0 x^*)\} \quad (10)$$

where ω_0 is defined in (4).

III. APPLICATIONS

In this section, we present several applications of our new technique in analysis of a broad class of modulation/detection/diversity schemes in a generalized fading environments. The CHFs for both fading amplitude and the SNR of several commonly used fading channel models (typical of both terrestrial and satellite communications systems) are tabulated in Table I. In Tables II and III, we derive the FTs for various forms (mathematical functions) of CEPs normally encountered in the communications systems analysis. Utilizing these tables and (7), we can immediately derive exact and easy-to-evaluate analytical expressions for the ASER (or ABER) of a wide range of coherent, differentially coherent, and noncoherent communication systems (for both single and multichannel reception) in a variety of fading environments.

A. Binary Signaling With Single-Channel Reception

In [25], Wojnar presents a general expression for CEP of binary signalling schemes in terms of an incomplete Gamma function, namely

$$P_e(\gamma) = \frac{\Gamma(a, b\gamma)}{2\Gamma(a)} = \frac{\Gamma[a, b\vartheta^2]}{2\Gamma(a)} \quad (11)$$

where

$$a = \begin{cases} 0.5, & \text{for coherent detection} \\ 1, & \text{for noncoherent/differentially coherent detection} \end{cases}$$

TABLE II
FOURIER TRANSFORM OF CEPs

$P_e(\gamma)$	$G_\gamma(\omega)=\mathrm{FT}[P_e(\gamma)]=\int_0^\infty P_e(\gamma)e^{-j\omega\gamma}\,d\gamma$
1. $a\exp(-b\gamma)$	$G_\gamma(\omega)=\frac{a}{b+j\omega}$
2. $a\mathrm{erfc}(\sqrt{b\gamma})$	$G_\gamma(\omega)=\frac{a}{j\omega}\left(1-\sqrt{\frac{b}{b+j\omega}}\right)$
3. $a\mathrm{erfc}(\sqrt{b\gamma})-c\mathrm{erfc}^2(\sqrt{b\gamma})$	$G_\gamma(\omega)=\frac{a}{j\omega}\left(1-\sqrt{\frac{b}{b+j\omega}}\right)-\frac{c}{j\omega}\left(1-\frac{4}{\pi}\frac{\tan^{-1}(\sqrt{1+j\omega/b})}{\sqrt{1+j\omega/b}}\right)$
4. $\frac{\Gamma(a,b\gamma)}{2\Gamma(a)}$	$G_\gamma(\omega)=\frac{1}{2j\omega}\left[1-\left(\frac{b}{b+j\omega}\right)^a\right]$
5. $\gamma^{a-1}\exp(-b\gamma)$	$G_\gamma(\omega)=\frac{\Gamma(a)}{[b^2+\omega^2]^{a/2}}\exp[-ja\tan^{-1}(\omega/b)]$
6. $e^{-a\gamma}\ {}_1F_1(b;c;d\gamma),\ a>d$	$G_\gamma(\omega)=\frac{1}{(a+j\omega)^2}\ {}_2F_1\left(b,1;c;\frac{d}{a+j\omega}\right)$
7. $\int_0^\eta a(\theta)\exp[-\gamma b(\theta)]\,d\theta$	$G_\gamma(\omega)=\int_0^\eta \frac{a(\theta)}{b(\theta)+j\omega}\,d\theta$
8. $Q(a\sqrt{\gamma},b\sqrt{\gamma})$ $-\frac{\eta}{1+\eta}I_0(ab\gamma)\exp\left[-\gamma\left(\frac{a^2+b^2}{2}\right)\right]$	$G_\gamma(\omega)=\frac{4b^2}{\kappa_2(\kappa_2-\kappa_1)}-\frac{2\eta}{(1+\eta)\kappa_2}$ where $\kappa_1=2j\omega+a^2-b^2$ $\kappa_2=\sqrt{[(a+b)^2+2j\omega][(a-b)^2+2j\omega]}$
9. $Q_M(a\sqrt{\gamma},b\sqrt{\gamma})=Q(a\sqrt{\gamma},b\sqrt{\gamma})$ $+\exp\left[-\gamma\left(\frac{a^2+b^2}{2}\right)\right]\sum_{n=1}^{M-1}I_n(ab\gamma)\left(\frac{b}{a}\right)^n$ where $b>a$	$G_\gamma(\omega)=\frac{4b^2}{\kappa_2(\kappa_2-\kappa_1)}+\sum_{n=1}^{M-1}\frac{2}{\kappa_2}\left(\frac{2b^2}{\kappa_2+\kappa_3}\right)^n$ where $\kappa_3=2j\omega+a^2+b^2$

TABLE III
FOURIER TRANSFORM OF CEPs

$P_e(\vartheta)$	$G_\vartheta(\omega)=\mathrm{FT}[P_e(\vartheta)]=\int_0^\infty P_e(\vartheta)e^{-j\omega\vartheta}\,d\vartheta$
1. $a\exp(-b\vartheta^2)$	$G_\vartheta(\omega)=\frac{a}{2b}\left[\sqrt{\pi b}\exp\left(\frac{-\omega^2}{4b}\right)-j\omega\ {}_1F_1\left(1;\frac{3}{2};\frac{-\omega^2}{4b}\right)\right]$
2. $a\mathrm{erfc}(\sqrt{b}\vartheta)$	$G_\vartheta(\omega)=\frac{a}{\sqrt{\pi b}}\ {}_1F_1\left(1;\frac{3}{2};\frac{-\omega^2}{4b}\right)-\frac{ja}{\omega}\left[1-\exp\left(\frac{-\omega^2}{4b}\right)\right]$
3. $a\mathrm{erfc}(\sqrt{b}\vartheta)-c\mathrm{erfc}^2(\sqrt{b}\vartheta)$	$G_\vartheta(\omega)=\frac{(a-2c)}{\sqrt{\pi b}}\ {}_1F_1\left(1;\frac{3}{2};\frac{-\omega^2}{4b}\right)+\frac{2c}{\sqrt{2\pi b}}\ {}_1F_1\left(1;\frac{3}{2};\frac{-\omega^2}{8b}\right)\exp\left(\frac{-\omega^2}{8b}\right)$ $+\frac{j(c-a)}{\omega}\left[1-\exp\left(\frac{-\omega^2}{4b}\right)\right]-\frac{j\omega c}{2b\pi}\left[\ {}_1F_1\left(1;\frac{3}{2};\frac{-\omega^2}{8b}\right)\right]$
4. $\frac{\Gamma(a,b\vartheta^2)}{2\Gamma(a)}$	$G_\vartheta(\omega)=\frac{\Gamma(2a)}{[\Gamma(a)]^2 2^{2a}}\sqrt{\frac{\pi}{b}}\ {}_1F_1\left(\frac{1}{2}+a;\frac{3}{2};\frac{-\omega^2}{4b}\right)$ $-\frac{j}{\omega}\left[\frac{1}{2}-\frac{\Gamma(2a)\sqrt{\pi}}{\Gamma(a)\Gamma(a+1/2)2^{2a}}\ {}_1F_1\left(a;\frac{1}{2};\frac{-\omega^2}{4b}\right)\right]$
5. $\int_0^\eta a(\theta)\exp[-\vartheta^2 b(\theta)]d\theta$	$G_\vartheta(\omega)=\frac{1}{2}\int_0^\eta \frac{a(\theta)}{b(\theta)}\left\{\sqrt{\pi b(\theta)}\exp\left(\frac{-\omega^2}{4b(\theta)}\right)-j\omega\ {}_1F_1\left(1;\frac{3}{2};\frac{-\omega^2}{4b(\theta)}\right)\right\}d\theta$
6. $\int_0^\eta a\exp\left[\frac{-b\vartheta^2}{\sin^2(\theta+c)}\right]d\theta$	$G_\vartheta(\omega)=\frac{a\sqrt{\pi}\cos c}{2\sqrt{b}}\exp\left(\frac{-\omega^2}{4b}\sin^2 c\right)\ {}_1F_1\left(1;\frac{3}{2};\frac{-\omega^2\cos^2 c}{4b}\right)$ $-\frac{a\sqrt{\pi}\cos(\eta+c)}{2\sqrt{b}}\exp\left[\frac{-\omega^2}{4b}\sin^2(\eta+c)\right]\ {}_1F_1\left(1;\frac{3}{2};\frac{-\omega^2\cos^2(\eta+c)}{4b}\right)$ $-\frac{j\omega a}{2}\int_0^\eta \frac{\sin^2(\theta+c)}{b}\ {}_1F_1\left(1;\frac{3}{2};\frac{-\omega^2\sin^2(\theta+c)}{4b}\right)d\theta$
7. $\int_0^\eta a\exp\left[\frac{-b\vartheta^2}{\cos^2(\theta+c)}\right]d\theta$	$G_\vartheta(\omega)=\frac{-a\sqrt{\pi}\sin c}{2\sqrt{b}}\exp\left(\frac{-\omega^2}{4b}\cos^2 c\right)\ {}_1F_1\left(1;\frac{3}{2};\frac{-\omega^2\sin^2 c}{4b}\right)$ $+\frac{a\sqrt{\pi}\sin(\eta+c)}{2\sqrt{b}}\exp\left[\frac{-\omega^2}{4b}\cos^2(\eta+c)\right]\ {}_1F_1\left(1;\frac{3}{2};\frac{-\omega^2\sin^2(\eta+c)}{4b}\right)$ $-\frac{j\omega a}{2}\int_0^\eta \frac{\cos^2(\theta+c)}{b}\ {}_1F_1\left(1;\frac{3}{2};\frac{-\omega^2\cos^2(\theta+c)}{4b}\right)d\theta$

