A Simplified Clipping and Filtering Technique for PAR Reduction in OFDM Systems

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***Abstract*—The existing iterative clipping and filtering techniques require several iterations to mitigate the peak regrowth. In this letter, we analyze the conventional clipping and filtering using a parabolic approximation of the clipping pulse. We show that the clipping noise obtained after several clipping and filtering iterations is approximately proportional to that generated in the first iteration. Therefore, we scale the clipping noise generated in the first iteration to get a new clipping and filtering technique that, with three fast Fourier transform/inverse fast Fourier transform (FFT/IFFT) operations, obtains the same PAR reduction as that of the existing iterative techniques with $2K+1$ FFT/IFFT operations, where K represents the number of iterations.**

***Index Terms*—Clipping and filtering, orthogonal frequency division multiplexing (OFDM), peak-to-average power ratio (PAR) reduction, scaling.**

I. Introduction

A main drawback of orthogonal frequency division multiplexing (OFDM) systems is the high peak-to-average power ratio (PAR) [1]. Large PAR requires a linear high power amplifier (HPA), which, however, is not efficiently used. If the linear range of HPA is not sufficient, large PAR leads to in-band distortion and out-of-band radiation [1]. To reduce the PAR, various techniques have been proposed, including clipping and filtering [2]–[4], multiple signal representation [5], [6], and coding [7]. Among these techniques, clipping and filtering (CF) is perhaps the simplest way for PAR reduction. Note that, compared to the in-band distortion, the out-of-band radiation is more critical since it severely interferes with communications in adjacent frequency bands. CF techniques eliminate the out-of-band radiation by clipping the time-domain signal to a predefined level and subsequently filtering it. The relatively small in-band distortion is combatted using low-order signal constellation, coding, and/or clipping noise cancellation techniques [8], [9].

To suppress peak regrowth due to filtering, iterative clipping and filtering (ICF) techniques can be used [3], [4]. Their convergence rate decreases significantly after the first few iterations. Note that each iteration requires two fast Fourier transform/inverse fast Fourier transform (FFT/IFFT), and after the last iteration, one extra IFFT is required to convert the clipped OFDM symbol to time domain. As a K-iteration process requires $2K+1$ FFT/IFFT, the increased number of iterations implies increased computational complexity, especially when the number of subcarriers is very large. The convergence rate can be improved by setting the clipping threshold to a level slightly lower than the required level [3], [10]. The steepest gradient method [11] and the active-set approach [12] are also proposed for tone reservation, which uses virtual subcarriers to suppress the high PAR. Nevertheless, these approaches still need several iterations to ensure low PAR regrowth. When passband OFDM signal is considered, some of these approaches (e.g., [12]) preprocess the complex signal to separate it to real numbers, which increases the computational complexity.

In this letter, we propose a new CF technique that, in one iteration, obtains the same PAR reduction as that of ICF with several iterations. We analyze CF by approximating each clipping pulse as a parabolic function. Our analysis shows that the clipping noise obtained after several CF iterations is approximately proportional to that generated in the first iteration. Therefore, we can scale the clipping noise generated in the first iteration to approximate that obtained after several iterations. The same PAR reduction can then be obtained with only three FFT/IFFT, which significantly reduces the computational complexity.

This letter is organized as follows: Section II characterizes the OFDM system. The new technique is proposed in Section III. In Section IV, simulation results are shown and compared with the previous ICF techniques. The conclusion of this letter is given in Section V.

