

Performance of Space-Frequency Codes in MIMO Channels with Frequency Offset

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Abstract— We investigate the pair-wise error probability (PEP) performance of space-frequency (SF) codes over quasi-static, frequency-selective Rayleigh fading channels with frequency offset (FO). The PEP expression shows that the conventional SF code design criteria are remained valid. If the normalized FO (NFO) is less than 1%, the performance loss is negligible. The loss increases with FO and with signal to noise ratio (SNR). We show that diversity can be used to mitigate the effects of FO. However, with large FO, the PEP is no longer exponentially decays with SNR. Therefore, we propose a SF coding scheme to combat effectively with even very high NFO (20%). Simulation results confirm our theoretical evaluations.

I. INTRODUCTION

The space time (ST) codes proposed by Tarokh *et al.* [1] for coherent systems over quasi-static flat fading channels achieves the maximum diversity order (DO) $d = MN$, where M and N are the number of Tx (Tx) and Rx (Rx) antennas. The ST codes can achieve at least the designed DO in the FSF channels [1], but are not guaranteed to exploit all the available frequency diversity advantage of FSF channels. To overcome this, the authors in [2], [3], [4] and references therein propose new ST coding methods so that the maximum achievable DO is obtainable. It is found that in FSF channels, the maximum DO is $d = LMN$ where L is the number of paths of channels. There are two approaches to design codes that can achieve this maximum DO [5]. In the first method, ST codes are constructed in the time domain, whereas in the second method called space-frequency (SF) coding, the source symbols are encoded across the subcarriers (or sub-carriers) of orthogonal frequency division multiplexing (OFDM) symbols in the frequency domain. Su *et al.* [6] derive SF code criteria which show an explicit relation between the SF code matrix and the characteristic parameters of FSF fading channels such as path delays and delay power distribution.

Like other error control coding methods, the performance of SF codes depends on the propagation environment [7]. Moreover, the performance of SF codes and OFDM may be affected by underlying factors such as frequency offset (FO), timing error, to name a few. A residual FO exists due to carrier synchronization mismatch and Doppler shift [8]. Residual FO breaks down the orthogonality among sub-carriers, hence inter-carrier interference (ICI) is produced and bit error rate (BER) is increased consequently. Several papers have analyzed the effects of FO on the BER performance. For

example in [9], BER is calculated for single input single output (SISO) OFDM systems with various modulation schemes. The authors in [10], [11] provide BER expressions of MIMO-OFDM employing Alamouti's scheme [12]. However, to the best of our knowledge, the impact of FO on the SF code design criteria and the performance have not been investigated. This interesting question will be addressed in our paper.

We analytically show that the SF code design criteria still hold in the case of FO. The performance loss is negligible if the NFO is small. This loss is increased rapidly with the increase of NFO and with signal-to-noise ratio (SNR). When NFO is large, PEP performance of SF codes no longer decays exponentially. To mitigate the effect of FO, we generalize the work of [13] for MIMO-OFDM with SF codes. The resulting SF codes not only achieve the same diversity and coding gains as the SF codes derived in [6] but also are capable of ICI self-cancellation. Simulations have showed that the ICI self-cancellation SF codes still yields good performance when the FO is extremely high, about 20%.

The paper has seven sections. The model of MIMO-OFDM systems with FO is presented in Section II. Section III reviews the design criteria of SFC codes. In Section IV, we derive the PEP performance of SF codes in the presence of FO. In Section V, we propose a class of SF codes that reduce effectively FO effects to PEP. Section VI provides simulation results to support the theoretical analysis. We summarize the paper in Section VII.

II. FREQUENCY OFFSET IN OFDM SYSTEMS

This section briefly introduces the OFDM system with FO model to be analyzed for remaining parts of the paper.