and

$$b=\begin{cases}0.5, & \text{for orthogonal FSK}\\ 1, & \text{for antipodal PSK.}\end{cases}$$

Therefore, the ABER for the single-channel reception case can be obtained using (7), Table I, and either Table II (entry 4) or Table III (entry 4). It appears simpler to attain the final result from the FT identity in Table II. For $L=1$, it is evident that we can replace $\Psi_\vartheta(\omega,1)=\Psi_\alpha(\omega)$ given in Table I, since $\vartheta\equiv\alpha$.

Now, let us consider the CEP of the exponential form, $P_e(\gamma)=0.5\exp(-b\gamma)$, which corresponds to DPSK and NCFSK modulation formats. Substituting the first entry from Table II into (5), and then applying Cauchy's theorem, we get a closed-form solution for the ABER

$$\overline{P_e}(1)=\frac{1}{2}\left[\frac{1}{2\pi j}\int_{-\infty}^{\infty}\frac{1}{\omega-jb}\Psi_\gamma(\omega)d\omega\right]$$

$$=\frac{1}{2}\Psi_\gamma(\omega)\Big|_{\omega=jb}$$

$$=\frac{1}{2}\phi_\gamma(b) \tag{12}$$

given that $\Psi_\gamma(\omega)\equiv\Psi_\gamma(\omega,1)$ and $\phi_\gamma(\omega)\equiv\phi_\gamma(\omega,1)$ without any loss of generality. Notice that the final result is simply the MGF of the SNR, and this formula holds for any fading channel models. As a result, shadowing cases can be treated easily.

At this point, we would also like to highlight that the PDF method can sometimes lead directly to a closed-form solution. For instance, if the PDF of the SNR is Gamma distributed, then using [26, eq. (6.455.1)] we can immediately get the ABER [for CEP given by (11)] in Nakagami-m fading

$$\overline{P_e}(1)=\frac{\Gamma(m+a)m^{m-1}(b\bar{\gamma})^a}{2\Gamma(a)\Gamma(m)(b\bar{\gamma}+m)^{m+a}}\times{}_2F_1\left(1,m+a;m+1;\frac{m}{m+b\bar{\gamma}}\right) \tag{13}$$

where $\bar{\gamma}$ denotes the average SNR per bit. Now utilizing identities [26, eqs. (9.131.1) and (8.391)], (13) can be rewritten more compactly as

$$\overline{P_e}(1) = \frac{1}{2} I_{\frac{m}{m+b\bar{\gamma}}}(m, a) \tag{14}$$

which is identical to [25, eq. (17)] where $I_x(.,.)$ denotes the incomplete beta function ratio.

B. M-ary Modulation With Single Channel Reception

In [17], Craig derived an exact average probability of symbol error for arbitrary M-ary two-dimensional (2-D) signal constellations in AWGN using geometric relations. The CEP is expressed as the weighted sum of probabilities for all decision subregions of every possible signal point, namely

$$P_e(\gamma) = \sum_{k=1}^{S} \frac{W_k}{2\pi} \int_0^{\eta_k} \exp\left[\frac{-\gamma a_k \sin^2(b_k)}{\sin^2(\theta + b_k)}\right] d\omega \tag{15}$$

where S is the total number of signal points or decision subregions, W_k is the *a priori* probability that the kth symbol is transmitted, η_k, a_k, and b_k are parameters relating to decision subregion k and are independent of γ. The corresponding ASER can be readily shown as

$$\begin{aligned}\overline{P_e}(L) &= \sum_{k=1}^{S} \frac{W_k}{2\pi} \int_0^{\eta_k} \left[\frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{\Psi_\gamma(\omega)}{\frac{a_k \sin^2(b_k)}{\sin^2(\theta+b_k)} + j\omega}\right] d\theta \\ &= \sum_{k=1}^{S} \frac{W_k}{2\pi} \int_0^{\eta_k} \left[\Psi_\gamma(\omega)\big|_{\omega = \frac{j a_k \sin^2(b_k)}{\sin^2(\theta+b_k)}}\right] d\theta \\ &= \sum_{k=1}^{S} \frac{W_k}{2\pi} \int_0^{\eta_k} \phi_\gamma\left(\frac{a_k \sin^2(b_k)}{\sin^2(\theta + b_k)}\right) d\theta \end{aligned} \tag{16}$$

by substituting entry 7 of Table II into (5) and then applying Cauchy's theorem (after interchanging the order of integration). Therefore, it is evident that, if the CEP can be expressed in a desirable exponential form, the solution obtained from the CHF method collapses to the MGF approach.[3] Aside from this, the CHF approach may also yield an alternative, exact integral expression for the ASER. This latter observation will become apparent in our next example where the CEP has both exponential and nonexponential representations.