II. Characterization of OFDM System

In OFDM systems, the time-domain signal $x(t)$ can be written as

$$x(t)=\frac{1}{\sqrt{N}}\sum_{k=-\frac{N}{2}}^{\frac{N}{2}-1} X_{\langle k+N\rangle} e^{\frac{j2\pi kt}{T}},\quad 0\le t\le T \tag{1}$$

where $\langle k+N\rangle$ denotes $(k+N)$ modulo N, $\mathbf{X}=[X_0,X_1,\ldots,X_{N-1}]$ represents the input OFDM symbol, N is the number of subcarriers, and T is the data symbol period. The PAR of $x(t)$ can be defined as

$$\xi \triangleq \frac{\max_{t\in[0,T)} |x(t)|^2}{P_{av}} \tag{2}$$

where P_{av} is the average power defined as $P_{av}\triangleq E\{|x(t)|^2\}$ and $P_{av}=E\{|X_k|^2\}$ based on Parseval's theorem. Note that ξ can be very large. For example, when phase shift keying (PSK) symbols are used, ξ can be as large as N. However, since such a large PAR occurs rarely, it is more useful to consider ξ as a random variable and use a statistical description given by the

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complementary cumulative density function (CCDF), defined as the probability that PAR exceeds ξ_p, i.e., $\Pr[\xi > \xi_p] = P_c$.

To approximate PAR in (2) in discrete time domain, an oversampled version of (1) can be used. In this case, the oversampled time-domain signal x_n can be written as

$$x_n = \frac{1}{\sqrt{N}} \sum_{k=-\frac{N}{2}}^{\frac{N}{2}-1} X_{\langle k+N \rangle} e^{j\frac{2\pi}{JN}nk}, \quad n=0,\ldots,JN-1 \quad (3)$$

where J is the oversampling factor. Usually, $J \geq 4$ is used to capture the peaks of the continuous time-domain signal. The oversampled time-domain signal x_n can be obtained by using a length-(JN) IFFT operation, with extending $\mathbf{X}$ to a length-(JN) vector by inserting $(J-1)N$ zeros in the middle, i.e.,

$$\mathbf{X}_{\text{ext}} = \left[X_0, \ldots, X_{\frac{N}{2}-1}, 0, \ldots, 0, X_{\frac{N}{2}}, \ldots, X_{N-1}\right].$$

III. New CF Algorithm

Based on the central limit theory, $x(t)$ can be approximated as a complex Gaussian process when N is large. Assume that $x(t)$ has zero mean and variance σ^2. Then, its magnitude $r(t) = |x(t)|$ is a Rayleigh process, and the real and imaginary parts of $x(t)$, denoted as $x_i(t)$ and $x_q(t)$, respectively, are identically distributed (real) Gaussian signals with zero mean and variance σ^2.

Now, let us consider clipping $x(t)$ using a soft limiter (SL). The clipped signal can be written as

$$\hat{x}(t) = \begin{cases} Ae^{j\phi(t)}, & |x(t)| > A \\ x(t), & |x(t)| \leq A \end{cases} \quad (4)$$

where $\phi(t)$ represents the phase of $x(t)$, and A is the clipping level. When A is large, the clipping occurs rarely, and the clipping noise $x(t) - \hat{x}(t)$ is a series of pulses. In this case, each pulse can be approximated as a parabolic arc [13]. Based on this approximation, our main contribution is as follows.

Theorem 1: Let A represent the clipping level, and define β as

$$\beta \triangleq \frac{\text{total filtered clipping noise after } K \text{ iterations}}{\text{filtered clipping noise generated in the first iteration}}.$$

Then, when A is large, the mean of β, denoted as $\bar{\beta}$, can be approximated as

$$\bar{\beta} \approx \frac{1-(1-\bar{\alpha})^{\frac{3K}{2}}}{1-(1-\bar{\alpha})^{\frac{3}{2}}} \quad (5)$$

where

$$\bar{\alpha} = \frac{2\sqrt{2}}{\sqrt{3\pi}} \frac{1}{\frac{A}{\sigma}}. \quad (6)$$

The proof is shown in the Appendix.

Using Theorem 1, the clipping noise in each iteration is similar, and the total filtered clipping noise can be approximated by scaling the filtered clipping noise generated in the first iteration. Then, the new CF algorithm can be stated as follows.

