Performance Analysis of Three-Branch Selection Combining Over Arbitrarily Correlated Rayleigh-Fading Channels

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***Abstract*—The recent literature has thoroughly treated two-branch selection combining (SC) over correlated Rayleigh fading and three-branch SC over *exponentially* correlated Rayleigh fading. However, a long-standing open problem involves the three-branch SC performance over arbitrarily correlated Rayleigh fading. We solve this problem completely by deriving new infinite series expressions for the cumulative distribution function, the probability density function, and the moment generating function (mgf) of the three-branch SC output signal-to-noise ratio (SNR). The output mgf can be used to derive the average symbol-error rate for any two-dimensional digital modulations. The outage probability and the higher moments of the SC output SNR are also derived. These analytical results are canonical, in that the three-branch SC performance is now completely solved for arbitrary correlation. Some previous results are shown to be special cases of our new results.**

***Index Terms*—Correlated Rayleigh fading, diversity, moment generating function (mgf), selection combining (SC).**

I. Introduction

Diversity combining techniques such as maximal ratio combining (MRC), equal gain combining (EGC), and selection combining (SC) are commonly used to mitigate the effects of multipath fading on wireless communication systems. The maximum diversity benefits occur when the diversity branches experience independent and identical fading. However, in practice, fading is not independent due to the insufficient antenna spacing. Hence, characterizing the diversity system performance over correlated fading channels is important from both a theoretical and practical viewpoint.

The two-branch SC performance over various correlated fading channels has been comprehensively studied in the literature (see [1]–[6]). For multibranch $(L > 2)$ SC, general methods remain elusive and most of the available papers deal with restricted correlation models. For example, Mallik and Win [7] analyze the generalized SC (GSC) over equally correlated Nakagami-m-fading channels using the symmetric property of the covariance matrix of equally correlated gamma random variables (RVs). Chen and Tellambura [8], [9] analyze the SC performance over various equally correlated fading channels by using a novel representation of the channel gains. More recently, the three-branch SC performance has been studied over exponentially correlated Nakagami-m- and Rayleigh-fading channels [10]–[12]. However, for arbitrarily correlated fading, we are able to find only two papers that address the multibranch SC performance. Exceptionally, Ugweje and Aalo [13] derive the probability density function (pdf) of the SC output signal-to-noise ratio (SNR) over correlated Nakagami-m fading as a multiple series of generalized Laguerre polynomials, using the previous results [14]. This approach becomes fairly complicated when $L > 2$. Zhang and Lu [15] derive a general approach for multibranch SC over correlated fading by expressing the joint pdf as an L-dimensional inverse integral of the joint characteristic function (chf) and manipulating a resulting kernel expression. Their results require L-dimensional infinite-range integrals. Karagiannidis *et al.* [16] use Green's matrix approximation to evaluate the multivariate Nakagami-m-fading distribution.

Here, we use the results of [17], [18] to derive cumulative distribution function (cdf), the pdf, and the moment generating function (mgf) of the three-branch SC output SNR and evaluate the error rate performance, the output moments and the outage probability of three-branch SC over arbitrarily correlated Rayleigh fading. Our new results are in the form of infinite series, which may be more suitable for computation than infinite-range integrals.

The rest of this paper is organized as follows. Section II derives the cdf and the pdf of the three-branch SC output SNR. Section III evaluates the error rate performance, the output moments, and the outage probability of SC. Section IV provides numerical results and concludes this paper.

II. Three-Branch SC Output Statistics

For three arbitrarily correlated Rayleigh amplitudes $\boldsymbol{\alpha} = (\alpha_1, \alpha_2, \alpha_3)$, the joint pdf can be found in [17, p. 559]. Representing the modified Bessel function by an infinite series [19, eq. (9.6.10)] and integrating the joint pdf, we obtain the joint cdf of $\boldsymbol{\alpha}$ as

$$\begin{aligned} F_{\boldsymbol{\alpha}}(x_1, x_2, x_3) &= \frac{\det(\boldsymbol{\Phi})}{\phi_{11}\phi_{22}\phi_{33}} \sum_{m=0}^{\infty} \varepsilon_m (-1)^m \cos(m\chi) \\ &\quad \times \sum_{i,j,k=0}^{\infty} \nu_{12}^{i+\frac{m}{2}} \nu_{23}^{j+\frac{m}{2}} \nu_{31}^{k+\frac{m}{2}} \frac{\gamma\left(i+k+m+1, x_1^2\phi_{11}\right)}{i!(k+m)!} \\ &\quad \times \frac{\gamma\left(i+j+m+1, x_2^2\phi_{22}\right)\gamma\left(j+k+m+1, x_3^2\phi_{33}\right)}{j!k!(i+m)!(j+m)!} \end{aligned} \tag{1}$$