The CEP for noncoherent detection of equal energy, equiprobable, binary nonorthogonal (correlated) complex signals is given by [10, eq. (5-2-70)]

$$\begin{aligned}P_e(\gamma) &= Q(a\sqrt{\gamma}, b\sqrt{\gamma}) - \frac{1}{2} I_0(ab\gamma) \exp\left[-\frac{\gamma}{2}(a^2 + b^2)\right] \\ &= \frac{1}{2\pi} \int_0^{\pi} \frac{1 - \left(\frac{a}{b}\right)^2}{1 + 2\left(\frac{a}{b}\right)\sin\theta + \left(\frac{a}{b}\right)^2} \\ &\quad \times \exp\left[-\frac{\gamma b^2}{2}\left[1 + 2\left(\frac{a}{b}\right)\sin\theta + \left(\frac{a}{b}\right)^2\right]\right] d\theta \end{aligned} \tag{17}$$

[3]The trigonometric integral in (16) can be expressed in closed-form in Rayleigh and Nakagami-m channels with positive integer fading severity index [27].

where $a = \sqrt{(1 - \sqrt{1 - |\rho|^2})/2}$, $b = \sqrt{(1 + \sqrt{1 - |\rho|^2})/2}$ and $0 \leq |\rho| \leq 1$ is the magnitude of the cross-correlation coefficient between the two signals. Using the FT identity in Table II (entry 8) in conjunction with (7), we can show that the ABER for the CEP given in (17) (expressed in terms of first-order Marcum-Q and modified Bessel functions) is simply

$$\overline{P_e}(1) = \frac{1}{\pi} \int_0^{\frac{\pi}{2}} \frac{\Psi_\gamma(\tan\theta)}{\cos^2\theta} \left[\frac{4b^2}{\tilde{\kappa}_2(\tilde{\kappa}_2 - \tilde{\kappa}_1)} - \frac{1}{\tilde{\kappa}_2}\right] d\theta \tag{18}$$

where $\tilde{\kappa}_1 = 2j\tan\theta + a^2 - b^2$, $\tilde{\kappa}_2 = \sqrt{[\tilde{\kappa}_3 + 2ab][\tilde{\kappa}_3 - 2ab]}$, and $\tilde{\kappa}_3 = 2j\tan\theta + a^2 + b^2$.

Alternatively, using entry 7 of Table II and then applying Cauchy's theorem, we obtain

$$\begin{aligned}\overline{P_e}(1) &= \frac{1}{2\pi} \int_0^{\pi} \frac{1 - \left(\frac{a}{b}\right)^2}{1 + 2\left(\frac{a}{b}\right)\sin\theta + \left(\frac{a}{b}\right)^2} \\ &\quad \times \Psi_\gamma\left[\frac{jb^2}{2}\left[1 + 2\left(\frac{a}{b}\right)\sin\theta + \left(\frac{a}{b}\right)^2\right]\right] d\theta. \end{aligned} \tag{19}$$

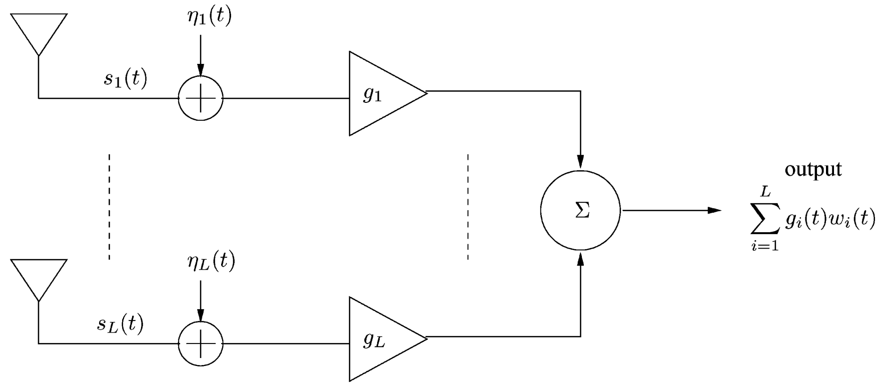
Notice that both (18) and (19) are exact solutions but that they are in completely different forms. For the special case of $a = 0$, it can easily shown that (18) and (19) reduce to a familiar result in [1] for the single-channel reception of binary orthogonal FSK $[b = 1]$ and differentially coherent PSK $[b = \sqrt{2}]$ systems. Furthermore, by substituting $[a = \sqrt{2 - \sqrt{2}}, b = \sqrt{2 + \sqrt{2}}]$ in (18), we obtain an alternative and exact integral expression for ABER of DQPSK with Gray coding over generalized fading channels.

We will avoid a repetition of derivations for each different modulation format and conclude this subsection noting that the ASER expressions for M-ary linearly modulated signals (MPSK, MQAM, star-QAM), MDPSK, MFSK in a variety of fading environments, can be derived in a manner similar to the above two examples. These results make use of (7), CHFs listed in Table I, FT identities listed in Table II, and Cauchy's theorem when an exponential representation for the CEP is available. A list of CEPs for common modulation formats may be found in [12].

C. Predetection Diversity Techniques

A unique feature of our CHF approach is that it facilitates, under a single common framework, the unified analysis of a wide range of modulation formats in a variety of fading environments, for all commonly used diversity-combining techniques. Hitherto, this task resisted simple solutions because of the difficulties arising from coherent EGC receiver analysis and mixed fading scenarios. Although the methodology applies to hybrid diversity systems as well, in this subsection, we will restrict our analysis to four basic predetection diversity schemes (see Fig. 1).

Observing (7) and our previous examples in Sections III-A and III-B, it is apparent that only knowledge of $\Psi_\gamma(., L)$ or $\Psi_\vartheta(., L)$ (whichever is applicable), for each type of diversity-combining scheme, is further required in order to evaluate



MRC: $g_i = \alpha_i \exp(-j\varepsilon_i)$

EGC: $g_i = \exp(-j\varepsilon_i)$

SDC: $g_i = \begin{cases} g_i = \alpha_i \exp(-j\varepsilon_i) & i = k \\ 0 & i \neq k \end{cases}$ $\quad k = \text{argmax}\{\gamma_i\}$

SWC: $g_i = \begin{cases} g_i = \alpha_i \exp(-j\varepsilon_i) & i = k \\ 0 & i \neq k \end{cases}$ $\quad$ where $\gamma_k \geq \xi$

Fig. 1. Predetection diversity systems, where $g_i = \alpha, e^{-\sigma_i}$ are complex channel gains and $\omega_i(t) = g_i s_i(t) + \eta_i(t), i = 1 \cdots L$.

the error performance of binary and M-ary modulation formats with diversity reception. Fortunately, these CHFs can be determined quite easily for many cases of practical interest.