1) Convert the OFDM symbol to time domain as $x_n = \text{IFFT}(X_k)$. Note that oversampling is needed.
2) Clip x_n to the threshold A and calculate the clipping noise f_n.
3) Convert f_n to frequency domain to obtain F_k by doing $\text{FFT}(f_n)$ and keep the first and last $N/2$ items of F_k to obtain the in-band distortion $\hat{F}_k$.
4) The clipped OFDM signal then becomes $\tilde{X}_k \approx X_k - \bar{\beta}\hat{F}_k$.
5) Convert $\tilde{X}_k$ to time domain and transmit it.

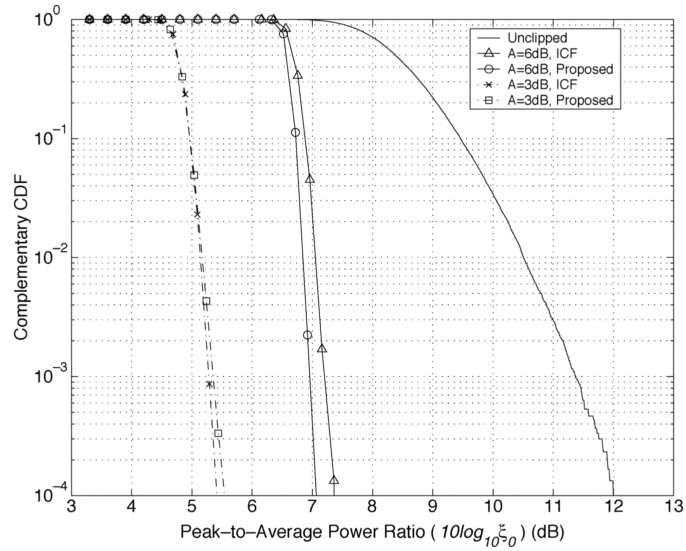

Fig. 1. PAR CCDF comparison of the proposed technique and ICF.

IV. Simulation Results

In this section, the PAR reduction and bit error rate (BER) performance are evaluated by simulations. In our simulations, the number of subcarriers is $N = 256$, and unitary quadrature phase shift keying (QPSK) symbols (i.e., $|X_k| = 1$) are input to the OFDM system. Our proposed technique uses $K = 3$ to approximate the ICF technique with three iterations. We consider 6 dB and 3 dB clipping cases, and A is small in the 3-dB clipping case. However, our simulation results show that our proposed technique can also be used together with clipping noise cancellation (CNC) techniques [8], [9].

Fig. 1 compares the PAR reduction of our proposed technique and that of ICF. Their PAR reduction is close. For the 6-dB clipping case, our proposed technique is about 0.3 dB better than ICF at the PAR CCDF of 10^{-4}.

To investigate BER performance degradation and out-of-band distortion, we consider passing the clipped signal through a solid-state power amplifier (SSPA) with limited linear range. The input/output relationship of SSPA can be written as [1]

$$y(t) = \frac{|x(t)|}{\left(1+\left(\frac{|x(t)|}{C}\right)^{2p}\right)^{\frac{1}{2p}}} e^{j\phi(t)}$$

where $x(t) = |x(t)|e^{j\phi(t)}$ is the input, and $y(t)$ is the output of SSPA. The SSPA becomes the SL when $p \to \infty$. Usually, $p = 3$ for practical SSPA.

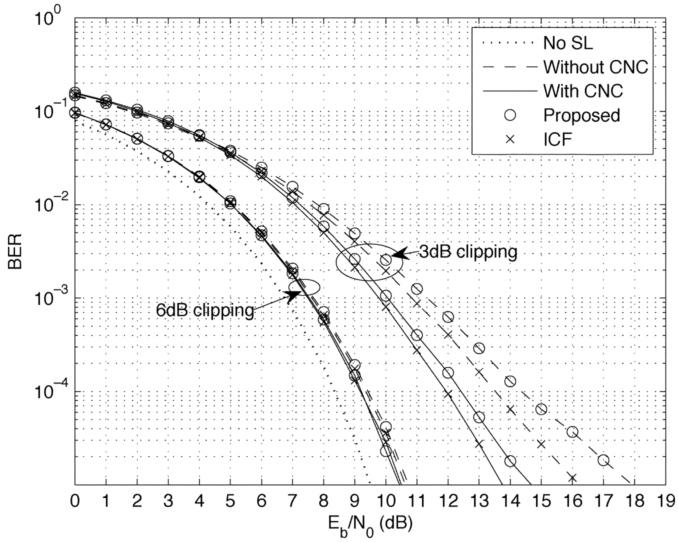

Fig. 2. BER performance comparison of the proposed technique and ICF.