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where $\chi = \chi_{12} + \chi_{23} + \chi_{31}$, $\nu_{ij} = |\phi_{ij}|^2/\phi_{ii}\phi_{jj}$ for $i, j \in \{1, 2, 3\}$, ε_m is the Neumann factor ($\varepsilon_0 = 1$, $\varepsilon_m = 2$ for $m = 1, 2, \cdots$), $\gamma(a, x)$ is the incomplete gamma function defined as [19, eq. (6.5.2)], and $\mathbf{\Phi}$ is the inverse covariance matrix of the underlying zero-mean complex Gaussian RVs $\boldsymbol{G} = (G_1, G_2, G_3)$

$$\mathbf{\Phi} = \mathbf{\Psi}^{-1} = \begin{bmatrix} \phi_{11} & \phi_{12} & \phi_{13} \\ \phi_{12}^* & \phi_{22} & \phi_{23} \\ \phi_{13}^* & \phi_{23}^* & \phi_{33} \end{bmatrix}, \quad \phi_{ik} = |\phi_{ik}| e^{j\chi_{ik}} \tag{2}$$

where $\mathbf{\Psi}$ is the covariance matrix of $\boldsymbol{G}$ with elements $\psi_{ij} = E(G_i G_j^*)$ and $E(x)$, $|x|$ and x^* denote the average, the amplitude, and the complex conjugate of x, respectively.

In SC, the branch with the largest SNR is selected as the output

$$\gamma_{\mathrm{sc}} = \max(\gamma_1, \gamma_2, \gamma_3) \tag{3}$$

where γ_k is the instantaneous SNR of the kth branch, which is given by

$$\gamma_k = |G_k|^2 \frac{E_s}{N_0} = \alpha_k^2 \frac{E_s}{N_0}, \quad k = 1, 2, 3 \tag{4}$$

where G_k is the channel gain associated with the kth branch, which is assumed to be zero-mean complex Gaussian distributed, E_s is the energy of the transmitted signal, and N_0 is the power spectral density of additive white Gaussian noise (AWGN).

A. CDF of the Three-Branch SC Output SNR

The cdf of the three-branch SC output SNR can be readily derived using (1) as

$$\begin{aligned} F_{\mathrm{sc}}(x) &= \Pr(\gamma_{\mathrm{sc}} \le x) \\ &= \Pr(\gamma_1 \le x, \gamma_2 \le x, \gamma_3 \le x) \\ &= F_{\boldsymbol{\alpha}}\left(\sqrt{\frac{x\psi_{11}}{\bar{\gamma}_1}}, \sqrt{\frac{x\psi_{22}}{\bar{\gamma}_2}}, \sqrt{\frac{x\psi_{33}}{\bar{\gamma}_3}}\right) \end{aligned} \tag{5}$$

where $\bar{\gamma}_k$ is the average SNR of the kth branch. Equation (5) holds for any arbitrary 3×3 correlation matrix. It reduces to previous results for three special cases.

1) Independent Rayleigh Fading: The covariance matrix $\mathbf{\Psi}$ for independent Rayleigh channel gains $\boldsymbol{G}$ is diagonal, i.e., $\psi_{ij} = 0$ for $i \ne j$ and $\psi_{ii} = E(\alpha_i^2)$. Thus, the inverse covariance matrix $\mathbf{\Phi}$ is also diagonal with elements $\phi_{ij} = 0$ for $i \ne j$ and $\phi_{ii} = 1/E(\alpha_i^2)$. Noticing that $\gamma(1, x) = 1 - e^{-x}$, we can readily obtain the cdf of the three-branch SC output SNR over independent Rayleigh fading as

$$F_{\mathrm{sc}}(x) = \left(1 - e^{-\frac{x}{\bar{\gamma}_1}}\right)\left(1 - e^{-\frac{x}{\bar{\gamma}_2}}\right)\left(1 - e^{-\frac{x}{\bar{\gamma}_3}}\right) \tag{6}$$

which is equivalent to the well-known result [1, eq. (10-4-9)].