1) Independent Fading:

a) Maximal-ratio diversity (MRC): From the definition of CHF, it is straightforward to show that

$$\Psi_{\gamma}^{(MRC)}(\omega, L) = E\left[\exp\left(j\omega \sum_{k=1}^{L} \gamma_k\right)\right] = \prod_{k=1}^{L} \Psi_{\gamma_k}(\omega) \tag{20}$$

where $E[x]$ denotes the expectation (statistical average) of random variable x, and $\Psi_{\gamma_k}(.)$ corresponds to the CHF of the SNR of the kth diversity branch. Substituting (20) into (7), while utilizing both the CHF of the SNR (for single-channel reception) in Table I and the FT of the CEPs listed in Table II, we obtain an exact analytical expression for the ASER with MRC diversity in generalized fading channels.

b) Equal-gain diversity (EGC): Since the envelope of the EGC combiner output is $\vartheta = (1/\sqrt{L}) \sum_{k=1}^{L} \alpha_k$, its CHF is given by

$$\Psi_{\vartheta}^{(EGC)}(\omega, L) = E\left[\exp\left(\frac{j\omega}{\sqrt{L}} \sum_{k=1}^{L} \alpha_k\right)\right] = \prod_{k=1}^{L} \Psi_{\alpha_k}\left(\frac{\omega}{\sqrt{L}}\right) \tag{21}$$

where $\Psi_{\alpha_k}(.)$is the CHF of the fading envelope of the kth diversity branch. In a similar fashion to the MRC case, the exact ASER with EGC diversity can be computed quite easily by substituting (21) into (7), and utilizing the FT of CEPs listed in Table III, instead of Table II, to yield (24)–(26), shown at the bottom of the next page.

c) Selection diversity (SDC): The CDF of the SNR at SDC combiner output is simply the product of CDFs of the SNR of individual diversity branches. Exploiting the FT of a derivative property, we get

$$\Psi_{\gamma}^{(SDC)}(\omega, L) \cong \sum_{i=1}^{N} w_i \prod_{k=1}^{L} F_{\gamma_k}\left(\frac{j\chi_i}{\omega}\right) + R_N \tag{22}$$

where $F_{\gamma_k}(.)$ denotes the CDF of the SNR of the kth diversity branch (can be expressed in closed-form for Rayleigh, Rician, and Nakagami-m channels [7]), χ_i and w_i are the ith abscissa and weight, respectively, of the Nth-order Laguerre polynomial, and R_N is the remainder term. Substituting (22) into (7) and using $G_{\gamma}(.)$ tabulated in Table II, we obtain a simple expression for characterizing the SDC receiver performance in different fading environments.

d) Switched diversity (SWC): The derivation of CHF of γ in a SWC scheme is slightly more involved, when compared to the ideal SDC or MRC. Using a discrete-time model, it can be shown that the CHF of the SNR at the output of a dual-branch switch-and-stay diversity combiner is given by [7, eq. (10)]

$$\Psi_{\gamma}^{(SWC)}(\omega, L) = \sum_{k=1, l \neq k}^{2} \frac{F_{\gamma_l}(\xi)}{F_{\gamma_k}(\xi) + F_{\gamma_l}(\xi)} \times \left[\lambda_k(\xi, \omega) + \Psi_{\gamma_l}(\omega) F_{\gamma_k}(\xi)\right] \tag{23}$$

where ξ denotes the fixed switching threshold and $\lambda_k(\xi, \omega) = \int_{\xi}^{\infty} \exp(j\omega\gamma) f_{\gamma_k}(\gamma) d\gamma$ is the marginal CHF of the SNR of the kth diversity branch (which can be expressed in closed-form for Rayleigh, Rician, and Nakagami-m channels [7]).

It is interesting to note that the final ASER (or ABER) expression for SWC and SDC systems is identical to the MRC case, with the single exception that the expression for $\Psi_{\gamma}^{(MRC)}(., L)$ is now replaced with $\Psi_{\gamma}^{(SWC)}(., L)$ and $\Psi_{\gamma}^{(SDC)}(., L)$, respectively.

2) Correlated Fading: Note that (7) also applies to correlated fading. For instance, the error rate performance of an L-branch MRC receiver, in arbitrarily correlated Nakagami-m, Rician, or Rayleigh fading channels, can be readily evaluated by substituting (B2) or (B3) into (7).

In our next example, we will summarize the CHF of the SNR at the combiner output of dual-diversity MRC, SDC and SWC systems, in a correlated Nakagami-m channel with arbitrary parameters. These closed-form CHFs (shown in (24)–(26) at the bottom of the next page) may be used in conjunction with our generic expression (7) to unify the performance evaluation of a broad class of modulation formats. In these equations, ρ denotes the power correlation coefficient, ${}_2F_1(.,.;.;.)$ is the Gauss hypergeometric series, $\gamma(a,x) = \int_0^x \exp(-t)t^{a-1}dt$, $a > 0$ is the incomplete Gamma function and its companion (complementary incomplete Gamma function) is defined as $\Gamma(a,x) = \Gamma(a) - \gamma(a,x) = \int_0^x \exp(-t)t^{a-1}dt$, $A_{kl} = (\sqrt{\bar\gamma_l(1-\rho)/2})/(\bar\gamma_l/\bar\gamma_k - 1 - j\omega\bar\gamma_l(1-\rho))$ and $B_{kl} = (\sqrt{[j\omega\bar\gamma_k\bar\gamma_l(\rho-1)+\bar\gamma_k+\bar\gamma_l]^2 - 4\rho\bar\gamma_k\bar\gamma_l})/(j\omega\bar\gamma_k\bar\gamma_l(\rho-1)+\bar\gamma_l-\bar\gamma_k)$.

Lastly, we would like to point out that, by using two trigonometric integral identities, we were able to get exact closed-form expressions for MDPSK, $\pi/4$-DQPSK and arbitrary two-dimensional (2-D) signal constellations with polygonal decision boundaries (including MPSK, MQAM, and star-QAM) with MRC in independent and correlated Nakagami-m fading channels (positive integer m). This subject is discussed in detail in [27].

D. Binary Multichannel Signaling With Postdetection EGC Diversity

1) Case $a \geq 0$: In [10, Appendix B], Proakis presents a closed-form solution for the CEP of multichannel reception of binary signals, where the decision variable is a quadratic form in complex-valued Gaussian random variables, namely

$$\begin{aligned}P_e(\gamma) =& Q(a\sqrt{\gamma}, b\sqrt{\gamma}) - I_0(ab\gamma)\exp\left[-\frac{\gamma}{2}(a^2+b^2)\right]\\ &\times\left[1-\frac{1}{(1+\eta)^{2L-1}}\sum_{k=0}^{L-1}\binom{2L-1}{k}\eta^k\right]\\ &+\frac{\exp\left[-\frac{\gamma}{2}(a^2+b^2)\right]}{(1+\eta)^{2L-1}}\\ &\times\left\{\sum_{n=1}^{L-1} I_n(ab\gamma)\sum_{k=0}^{L-1-n}\binom{2L-1}{k}\right.\\ &\left.\times\left[\left(\frac{b}{a}\right)^n\eta^k - \left(\frac{a}{b}\right)^n\eta^{2L-1-k}\right]\right\}. \quad (27)\end{aligned}$$

When $L=1$, the third term does not contribute, and therefore (27) reduces to [10, eq. (B-21)] as expected. By substituting (27) into (1), and then applying (A4) (i.e., Parseval's theorem), we can show that the ABER with post-detection EGC is given by

$$\begin{aligned}\overline{P_e}(L) =& \frac{1}{2\pi}\int_{-\infty}^{\infty}\left\{\frac{4b^2}{\kappa_2(\kappa_2-\kappa_1)}\right.\\ &-\frac{2}{\kappa_2}\left[1-\frac{1}{(1+\eta)^{2L-1}}\sum_{k=0}^{L-1}\binom{2L-1}{k}\eta^k\right]\\ &+\frac{1}{(1+\eta)^{2L-1}}\sum_{n=1}^{L-1}\frac{2}{\kappa_2}\left(\frac{2ab}{\kappa_2+\kappa_3}\right)^n\\ &\times\sum_{k=0}^{L-1-n}\binom{2L-1}{k}\\ &\left.\times\left[\left(\frac{b}{a}\right)^n\eta^k - \left(\frac{a}{b}\right)^n\eta^{2L-1-k}\right]\right\}\\ &\times\Psi_\gamma(\omega,L)d\omega \quad (28)\end{aligned}$$

using identity [26, eq. (6.611.4)]

$$\int_0^\infty \exp(-\alpha x)I_\nu(\beta x)dx = \frac{\beta^\nu}{\sqrt{\alpha^2-\beta^2}[\alpha+\sqrt{\alpha^2-\beta^2}]^\nu},\quad \mathrm{Re}\,\nu > 1, \mathrm{Re}\,\alpha > |\mathrm{Re}\,\beta| \quad (29)$$

and $\kappa_1 = 2j\omega + a^2 - b^2$,

$$\kappa_2 = \sqrt{[(a+b)^2+2j\omega][(a-b)^2+2j\omega]}$$

and $\kappa_3 = 2j\omega + a^2 + b^2$. For independent fading, $\Psi_\gamma(\omega,L) = \Psi_\gamma^{(MRC)}(\omega,L)$. Similarly, correlated Rayleigh, Rician, or Nakagami-m fading cases can be handled using the CHFs listed in Appendix B.