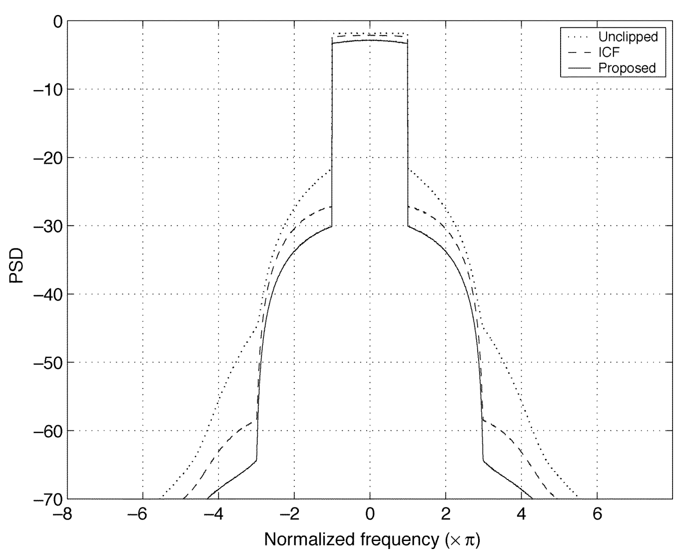

Fig. 3. Out-of-band radiation comparison of the proposed technique and ICF, the 3-dB clipping case.

For the 6-dB clipping case, the time-domain signal is first clipped with $A = 6$ dB and then is passed through an SSPA with $C = 6$ dB. For 3-dB clipping, both A and C are set to 3 dB. Fig. 2 compares the BER performance of our proposed technique and ICF. As a reference, the dotted curve shows the BER of the ideal case when the amplifier has enough linear range, and no PAR reduction technique is needed. Solid curves represent BER when using the CNC technique, while dashed curves represent the BER without CNC. "$\circ$" are for our proposed technique, and "$\times$" are for ICF.

For 6-dB clipping, the BER performance of these two techniques are very close, and there is little need to use CNC techniques. The BER performance in this case is only about 1 dB worse than the ideal case. On the other hand, in 3-dB clipping, both techniques are quite worse than the ideal case (6.5–8.5 dB at the BER of 10^{-5}) when no CNC is used, while the gap between these two techniques at the BER of 10^{-5} is about 2 dB. However, when CNC is used, the BER performance of these two techniques is improved by 2.5–3.5 dB, and our proposed technique is only about 1 dB worse than ICF. Therefore, the performance of our proposed technique is comparable to that of ICF. However, since the new technique only performs one iteration, its computational complexity is significantly lower than that of ICF at both the transmitter and the receiver.[1]

As an example, the out-of-band radiation comparison for the 3-dB clipping case is shown in Fig. 3. Here, pulse shaping is not considered for simplicity. We can see that ICF leads to about 6-dB lower out-of-band radiation than that of without using any PAR reduction technique, and our proposed technique leads to about 2.5-dB lower out-of-band radiation than ICF.

V. Conclusion

In this letter, we propose a new CF technique that, in one iteration, obtains the same PAR reduction as that of the previous ICF technique with several iterations. The computational complexity is, therefore, significantly reduced. Simulations show that, in relatively large clipping cases, our proposed technique has better PAR reduction and out-of-band radiation than and very close BER performance to ICF. In terms of out-of-band radiation and BER, our proposed technique (in conjunction with CNC) performs similar to ICF in deep clipping cases.