2) Exponentially Correlated Rayleigh Fading: The exponential correlation model is used to describe the correlation among equally spaced linear antenna arrays [20]. The normalized covariance matrix of this model is described as $\psi_{ij} = \rho^{|i-j|}$, where $0 \le \rho < 1$. It can then be shown that $\phi_{13} = 0$ and (5) can be simplified considerably for the cdf of the three-branch SC over exponentially correlated Rayleigh fading

$$F_{\mathrm{sc}}(x) = \frac{1-\rho^2}{1+\rho^2} \sum_{i,j=0}^{\infty} \frac{\left(\frac{\rho^2}{1+\rho^2}\right)^{i+j}}{(i!)^2 (j!)^2} \gamma\left(i+1, \frac{x}{(1-\rho^2)\bar{\gamma}}\right) \times \gamma\left(i+j+1, \frac{(1+\rho^2)x}{(1-\rho^2)\bar{\gamma}}\right) \gamma\left(j+1, \frac{x}{(1-\rho^2)\bar{\gamma}}\right) \tag{7}$$

which is equivalent to recently derived [10, eq. (4)], when simplified for Rayleigh fading.

3) Equally Correlated Rayleigh Fading: The constant correlation model is valid for a set of closely placed antennas. The normalized covariance matrix of this model is described as $\psi_{ij} = \rho$ for $i \ne j$ and $\psi_{ii} = 1$, where $-1/2 \le \rho < 1$. It can be shown that $\chi_{12} + \chi_{23} + \chi_{31} = 3\pi$. Thus, the output cdf of the three-branch SC over equally correlated Rayleigh fading can be derived from (5) as

$$\begin{aligned} F_{\mathrm{sc}}(x) = \frac{(1-\rho)(1+2\rho)^2}{(1+\rho)^3} \sum_{m=0}^{\infty} \varepsilon_m \sum_{i,j,k=0}^{\infty} \left(\frac{\rho}{1+\rho}\right)^{\beta} &\times \frac{\gamma\left(i+k+m+1, \frac{(1+\rho)x}{(1+\rho-2\rho^2)\bar{\gamma}}\right)}{i!j!k(i+m)!(j+m)!(k+m)!} \\ &\times \gamma\left(i+j+m+1, \frac{(1+\rho)x}{(1+\rho-2\rho^2)\bar{\gamma}}\right) \\ &\times \gamma\left(j+k+m+1, \frac{(1+\rho)x}{(1+\rho-2\rho^2)\bar{\gamma}}\right) \end{aligned} \tag{8}$$

where $\beta = 2(i+j+k) + 3m$. We have checked (8) numerically against an alternative cdf expression [8, eq. (13)]. Both the methods give exactly the same numerical values.

TABLE I
NUMBER OF TERMS NEEDED IN (8) TO ACHIEVE FIVE SIGNIFICANT FIGURE ACCURACY

	$x/\bar{\gamma} = 0$dB	$x/\bar{\gamma} = 5$dB
$\rho = 0.2$	$M = 2, I = J = K = 3$	$M = 3, I = J = K = 4$
$\rho = 0.5$	$M = I = J = K = 4$	$M = I = J = K = 5$
$\rho = 0.8$	$M = 4, I = J = K = 6$	$M = 7, I = J = K = 10$

The convergence rate of the cdf series (5) depends not only on the correlation among branch signals $\mathbf{\Psi}$ but also on the normalized branch SNRs $x/\bar{\gamma}_k$. Without loss of generality, we investigate the convergence property of the cdf series for the equally correlated case. Table I lists the number of terms required in each sum of (8) to achieve five significant figure accuracy for different correlations ρ and the normalized branch SNRs $x/\bar{\gamma}$. The total number of terms required is equal to $M \times I \times J \times K$, where M, I, J, and K denote the number of terms required in the variables m, i, j, and k, respectively. We find that the cdf series (8) converges much faster as the correlation or the normalized branch SNR decreases.