$$\Psi_\gamma^{(MRC)}(\omega,2) = \left(\frac{4m^2\bar\gamma_1\bar\gamma_2(1-\rho)}{[m(\bar\gamma_1+\bar\gamma_2)-j\omega 2\bar\gamma_1\bar\gamma_2(1-\rho)]^2 - m^2[(\bar\gamma_1-\bar\gamma_2)^2+4\bar\gamma_1\bar\gamma_2\rho]}\right)^m \quad (24)$$

$$\Psi_\gamma^{(SDC)}(\omega,2) = \frac{2^{2m}\Gamma(2m)}{\Gamma(m)\Gamma(m+1)}\sum_{k=1,l\neq k}^{2}\frac{(A_{kl})^{2m}}{[\bar\gamma_k B_{kl}(1+B_{kl})]^m}\,{}_2F_1\left[1-m, m; 1+m; \frac{1}{2}\left(1-\frac{1}{B_{kl}}\right)\right] \quad (25)$$

$$\begin{aligned}\Psi_\gamma^{(SWC)}(\omega,2) =& \frac{1}{\Gamma(m)\left[\gamma\left(m,\frac{m\xi}{\bar\gamma_l}\right)+\gamma\left(m,\frac{m\xi}{\bar\gamma_2}\right)\right]}\\ &\times\sum_{k=1,l\neq k}^{2}\frac{\gamma\left(m,\frac{m\xi}{\bar\gamma_l}\right)\Gamma\left[m,\xi\left(\frac{m}{\bar\gamma_k}-j\omega\right)\right]+\gamma\left(m,\frac{m\xi}{\bar\gamma_k}\right)\gamma\left[m,\frac{m\xi(m-j\omega\bar\gamma_k)}{\bar\gamma_l(m-j\omega\bar\gamma_k(1-\rho))}\right]}{\left(1-\frac{j\omega\bar\gamma_k}{m}\right)^m} \quad (26)\end{aligned}$$

Now, using variable substitution $\omega = \tan\theta$ and taking advantage of the fact that the resulting integrand is an even function of θ, (28) can be restated as

$$\begin{aligned}\overline{P_e}(L) &= \frac{1}{\pi}\int_0^{\frac{\pi}{2}} \frac{\Psi_\gamma(\tan\theta, L)}{\cos^2\theta} \\ &\times \left\{ \frac{4b^2}{\tilde{\kappa}_2(\tilde{\kappa}_2 - \tilde{\kappa}_1)} - \frac{2}{\tilde{\kappa}_2}\left[1 - \frac{1}{(1+\eta)^{2L-1}}\sum_{k=0}^{L-1}\binom{2L-1}{k}\eta^k\right] \right. \\ &\quad + \frac{1}{(1+\eta)^{2L-1}}\sum_{n=1}^{L-1}\frac{2}{\tilde{\kappa}_2}\left(\frac{2ab}{\tilde{\kappa}_2+\tilde{\kappa}_3}\right)^n \sum_{k=0}^{L-1-n}\binom{2L-1}{k} \\ &\quad \left. \times \left[\left(\frac{b}{a}\right)^n \eta^k - \left(\frac{a}{b}\right)^n \eta^{2L-1-k}\right]\right\} d\theta \qquad (30)\end{aligned}$$

where $\tilde{\kappa}_1$, $\tilde{\kappa}_2$, and $\tilde{\kappa}_3$, are defined in (18).

Moreover, if we interchange the order of the summation in the second term of (30) and replace $l = L - k$, we obtain an equivalent form for the ABER

$$\begin{aligned}\overline{P_e}(L) &= \frac{1}{\pi}\int_0^{\frac{\pi}{2}} \frac{\Psi_\gamma(\tan\theta, L)}{\cos^2\theta} \\ &\times \left\{ \frac{4b^2}{\tilde{\kappa}_2(\tilde{\kappa}_2 - \tilde{\kappa}_1)} - \frac{2}{\tilde{\kappa}_2}\left[1 - \frac{1}{(1+\eta)^{2L-1}}\sum_{k=0}^{L-1}\binom{2L-1}{k}\eta^k\right] \right. \\ &\quad + \frac{1}{(1+\eta)^{2L-1}}\sum_{l=1}^{L}\binom{2L-1}{L-l}\sum_{n=1}^{l-1}\frac{2^{n+1}}{\tilde{\kappa}_2(\tilde{\kappa}_2+\tilde{\kappa}_3)^n} \\ &\quad \left. \times [b^{2n}\eta^{L-l} - a^{2n}\eta^{L-1+l}]\right\} d\theta. \qquad (31)\end{aligned}$$

For the special case of $\eta = 1$, the authors in [6] express the CEP given in (27) in terms of a finite series of generalized Marcum-Q function [6, eq. (68)], namely

$$\begin{aligned}P_e(\gamma) = \frac{1}{2} + \frac{1}{2^{2L-1}}\sum_{k=1}^{L}\binom{2L-1}{L-k} \\ \times [Q_k(a\sqrt{\gamma}, b\sqrt{\gamma}) - Q_k(b\sqrt{\gamma}, a\sqrt{\gamma})]. \qquad (32)\end{aligned}$$

By utilizing the identity [28, p. 528],

$$\begin{aligned}Q_M(\alpha,\beta) + Q_M(\beta,\alpha) = 1 + \exp\left[-\frac{1}{2}(\alpha^2+\beta^2)\right] \\ \times \sum_{n=1-M}^{M-1}\left(\frac{\beta}{\alpha}\right)^n I_n(\alpha\beta) \qquad (33)\end{aligned}$$

in (32) and then using the FTs listed in Table II, we found that the resulting ABER expression is identical to (31) with $\eta = 1$ because $\sum_{k=1}^{L}\binom{2L-1}{L-k} = 2^{2(L-1)}$.

2) Case $a = 0$: We will now focus our attention on the special case of $a = 0$, which corresponds to both binary orthogonal square-law detected FSK and binary differential-detected PSK with multichannel reception. The performance of these receiver structures have been studied by a number of authors (e.g., [6], [10], [29]–[31]). Recently, Simon and Alouini expressed the CEP as a single finite range integral [6, eq. (71)] by utilizing the trigonometric integral representations for the generalized Marcum-Q function [28, Appendix C]. However, this integral representation becomes void[4] if $a = 0$ for any $L > 1$, which limits its usefulness for the analysis of multichannel reception in both nonfading and fading channels. That aside, this particular case is interesting because the *exact* CEP can be expressed using different mathematical functions that, further highlighting the generality of our approach, can all be handled by our CHF technique.