Appendix
Proof of Theorem 1

Assume a clipping occurs at $-(\tau/2) \le t \le \tau/2$. By using Taylor's series expansion, the clipping pulse can be written as

$$f(t) = x(t) - x\left(-\frac{\tau}{2}\right) \approx \dot{x}\left(-\frac{\tau}{2}\right)t + \frac{1}{2}\ddot{x}\left(-\frac{\tau}{2}\right)t^2. \quad (7)$$

We can see that[2] $\tau \approx -(2\dot{x}(-(\tau/2))/\ddot{x}(-(\tau/2)))$. By defining $b \triangleq -\ddot{x}(-(\tau/2))$, $f(t)$ can be written as

$$f(t) = -\frac{1}{2}bt^2 + \frac{1}{8}b\tau^2, \quad -\frac{\tau}{2} \le t < \frac{\tau}{2}. \quad (8)$$

Here, both b and τ are random variables. In [14] and [15], the clipping of a real Gaussian signal is considered, and b was approximated to be proportional to $x(\tau/2)$. However, we will not use such an assumption in our case, and it will be seen that b has no effect on our result.

The clipping pulse is the consequence of upward level crossing (at level A) of $r(t) = |x(t)|$. The level crossing rate (expected number of crossings of level A per second) can be found as [16]

$$\lambda_A = \frac{\dot{\sigma}}{\sqrt{2\pi}}\frac{A}{\sigma^2}e^{-\frac{A^2}{2\sigma^2}} \quad (9)$$

where $\dot{\sigma}^2$ is the power of the first derivative of $x_i(t)$. Since the power density spectrum of $x(t)$ is (approximately) constant over the frequency band $[-(N/2T), N/2T]$

$$\frac{\dot{\sigma}^2}{\sigma^2} = \frac{(\pi N)^2}{3T^2}.$$

[1]As in [9], the same number of iterations have to be performed at the receiver to cancel the clipping noise.

[2]Strictly speaking, τ is a real number, while $-(2\dot{x}(-(\tau/2))/\ddot{x}(-(\tau/2)))$ is complex. However, since $|x(-(\tau/2))| = |x(\tau/2)| = A$, one can expect that the imaginary part of $-(2\dot{x}(-(\tau/2))/\ddot{x}(-(\tau/2)))$ is very small, and this approximation is valid.

Now, denote the expect value of τ as $\bar{\tau}$. Since $\lambda_A \bar{\tau} = \Pr(r(t) > A)$, $\bar{\tau}$ can be calculated as

$$\bar{\tau} = \frac{\Pr(r(t) > A)}{\lambda_A} = \frac{\sigma\sqrt{2\pi}}{\frac{\dot{\sigma} A}{\sigma}} = \sqrt{\frac{6}{\pi}} \frac{T}{N\left(\frac{A}{\sigma}\right)}. \quad (10)$$

The frequency spectrum of $f(t)$ can be written as

$$F(\omega) = \frac{b\tau}{\omega^2}\left(\text{sinc}\frac{\omega\tau}{2} - \cos\frac{\omega\tau}{2}\right). \quad (11)$$

$F(\omega)$ consists of a large portion of out-of-band radiation and has to be filtered. Suppose that an ideal lowpass filter with the cutoff frequency $\omega_c = 2\pi f_c = 2\pi(N/2T)$ is used. For the in-band part of $F(\omega)$, $\omega \le \omega_c$, we have $\omega\tau/2 \le \omega_c\tau/2 \ll 1$, when A/σ is large. Therefore, $F(\omega)$ can be approximated as

$$F(\omega) \approx \frac{b\tau^3}{12}, \quad |\omega| \le \omega_c \quad (12)$$

by using $\text{sinc}\theta - \cos\theta \approx \theta^2/3$, when $\theta \ll 1$ [14]. Taking the inverse Fourier transform of (12), the filtered clipping pulses in time domain becomes

$$\hat{f}(t) = \frac{1}{2\pi}\int_{-\omega_c}^{\omega_c} \frac{b\tau^3}{12} e^{j\omega t} d\omega = \frac{b\tau^3 f_c}{6}\text{sinc}2\pi f_c t. \quad (13)$$