B. PDF of the Three-Branch SC Output SNR

Differentiating (5) yields the corresponding pdf of the SC output SNR

$$p_{\mathrm{sc}}(x) = \frac{\det(\mathbf{\Phi})}{\phi_{11}\phi_{22}\phi_{33}} \sum_{m=0}^{\infty} \varepsilon_m (-1)^m \cos m\chi \sum_{i,j,k=0}^{\infty} \nu_{12}^{i+\frac{m}{2}} \times \nu_{23}^{j+\frac{m}{2}} \nu_{31}^{k+\frac{m}{2}} \frac{I(1,2,3) + I(2,3,1) + I(3,1,2)}{i!j!k!(i+m)!(j+m)!(k+m)!} \tag{9}$$

where

$$I(i,j,k)=b_i^{\delta_i+1}x^{\delta_i}e^{-b_i x}\gamma(\delta_j+1,b_j x)\gamma(\delta_k+1,b_k x) \quad (10)$$

and $b_i = \psi_{ii}\phi_{ii}/\bar{\gamma}_i$ for $i = 1,2,3$, $\delta_1 = i+k+m$, $\delta_2 = i+j+m$, and $\delta_3 = j+k+m$.

III. Performance Analysis of Triple-Branch SC

We now derive the average symbol-error rate (SER) or bit-error rate (BER) of various digital modulations with three-branch SC over arbitrarily correlated Rayleigh fading. The outage probability and the higher moments of the SC output SNR are also derived. Channel fading is assumed to be frequency nonselective and changing slowly enough so that the channel parameters remain constant for the duration of the signaling interval.

A. Error-Rate Performance

The mgf of a diversity combiner output SNR is the key to the performance analysis of digital modulations over wireless-fading channels [21], [22]. Noticing the relation between the incomplete gamma function and the confluent hypergeometric function [23, eq. (9.236.4)] and applying the integral identity [24, eq. (C.1)], we obtain the mgf of the three-branch SC output SNR $\gamma_{\rm sc}$ as

$$\begin{aligned} M_{\rm sc}(s)=E(e^{-s\gamma_{\rm sc}})&=\frac{\det(\mathbf{\Phi})}{\phi_{11}\phi_{22}\phi_{33}}\sum_{m=0}^{\infty}\varepsilon_m(-1)^m\cos m\chi \\ &\times\sum_{i,j,k=0}^{\infty}\frac{\nu_{12}^{i+\frac{m}{2}}\nu_{23}^{j+\frac{m}{2}}\nu_{31}^{k+\frac{m}{2}}b_1^{\delta_1+1}b_2^{\delta_2+1}b_3^{\delta_3+1}\Gamma(\delta)}{(b+s)^{\delta}i!j!k!(i+m)!(j+m)!(k+m)!} \\ &\times\left[\frac{F_A\left(\delta;1,1;\delta_2+2,\delta_3+2;\frac{b_2}{b+s},\frac{b_3}{b+s}\right)}{(\delta_2+1)(\delta_3+1)}\right. \\ &\quad+\frac{F_A\left(\delta;1,1;\delta_1+2,\delta_3+2;\frac{b_1}{b+s},\frac{b_3}{b+s}\right)}{(\delta_1+1)(\delta_3+1)} \\ &\quad\left.+\frac{F_A\left(\delta;1,1;\delta_1+2,\delta_2+2;\frac{b_1}{b+s},\frac{b_2}{b+s}\right)}{(\delta_1+1)(\delta_2+1)}\right] \end{aligned} \quad (11)$$

where $\delta = \delta_1+\delta_2+\delta_3+3 = 2(i+j+k+m)+3$, $b = b_1+b_2+b_3 = \sum_{i=1}^{3}\psi_{ii}\phi_{ii}/\bar{\gamma}_i$, and $F_A(\alpha;\beta_1,\cdots,\beta_n;\gamma_1,\cdots,\gamma_n;z_1,\cdots,z_n)$ denotes the nth order Appell hypergeometric function defined as [23, eq. (9.19)].