From [10, eq. (14-4-24)], we have

$$P_e(\gamma) = \frac{1}{2^{2L-1}}\exp(-g\gamma)\sum_{k=0}^{L-1}\frac{1}{k!}\sum_{n=0}^{L-1-k}\binom{2L-1}{n}(g\gamma)^k \qquad (34)$$

where $g = 1/2$ for orthogonal binary FSK, and $g = 1$ for differential antipodal binary PSK. Note that (34) can also be derived from (27) by letting $\eta = 1$, $a = 0$, and $b = \sqrt{2g}$ and recognizing that $I_0(0) = 1$, $Q(0, b\sqrt{\gamma}) = \exp(-b^2\gamma/2)$ and $\lim_{a\to 0} I_n(ab\gamma)(b/a)^n = (1/n!)(b^2\gamma/2)^n$. Substituting (34) into (1), and recognizing $f(t) \leftrightarrow F(s)$ and $(gt)^k \exp(-gt)f(t) \leftrightarrow (-s)^k(d^k/ds^k)F(s)|_{s=g}$ are Laplace transform pairs, the ABER is given by

$$\begin{aligned}\overline{P_e}(L) = \frac{1}{2^{2L-1}}\sum_{k=0}^{L-1}\frac{1}{k!}\sum_{n=0}^{L-1-k} \\ \times \binom{2L-1}{n}(-g)^k\frac{d^k}{ds^k}\phi_\gamma(s,L)|_{s=g}. \qquad (35)\end{aligned}$$

In fact, this is what Patenaude *et al.* [29] have done for the special case of Nakagami-m fading.

The general result above (obtained directly from the classical representation of the CEP) holds for *any* fading channel models (including correlated fading as well as mixed-fading) and can be used to calculate the ABER as long as the MGF $\phi_\gamma(.)$ is known. Surprisingly, in [5] and [6], Simon and Alouini suggest that only their *approximate* solution to the above problem can handle the analysis in generalized fading channels. They state that, "...This is particularly true for the performance of binary orthogonal FSK and binary DPSK which cannot be obtained via the classical representation of (69) in the most general fading case, but which can be solved using the desirable conditional BER expression (71) as we will show next" [6, p. 1636]. A similar claim is also made in [5, p. 1874]. Nevertheless, since the evaluation of Lth order derivative in (35) can be quite cumbersome for large L, a single integral formula for the ABER may be preferable. The use of the MGF approach to arrive at [6, eq. (71)] may have lead to the difficulty in handling the case $a = 0$ easily. Therefore, we will now go on to demonstrate that, by using the CHF method, an *exact* single integral expression with finite integration limits for the ABER can be readily obtained from various forms of the CEP previously reported in literature. Furthermore, our *exact* expressions are considerably simpler than that of [6, eq. (76)] with $a/b = 10^{-3}$.

[4]Hence, one can only resort to an *approximate* solution by choosing a/b at the vicinity of zero (e.g., $a/b = 10^{-3}$).

The FT of CEP in the classical form (34) is listed in Table II. Using this FT identity, in conjunction with (7), we obtain

$$\overline{P_e}(L)=\frac{1}{2^{2L-1}}\sum_{k=0}^{L-1}\sum_{n=0}^{L-1-k}\binom{2L-1}{n}\times\frac{1}{\pi}\int_0^{\frac{\pi}{2}}\frac{1}{\cos^2\theta}\Re\left\{\frac{g^k}{[g^2+\tan^2\theta]^{\frac{(k+1)}{2}}}\times\exp\left[-j(k+1)\tan^{-1}\left(\frac{\tan\theta}{g}\right)\right]\times\Psi_\gamma(\tan\theta,L)\right\}d\theta. \quad (36)$$

The above result can also be derived directly, after some routine algebra manipulations, from (30) by substituting $[\eta=1, a=0, b=\sqrt{2g}]$. Also note that the imaginary part will be zero since the ABER is real.

In [31], Charash presented an alternative, exact CEP for multichannel noncoherent and differentially coherent binary signals involving confluent hypergeometric function, namely

$$P_e(\gamma)=\frac{\exp(-2g\gamma)}{2^L\Gamma(L)}\sum_{k=0}^{L-1}\frac{\Gamma(L+k)}{2^k\Gamma(k+1)}{}_1F_1(L+k;L;g\gamma). \quad (37)$$

Substituting the FT of CEP illustrated in (37) (see Table II) into (7), it can be shown that the corresponding ABER is given by

$$\overline{P_e}(L)=\frac{1}{2^L\Gamma(L)}\sum_{k=0}^{L-1}\frac{\Gamma(L+k)}{2^k\Gamma(k+1)}\frac{1}{\pi}\int_0^{\frac{\pi}{2}}\frac{\Psi_\gamma(\tan\theta,L)}{(2g+j\tan\theta)\cos^2\theta}\times{}_2F_1\left[L+k,1;L;\frac{g}{2g+j\tan\theta}\right]d\theta \quad (38)$$

which may be rewritten as

$$\overline{P_e}(L)=\frac{1}{2^L\Gamma(L)}\sum_{k=0}^{L-1}\frac{\Gamma(L+k)}{2^k\Gamma(k+1)}\sum_{n=0}^{k}(-g)^n\binom{k}{n}\frac{L-1}{L-1+n}\times\frac{1}{\pi}\int_0^{\frac{\pi}{2}}\Re\left\{\frac{\Psi_\gamma(\tan\theta,L)}{(2g+j\tan\theta)^{n+1}\cos^2\theta}\right\}d\theta \quad (39)$$

using Kummer's transformation formula.

Next, by interchanging the order of summation in (34) and subsequently replacing $l=L-n$, we obtain

$$P_e(\gamma)=\frac{1}{2^{2L-1}}\sum_{l=1}^{L}\binom{2L-1}{L-l}\frac{\Gamma(l,g\gamma)}{\Gamma(l)} \quad (40)$$

since $(\Gamma(1+n,x))/(\Gamma(1+n))=\exp(-x)\sum_{k=0}^{n}(x^k/k!)$ for $n\in\{0,1,2,\ldots\}$. This CEP representation appears to be new. Since the FT of the incomplete Gamma function can be expressed in a closed form, we obtain yet another exact, single integral expression with finite integration limits, for calculating the ABER over generalized fading channels

$$\overline{P_e}(L)=\frac{1}{2^{2(L-1)}\pi}\sum_{l=1}^{L}\binom{2L-1}{L-l}\times\int_0^{\frac{\pi}{2}}\Re\left\{\frac{\Psi_\gamma(\tan\theta,L)}{j\sin(2\theta)}\left[1-\left(\frac{g}{g+j\tan\theta}\right)^l\right]\right\}d\theta. \quad (41)$$