A. MIMO-OFDM System Model

We consider the MIMO-OFDM system with M Tx and N Rx antennas. The number of subcarriers in the OFDM modulator is K . The L -path quasi-static Rayleigh fading channel model is assumed identically for any link between Tx antenna m ($m = 1, \dots, M$) and Rx antenna n ($n = 1, \dots, N$). The channel impulse response in the time domain is [14]

$$h_{m,n}(t, \tau) = \sum_{l=0}^{L-1} \alpha_{m,n}(t, l) \delta(\tau - \tau_l) \quad (1)$$

where τ_l is the channel delay of the path l -th ($l = 0, \dots, L - 1$), $\delta(\cdot)$ denotes Dirac's delta function. The coefficients $\alpha_{m,n}(t, l)$'s are complex channel gains of the path l between Tx antenna m and Rx antenna n , they are modeled as zero-mean complex Gaussian random variables with variance $E[|\alpha_{m,n}(l)|^2] = \delta_l^2$. Assume that coefficients $\alpha_{m,n}(t, l)$ are constant during an OFDM symbol duration, but vary randomly from symbol to symbol. Thus during a symbol period, the time index t in (1) can be omitted. The total power of L -path channels is normalized, so that $\sum_{l=0}^{L-1} \delta_l^2 = 1$. The frequency response of the channel is

$$H_{m,n}(f) = \sum_{l=0}^{L-1} \alpha_{m,n}(l) e^{-j2\pi f \tau_l}, \quad j = \sqrt{-1}. \quad (2)$$

At the receiver, assume that Rx subcarriers are perfectly sampled, let $c_m(k)$ is the subcarrier k -th ($k = 0, \dots, K - 1$) being sent from Tx antenna m , the received signal at the Rx antenna n is $r_n(k)$

$$r_n(k) = \sum_{m=1}^M c_m(k) H_{m,n}(k) + z_n(k) \quad (3)$$

where $z_n(k)$'s are independent noise samples, which are modelled as zero-mean complex white Gaussian variables. The Tx power from each antenna is normalized to 1, resulting the noise variance per dimension is $M/(2\rho)$ where ρ is the average SNR at each Rx antenna.

B. Model of MIMO-OFDM Systems with FO

There is always a FO at the sampling points of received signal [15], [16]. In the SISO-OFDM system, the NFO ε is defined by $\varepsilon = \delta f / \Delta f$, where δf is the residual FO between the demodulated and transmitted subcarrier k -th, $\Delta f = 1/T_s$ is the subcarrier spacing, T_s is the OFDM symbol duration. The NFO is assumed the same for all subcarriers of one OFDM symbol, but can vary from symbol to symbol. In SISO systems, the k -th received subcarrier is expressed as follows:

$$r(k) = c(k)H(k)S(0) + I(k) + z(k) \quad (4)$$

where $I(k)$ is the inter-carrier interference (ICI) from the other subcarriers to received subcarrier k ; it is described by

$$I(k) = \sum_{p=0}^{K-1} \sum_{p \neq k} c(p)H(p)S(p-k). \quad (5)$$

Coefficients $S(k)$ in (5) are expressed as:

$$S(k) = \frac{\sin[\pi(k+\varepsilon)]}{K \sin[\frac{\pi}{K}(k+\varepsilon)]} \exp\left[j\pi\left(1 - \frac{1}{K}\right)(k+\varepsilon)\right]. \quad (6)$$

We generalize (4) for MIMO-OFDM systems. Let NFO of the transmission link from Tx antenna m and Rx antenna n is $\varepsilon_{m,n}$, (4) becomes:

$$r_n(k) = \sum_{m=1}^M c_m(k)H_{m,n}(k)S_{m,n}(0) + \sum_{m=1}^M I_{m,n}(k) + z_n(k). \quad (7)$$

III. SPACE-FREQUENCY CODE DESIGN CRITERIA

In the SF encoding process, the source data is two-dimensional encoded across the space (over multiple antennas) and frequency (over the subcarriers of OFDM symbols). An SF codeword may occupy several OFDM symbols [4], [17] or within one OFDM symbol [3], [5], [6]. It is found that the maximal DO can be achieved by coding over the subcarriers of only one OFDM symbol [3], [6], whereas in [4], [17] the maximal diversity order is gained by coding over multiple OFDM symbols, that obviously causes higher coding and decoding delay. We adopt the approach in [6] for our analysis.