The mgf approach [21], [22] can be readily employed to evaluate the error-rate performance of three-branch SC over arbitrarily correlated Rayleigh fading. From [25] and [26], the SER for any two-dimensional (2-D) coherent linear modulation over AWGN can be expressed in terms of finite-range exponential-type integrals. Averaging the conditional SER over the distribution of the combiner output SNR, we obtain the average SER as

$$\bar{P}_s=\frac{1}{2\pi}\sum_{k=1}^{N}W_k\int_0^{\eta_k}M_{\rm sc}\left(\frac{a_k\sin^2(\varphi_k)}{\sin^2(\theta+\varphi_k)}\right)d\theta \quad (12)$$

where N is the number of signal points, W_k is the *a priori* probability that kth signal point is transmitted, and a_k, η_k, and φ_k are parameters relating to decision subregion k.

For noncoherent digital modulations such as noncoherent binary frequency shift keying (NC-BFSK) and binary differential phase shift keying (BDPSK), the mgf approach can also be employed. For example, the BER for NC-BFSK and BDPSK over AWGN is given by [27, eq. (5-6-29)]. The average BER can then be obtained in terms of the output mgf (11) as

$$\bar{P}_s=\frac{1}{2}M_{\rm sc}(a) \quad (13)$$

where $a = 1$ for BDPSK and $a = 1/2$ for NC-BFSK. From (12) and (13), we find that the average SER of various digital modulations can be numerically computed using either a finite-range integral of $M_{\rm sc}(s)$ or just the $M_{\rm sc}(s)$.

B. Output Moments

The moments of a combiner output SNR can be used as an alternative performance measure to error-rate analysis. However, the average output SNR alone does not reveal enough information and the higher order moments can furnish additional information for system design. For example, if the variance of the output is small, large fades from the average is not likely (which follows from the Chebyshev inequality). The moments of three-branch SC output SNR over arbitrarily correlated Rayleigh fading can be obtained using the result we derived in (9)

$$\begin{aligned} E(\gamma_{\rm sc}^n) &=\frac{\det(\mathbf{\Phi})}{\phi_{11}\phi_{22}\phi_{33}}\sum_{m=0}^{\infty}\varepsilon_m(-1)^m\cos m\chi \\ &\times\sum_{i,j,k=0}^{\infty}\frac{\nu_{12}^{i+\frac{m}{2}}\nu_{23}^{j+\frac{m}{2}}\nu_{31}^{k+\frac{m}{2}}b_1^{\delta_1+1}b_2^{\delta_2+1}b_3^{\delta_3+1}\Gamma(\delta+n)}{b^{\delta+n}i!j!k!(i+m)!(j+m)!(k+m)!} \\ &\times\left[\frac{F_A\left(\delta+n;1,1;\delta_2+2,\delta_3+2;\frac{b_2}{b},\frac{b_3}{b}\right)}{(\delta_2+1)(\delta_3+1)}\right. \\ &\quad+\frac{F_A\left(\delta+n;1,1;\delta_1+2,\delta_3+2;\frac{b_1}{b},\frac{b_3}{b}\right)}{(\delta_1+1)(\delta_3+1)} \\ &\quad\left.+\frac{F_A\left(\delta+n;1,1;\delta_1+2,\delta_2+2;\frac{b_1}{b},\frac{b_2}{b}\right)}{(\delta_1+1)(\delta_2+1)}\right]. \end{aligned} \quad (14)$$

Using (14), we can obtain other useful performance measures, such as the central moments, the skewness, the Kurtosis, and the amount of fading. For brevity, we omit the details.

C. Outage Probability

Using (5) with (1), the outage probability of the three-branch SC output can be evaluated readily. The outage probability is defined as the probability that the output SNR γ falls below a preset threshold $\gamma_{\rm th}$

$$\begin{aligned} P_{\rm out} &= \Pr(\gamma\le\gamma_{\rm th}) \\ &= F_{\rm sc}(\gamma_{\rm th}) \\ &= F_{\boldsymbol{\alpha}}\left(\sqrt{\frac{\gamma_{\rm th}\psi_{11}}{\bar{\gamma}_1}},\sqrt{\frac{\gamma_{\rm th}\psi_{22}}{\bar{\gamma}_2}},\sqrt{\frac{\gamma_{\rm th}\psi_{33}}{\bar{\gamma}_3}}\right). \end{aligned} \quad (15)$$

IV. Numerical Results

Here, numerical and simulation results are provided to illustrate the impact of fading correlation on the SC performance.