E. M-ary Orthogonal Signaling With Postdetection EGC Diversity

The ASER performance of M-ary orthogonal signals, employing a square-law combining receiver, in fading channels is given by [10, eq. (14-4-44)], [32], [33]

$$\overline{P_e}(L)=\sum_{n=1}^{M-1}\binom{M-1}{n}(-1)^{n+1}\sum_{k=0}^{n(L-1)}\beta_{kn}\times\int_0^\infty\left[\int_0^\infty\left(\frac{u}{2}\right)^k\exp\left(\frac{-nu}{2}\right)f_U(u|\gamma)du\right]f_\gamma(\gamma)d\gamma \quad (42)$$

where $f_\gamma(\gamma)$ is the PDF of symbol SNR at the combiner output, $f_U(u|\gamma)$ is the conditional PDF of the decision variable U when the signal is present,

$$f_U(u|\gamma)=\frac{1}{2}\left(\frac{u}{\gamma}\right)^{\frac{L-1}{2}}\exp\left(-\frac{u+\gamma}{2}\right)I_{L-1}(\sqrt{u\gamma}) \quad (43)$$

and the coefficients β_{kn} may be computed using [32, eq. (32)]

$$\beta_{kn}=\sum_{i=k-L+1}^{k}\frac{\beta_{i(n-1)}}{(k-i)!}I_{[0,(n-1)(L-1)]}(i) \quad (44)$$

where $\beta_{00}=\beta_{0n}=1$, $\beta_{k1}=1/k!$, $\beta_{1n}=n$, and

$$I_{[a,b]}(i)=\begin{cases}1, & a\le i\le b\\ 0, & \text{otherwise.}\end{cases}$$

The conditional CHF of U (noncentral chi-square distribution) is given by [10, eq. (2-1-117)],

$$\Psi_U(\omega|\gamma)=\frac{1}{(1-2j\omega)^L}\exp\left(\frac{2j\omega\gamma}{1-2j\omega}\right). \quad (45)$$

Now, applying Parseval's theorem twice in (42) (in order to transform the two product integrals into frequency domain), we obtain

$$\begin{aligned}\overline{P_e}(L)=&\sum_{n=1}^{M-1}\binom{M-1}{n}(-1)^{n+1}\sum_{k=0}^{n(L-1)}\beta_{kn}\frac{1}{2\pi}\times\int_{-\infty}^{\infty}\left(\frac{1}{2}\right)^k\frac{\Gamma(k+1)}{\left[\left(\frac{n}{2}\right)^2+\omega^2\right]^{\frac{(k+1)}{2}}}\\&\times\exp\left[-j(k+1)\tan^{-1}\left(\frac{2\omega}{n}\right)\right]\times\left[\int_0^\infty\Psi_U(\omega|\gamma)f_\gamma(\gamma)d\gamma\right]d\omega\\=&\sum_{n=1}^{M-1}\binom{M-1}{n}(-1)^{n+1}\sum_{k=0}^{n(L-1)}\beta_{kn}\frac{1}{2\pi}\times\int_{-\infty}^{\infty}\frac{2\Gamma(k+1)}{[n^2+(2\omega)^2]^{\frac{(k+1)}{2}}}\\&\times\exp\left[-j(k+1)\tan^{-1}\left(\frac{2\omega}{n}\right)\right]\frac{1}{(1-2j\omega)^L}\times\left[\frac{1}{2\pi j}\int_{-\infty}^{\infty}\frac{1}{\upsilon-\frac{2\omega}{1-2j\omega}}\Psi_\gamma(\upsilon,L)d\upsilon\right]d\omega \quad (46)\end{aligned}$$

which may be simplified as

$$\overline{P_e}(L) = \frac{1}{2\pi}\sum_{n=1}^{M-1}\binom{M-1}{n}(-1)^{n+1}\sum_{k=0}^{n(L-1)}\beta_{kn} \times \int_{-\infty}^{\infty}\frac{\Gamma(k+1)}{[n^2+\omega^2]^{\frac{(k+1)}{2}}}\exp\left[-j(k+1)\tan^{-1}\left(\frac{\omega}{n}\right)\right] \times \frac{1}{(1-j\omega)^L}\Psi_\gamma\left(\frac{\omega}{1-j\omega}, L\right)d\omega \quad (47)$$

with the aid of Cauchy's theorem and also normalizing ω by 2.

Also note that the CHF of combiner output SNR for post-detection EGC is identical to the predetection MRC case. By replacing $1/(1-j\omega)^L$ in (47) with its polar representation, letting $\Psi_\gamma(x, L) = \phi_\gamma(-jx, L)$ and finally using variable substitution $\omega = \tan\theta$, we obtain a single integral expression that has finite integration limits

$$\overline{P_e}(L) = \frac{1}{\pi}\sum_{n=1}^{M-1}\binom{M-1}{n}(-1)^{n+1}\sum_{k=0}^{n(L-1)}\beta_{kn} \times \int_{o}^{\frac{\pi}{2}}\frac{\Gamma(k+1)(\cos\theta)^{L-2}}{[n^2+\tan^2\theta]^{\frac{(k+1)}{2}}} \times \exp\left\{j\left[L\theta-(k+1)\tan^{-1}\left(\frac{\tan\theta}{n}\right)\right]\right\} \times \phi_\gamma\left(\frac{-j\tan\theta}{1-j\tan\theta}, L\right)d\theta. \quad (48)$$

Note that (48) is equivalent to [34, eq. (10)].[5] The above development is particularly interesting because the MGF method cannot facilitate the analysis of multichannel MFSK systems, but the CHF method leads directly to a simple analytical solution. For the special case of $M = 2$ (binary orthogonal square-law detected FSK), (48) reduces to

$$\overline{P_e}(L) = \sum_{k=0}^{L-1}\frac{1}{\pi}\int_0^{\frac{\pi}{2}}(\cos\theta)^{L+k-1} \times \exp\left[j(L-k-1)\theta\right]\phi_\gamma\left(\frac{-j\tan\theta}{1-j\tan\theta}, L\right)d\theta \quad (49)$$

which is equivalent to (36) since it can be shown that these two expressions agree numerically.

Also, for $L = 1$, (48) reduces to

$$\overline{P_e}(1) = \frac{1}{\pi}\sum_{n=1}^{M-1}\binom{M-1}{n}(-1)^{n+1} \times \int_0^{\frac{\pi}{2}}\frac{\exp\left(j\left[\theta-\tan^{-1}\left(\frac{\tan\theta}{n}\right)\right]\right)}{\sqrt{n^2+\tan^2\theta}\cos\theta}\phi_\gamma\left(\frac{-j\tan\theta}{1-j\tan\theta}\right)d\theta. \quad (50)$$

However, it is more concise to obtain the corresponding ASER (envelope detection of MFSK with single channel reception) directly from [10, eq. (5-4-46)], viz.,

$$\overline{P_e}(1) = \sum_{n=1}^{M-1}\frac{(-1)^{n+1}}{n+1}\binom{M-1}{n}\Psi_\gamma\left(\frac{jn}{n+1}\right) \quad (51)$$

which is a generalization of Crepeau's results [35] for different fading environments.

F. Outage Rate of Error Probability

Outage probability is another performance criterion frequently used in cellular mobile radio communications systems. The outage rate of error probability P_{out} is defined as the probability that the instantaneous bit-error probability of the system will exceed a specified value (say P_e^*) and can be written as [14]

$$P_{\text{out}} = \int_0^{x^*} p_x(x, L)dx = F_x(x^*, L) \quad (52)$$

where x^* is obtained by solving $P_e(x^*) = P_e^*$ and $x \in \{\gamma, \vartheta\}$.

The solution γ^* and $\vartheta^* = \sqrt{\gamma^*}$ can be easily obtained in closed-form for several modulation formats and they are listed below.