Note that both $f(t)$ and $\hat{f}(t)$ reach their peaks at the same time instant $t = 0$. Moreover, because $\omega_c\tau/2 \ll 1$, $\hat{f}(t)$ is roughly constant within the duration of $f(t)$, $-(\tau/2) \le t \le \tau/2$. Therefore, we can approximate $\hat{f}(t)$ in terms of the peak value of $f(t)$ as

$$\hat{f}(t) \approx \left|\hat{f}(t)\right|_{\max} \approx \frac{b\tau^3 f_c}{6} = \alpha\,|f(t)|_{\max}, \quad -\frac{\tau}{2} \le t \le \frac{\tau}{2}$$

where α is defined as

$$\alpha \triangleq \frac{4}{3} f_c \tau = \frac{2}{3}\frac{N}{T}\tau. \quad (14)$$

By using (10), the expectation of α is

$$\bar{\alpha} = \frac{2\sqrt{2}}{\sqrt{3\pi}} \frac{1}{\frac{A}{\sigma}}. \quad (15)$$

In OFDM systems, the (unfiltered) clipping noise is usually a series of parabolic pulses, and the value of the filtered clipping noise at $t = 0$ is the summation of the values of all the filtered pulses at $t = 0$. However, (13) shows that the sidelobe peaks of $|\hat{f}(t)|$ are

$$\left|\hat{f}(t_k)\right| \approx \frac{b\tau^3 f_c}{6}\frac{2}{(2k+1)\pi}, \quad k = 1, 2, 3, \ldots$$

with t_k representing sidelobe peak occurrence time in accordance with

$$t_k \approx \frac{(2k+1)\frac{\pi}{2}}{2\pi f_c} = \frac{T}{N}\frac{2k+1}{2}.$$

Since N is large, $|\hat{f}(t_k)|$ decays very fast with time. On the other hand, denoting the average number of clipping pulses in one OFDM signal duration as N_p, then the average "distance" between two pulses can be calculated as

$$\Delta T_p = \frac{T}{N_p} = \frac{T}{\lambda_A T} = \frac{T}{N}\frac{1}{\sqrt{\frac{\pi}{6}}\frac{A}{\sigma}e^{-\frac{A^2}{2\sigma^2}}}.$$

Therefore, the effects of other filtered pulses on the considered pulse (occurs at $t = 0$) is negligible when A is large. For example, when $A = 6$ dB ≈ 1.9953, $\sigma^2 = 1/2$, $\Delta T_p \approx 26.2388 T/N$. The ratio of the sidelobe peak of a neighboring pulse at $t = 0$ over the mainlobe peak of $|\hat{f}(t)|$ is about $2/(51\pi) \approx 0.0125$, which can be ignored.

In the second iteration, the clipping pulse generated at $t = 0$ will still be the same as $f(t)$, except the pulse duration τ is reduced. By solving the equation $f(t) = \alpha|f(t)|_{\max}$, we get the clipping pulse duration as

$$\tau^{(1)} = \tau\sqrt{1-\alpha}. \quad (16)$$

By iteratively using (16), the clipping pulse duration after K iterations can be obtained.

The above analysis is valid when K is not large. In this case, (12) holds in each iteration. Therefore, the in-band clipping noise $F^{(i)}(\omega)$ generated in the ith iteration is proportional to $(\tau^{(i)})^3$. Then, the ratio of the total clipping noise after K iterations over that generated in the first iteration can be calculated as

$$\beta = \frac{F(\omega) + F^{(1)}(\omega) + \cdots + F^{(K-1)}(\omega)}{F(\omega)} = \frac{\sum_{i=0}^{K-1}\left(\tau^{(i)}\right)^3}{\tau^3} \quad (17)$$

where $\tau^{(0)} \triangleq \tau$.

Now, consider (16) and (17). The term τ in (16) will be cancelled in (17). Moreover, since $\tau \ll 1$, the variance of α is small compared to the mean of α. Therefore, we can replace α with its mean value $\bar{\alpha}$ and treat it as a constant. Then, we get (5).

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