For brevity, we consider only the performance of the balanced three-branch SC (the average SNRs at all the three branches are identical, i.e., $\bar{\gamma}_1 = \bar{\gamma}_2 = \bar{\gamma}_3 = \bar{\gamma}$). A linear antenna array of three antennas, with equal antenna spacing d, is considered so that the distance between the ith and jth antenna can be obtained as $|i-j|d$. Several statistical models have been proposed to describe the correlation between antennas. Clarke's 2-D isotropic scattering model assumes a uniform angle of arrival (AOA) distribution. The covariance between two antennas follows the zeroth-order Bessel function $J_0(x)$ [28]

$$\psi_{ij} = \bar{\gamma} J_0\left(2\pi \frac{|i-j|d}{\lambda}\right) \tag{16}$$

where λ is the carrier wavelength.

However, measurement data [29] suggests that a Gaussian AOA distribution is more realistic than the uniform AOA distribution for GSM systems in rural and suburban areas. The AOA is assumed to be Gaussian distributed [30] with mean ϕ_0 and variance (angular spread) σ_ϕ. Hence, the covariance matrix of the underlying complex Gaussian components can be computed using [31, eqs. (33) and (34)] for different antenna spacing d. For example, when $\phi_0 = 30°$ and $\sigma_\phi = 10°$, the covariance matrix is given by

$$\mathbf{\Psi} = \bar{\gamma} \begin{pmatrix} 1 & -0.60+0.45i & 0.13-0.30i \\ -0.60-0.45i & 1 & -0.60+0.45i \\ 0.13+0.30i & -0.60-0.45i & 1 \end{pmatrix} \tag{17}$$

for $d = 0.8\lambda$ where $i = \sqrt{-1}$.

Using our new results, we are able to evaluate the three-branch SC performance with the previous linear antenna arrays. The performance of two-branch SC, which combines the first two antennas, is plotted for comparison. Note that simulation results are provided for the SC performance as an independent check of our analytical results. We use the Cholesky decomposition approach [32] to generate correlated complex Gaussian variables and their amplitudes give the required Rayleigh envelopes.

Fig. 1 compares the outage probability of three-branch SC with that of two-branch SC for both the uniform AOA model and the Gaussian AOA model with $\phi_0 = 30°$ and $\sigma_\phi = 10°$. The covariance matrices for the previous two AOA models are determined using (16) and [31, eqs. (33) and (34)], respectively. The insufficient antenna spacing causes a significant performance loss. As antenna spacing d increases, the performance of the three-branch SC improves more rapidly than that of the two-branch SC. This effect is more pronounced at the high SNR region. That is, three-branch SC is more sensitive to the fading correlation than two-branch SC. Compared with the uniform AOA distribution model, the Gaussian AOA model describes a worse case of spacial correlation.

Fig. 2 plots the average error rate performance of SC using BDPSK modulation. The Gaussian AOA correlation model with $\phi_0 = 30°$ and $\sigma_\phi = 10°$ is used. The MRC performance [33] is plotted for comparison. The performance gain of three-branch

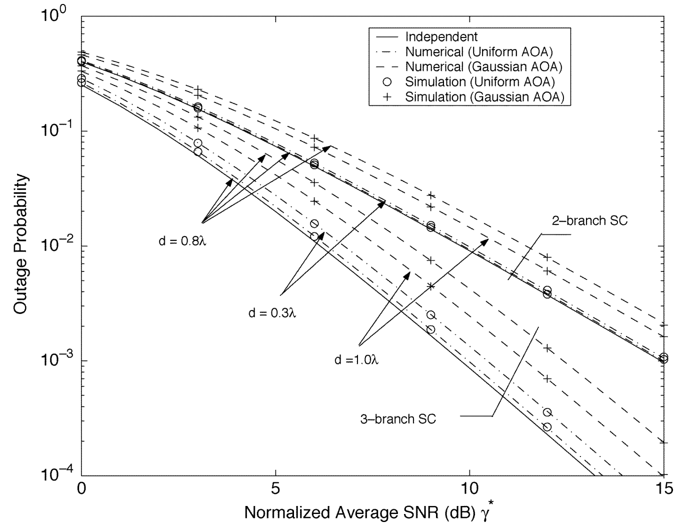

Fig. 1. Outage probability of the balanced SC with two spacial correlation models (the uniform AOA model and the Gaussian AOA model with $\phi_0 = 30°$ $\sigma_\phi = 10°$). The normalized average SNR is $\gamma^* = \bar{\gamma}/\gamma_{\mathrm{th}}$.