- Case 1) If $P_e(\gamma) = a\exp(-b\gamma)$, then $\gamma^* = (\ln(a/P_e^*)/b)$ or $\vartheta^* = \sqrt{\ln(a/P_e^*)/b}$.
- Case 2) If $P_e(\gamma) = a\text{erfc}(\sqrt{b\gamma})$, then $\gamma^* = (1/b)[\text{erfc}^{-1}(P_e^*/a)]^2$.
- Case 3) If $P_e(\gamma) = a\text{erfc}(\sqrt{b\gamma}) - c\text{erfc}^2(\sqrt{b\gamma})$, then $\gamma^* = (1/b)\{\text{erfc}^{-1}[(a-\sqrt{a^2-4cP_e^*})/2c]\}^2$.

However, the evaluation of (52) becomes difficult for diversity systems since we need to determine the PDF of the SNR at the combiner output. This problem can be circumvented by replacing the PDF with its Fourier inversion integral and then simplifying the resultant expression, using the inverse Fourier transform identity of a unit-step function. Thus, (9) and/or (10) provide a simple technique for calculating the outage probability for both single and multichannel reception cases.

IV. Conclusion

This paper makes a number of contributions: 1) we outline a direct method of computing ABER or ASER of different modulation formats, with/without diversity, in a single common framework; 2) we derive the FT for the CEPs frequently encountered in the analysis of digital communications systems (to be used in conjunction with our unified expressions); 3) we show that the CHF method encapsulates all results obtained using the MGF technique; and 4) we remove the limitations of MGF approach, reformulating the task of finding a desirable exponential form for the CEP to simply determining its FT. Selected applications of our new expressions are presented. Several previous results are also shown to be special cases of this analytical framework.

Appendix A
Parseval's Theorem

A simple proof of Parseval's theorem from the first principle is detailed below. This development reveals that no particular restriction is imposed on the application of (A3). Note that, if the time-domain functions are real, then the identity (A4) also applies.

[5] However, the sign for the imaginary term in equations (10), (12), and (14) in [34] should be a "+" instead of a "−" sign.

The frequency convolution theorem states that, if $\mathrm{FT}[f_1(t)] = G_1(\omega)$ and $\mathrm{FT}[f_2(t)] = G_2(\omega)$, then

$$\mathrm{FT}\left[f_1(t)f_2(t)\right] = \frac{1}{2\pi}\left[G_1(\omega)\otimes G_2(\omega)\right] = \frac{1}{2\pi}\int_{-\infty}^{\infty} G_1(y)G_2(\omega-y)dy \quad \text{(A1)}$$

where notation $\otimes$ denotes the convolution operation. In other words,

$$\int_{-\infty}^{\infty}\left[f_1(t)f_2(t)\right]e^{-j\omega t}dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} G_1(y)G_2(\omega-y)dy. \quad \text{(A2)}$$

Now, letting $\omega = 0$ in (A2) and then changing the dummy variable of integration, we obtain

$$\int_{-\infty}^{\infty} f_1(t)f_2(t)dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} G_1(\omega)G_2(-\omega)d\omega. \quad \text{(A3)}$$

If $f(t)$ is real, then the property $G(-\omega) = G^*(\omega)$ applies, where $G^*(\omega)$ denotes the complex conjugate of $G(\omega)$. Consequently, from (A3), we have

$$\int_{-\infty}^{\infty} f_1(t)f_2(t)dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} G_1(\omega)G_2^*(\omega)d\omega = \frac{1}{2\pi}\int_{-\infty}^{\infty} G_1^*(\omega)G_2(\omega)d\omega \quad \text{(A4)}$$

if $f_1(t)$ and $f_2(t)$ are real functions.

APPENDIX B
CORRELATED FADING STATISTICS

In this appendix, the CHFs for the sum of instantaneous SNRs (from L diversity branches) in both correlated Nakagami-m and correlated Rician fading environments are presented. Using these CHFs, *generically correlated* fading cases can be readily handled with our expressions (7) and (10), for the calculation of ASER (with predetection MRC and/or post-detection EGC) and outage analysis, respectively.

A. Correlated Nakagami-m Fading

In an arbitrarily correlated Nakagami-m fading environment (with the assumption that the fading severity index is common to all the diversity branches), the joint CHF of the instantaneous SNR may be written in the form [20]

$$\Psi(\omega_1,\ldots,\omega_L) = \det(I - jWR\Lambda)^{-m} \quad \text{(B.1)}$$

where I is the $L\times L$ identity matrix, Λ is a positive definite matrix of dimension L (determined by the branch covariance matrix), W and R are two diagonal matrices defined as $W = \mathrm{diag}(\omega_1,\ldots,\omega_L)$ and $R = \mathrm{diag}(\bar{\gamma}_1/m,\ldots,\bar{\gamma}_1/m)$, respectively, and m is the fading parameter. Consequently, the CHF of the combiner output SNR, $\gamma = \sum_{k=1}^{L}\gamma_k$, can be obtained from (B.1) by setting $\omega_1 = \ldots = \omega_L = \omega$, i.e.,

$$\Psi_\gamma(\omega, L) = \det(I - j\omega R\Lambda)^{-m} = \prod_{k=1}^{L}\frac{1}{(1-j\omega\lambda_k)^m} \quad \text{(B2)}$$

where λ_k is the kth eigen value of matrix $R\Lambda$. For the particular case of $m = 1$, (B2) reduces to the familiar expression in [16] for the CHF of SNR at the output of a MRC and/or a quadratic combiner (post-detection EGC) in correlated Rayleigh fading channels.

B. Correlated Rician Fading

Let x denote a complex normal random vector with mean $\boldsymbol{\mu} = [\mu_1,\ldots,\mu_L]^T$ and covariance $C = E[(x-\boldsymbol{\mu})(x-\boldsymbol{\mu})^H]$ (Hermitian positive definite). If x corresponds to the channel gain vector, then the instantaneous SNR at the output of a MRC and/or a quadratic (square-law) combiner is given by $\gamma = x^H x$. Then the CHF of γ in correlated Rician fading can be readily expressed in a Hermitian quadratic form [16, eq. (B-3-4)], namely

$$\Psi_\gamma(\omega, L) = \exp\left(\sum_{k=1}^{L}\frac{j\omega\lambda_k}{1-j\omega\lambda_k}|R_k|^2\right)\prod_{k=1}^{L}\frac{1}{(1-j\omega\lambda_k)} \quad \text{(B3)}$$

where λ_k is the kth eigen value of the covariance matrix C, and R_k is the kth element of vector R defined as

$$R = U^H C^{-\frac{1}{2}}\mu \quad \text{(B4)}$$

where $C^{-1/2}$ is the inverse of Hermitian square root of C and U^H can be obtained from the covariance matrix by singular value decomposition, viz., $C = U\Lambda U^H$. For the specific case of Rayleigh fading, R is a zero vector since $x_k (k = 1,\ldots,L)$ are zero-mean Gaussian RVs ($\mu_k = 0$). In this specific case, both (B2) and (B3) yield identical results, as anticipated.

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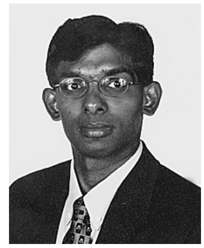

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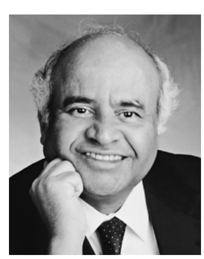

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