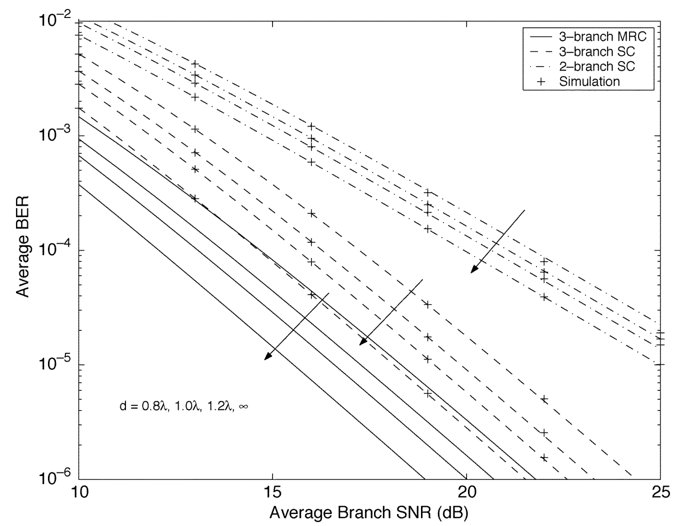

Fig. 2. Average BER of BDPSK with SC and MRC using the Gaussian AOA model with $\phi_0 = 30°$ and $\sigma_\phi = 10°$. The antenna spacing $d = 0.8\lambda, 1.0\lambda, 1.2\lambda$, and ∞.

MRC with respect to three-branch SC is about 2.5 dB, regardless of the antenna spacing. As the antenna spacing decreases, the correlation between branch signals increases and the diversity gain of three-branch SC with respect to two-branch SC diminishes.

Fig. 3 shows the effect of the fading correlation ρ on the normalized average output SNR γ^*_{out} of SC. For simplicity, we consider the balanced SC over equally correlated Rayleigh fading. As the fading correlation ρ increases, the average output SNR of three-branch SC decreases more rapidly than that of two-branch SC. This observation agrees with our expectation that three-branch SC is more sensitive to the fading correlation. However, the error-rate performance depends on all the moments of the output SNR. Caution must be exercised when using the average

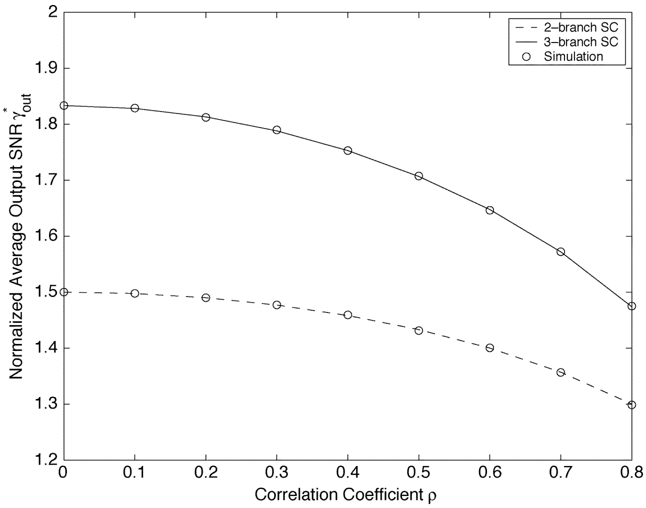

Fig. 3. Normalized average output SNR of the balanced SC over equally correlated Rayleigh fading. $\gamma^*_{\rm out} = \bar{\gamma}_{\rm out}/\bar{\gamma}$.

output SNR as a performance measure.[1] Performance measures that take into consideration the higher moments of the output are required.

V. Conclusion

We derived new expressions for the cdf, the pdf, and the mgf of the three-branch SC output SNR over arbitrarily correlated Rayleigh fading. Several previous results have turned out to be special cases of our new results. We also analyzed the outage probability, the error rate performance, and the output moments of three-branch SC. Numerical results, showing the effect of antenna spacing on the SC performance and comparing the performance of three-branch SC with that of two-branch SC, have been provided. We have thus solved the long standing problem of the three-branch SC performance over arbitrarily correlated Rayleigh fading.

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