The input data symbols are divided into b -symbol source words then parsed into blocks and mapped to SF codewords. An SF codeword can be represented by a $K \times M$ matrix

$$C = \begin{bmatrix} c_1(0) & c_2(0) & \dots & c_M(0) \\ c_1(1) & c_2(1) & \dots & c_M(1) \\ \vdots & \vdots & \ddots & \vdots \\ c_1(K-1) & c_2(K-1) & \dots & c_M(K-1) \end{bmatrix}. \quad (8)$$

At the receiver, the maximum likelihood (ML) decoder selects a codeword E if its metric Me is minimum

$$Me = \sum_{k=0}^{K-1} \sum_{n=1}^N \left| r_n(k) - \sum_{m=1}^M e_m(k) H_{m,n}(k) \right|^2. \quad (9)$$

Assume perfect channel state information (CSI) is available at the receiver but not at the Txter and perfect symbol timing, the PEP of a transmitted codeword C and erroneously decoded codeword E is upper bounded [6] as:

$$P(C \rightarrow E) \leq \binom{2\Gamma N - 1}{\Gamma N} \left(\prod_{m=1}^{\Gamma} \lambda_i \right)^{-N} \rho^{-\Gamma N}. \quad (10)$$

In (10), Γ is the rank of the matrix $Q = \Delta \circ R$, \circ denotes Hadamard product [18], $\lambda_i, i = 1, \dots, \Gamma$ are non-zero eigenvalues of Q ,

$$\Delta = (C - E)(C - E)^\dagger, \quad (11)$$

$$R = R_{m,n} = E [H_{m,n}H_{m,n}^\dagger] = W \text{diag}(\delta_0^2, \delta_1^2, \dots, \delta_{L-1}^2) W^\dagger, \quad (12)$$

where $R_{m,n} = E [H_{m,n}H_{m,n}^\dagger]$,

$$H_{m,n} = [H_{m,n}(0) H_{m,n}(1) \dots H_{m,n}(K-1)]^T, \quad (13)$$

$$W = \begin{bmatrix} 1 & 1 & \dots & 1 \\ w^{\tau_0} & w^{\tau_1} & \dots & w^{\tau_{L-1}} \\ \vdots & \vdots & \ddots & \vdots \\ w^{(K-1)\tau_0} & w^{(K-1)\tau_1} & \dots & w^{(K-1)\tau_{L-1}} \end{bmatrix}. \quad (14)$$

In (14), $w = e^{-j2\pi\Delta f}$, $\Delta f = 1/T_s$ is the subcarrier spacing, T_s is the OFDM symbol duration. X^T and X^\dagger denote transpose and transpose conjugate of matrix X .

From (10), the DO of SF codes is ΓN , maximum achievable DO is equal $\min(LMN, KN)$ [6].

IV. PERFORMANCE OF SF CODES WITH FO

We continue the analysis with two assumptions below:

- AS1: Residual NFO's $\varepsilon_{m,n}$ are identical independent distributed (i.i.d) random variables. Their values are independent of the channel coefficients.
- AS2: The ICI terms $I_{m,n}(k)$ in (8) are independent.

In practice, AS2 is not completely true since there is a degree of correlation between transmitted streams. In OFDM systems, K is typically 64 or larger. Therefore the central limit theorem can be applied to model the term $I_{m,n}(k)$ as a Gaussian random variable [16].

To investigate the PEP of SF codes with FO using formula (10), the channel coefficients $H_{m,n}^S(k)$ in (12) should be complex GRVs. This requirement can be met if $S_{m,n}(0)$ is deterministic or NFO is not a random variable (Case 1). In general case, $\varepsilon_{m,n}$ can be assumed to be independent and identically distributed random variables in the range $[E_1, E_2]$, their values can be changed from OFDM symbol to symbol (Case 2). However, the performance of SF codes with fixed values of FO is of greater interest since it provides a closer look at the performance of SF codes at specific FO values. For analytical tractability, we further make a third assumption:

- AS3: NFO $\varepsilon_{m,n}$ are constant and the same for all pair of indices (m, n) : $\varepsilon_{m,n} = \varepsilon_0$.

Eq. (7) is rewritten with a slight modification. The channel coefficient and the $S_{m,n}(0)$ are grouped as

$$r_n(k) = \sum_{m=1}^M c_m(k) [H_{m,n}(k)S_{m,n}(0)] + \sum_{m=1}^M I_{m,n}(k) + z_n(k). \quad (15)$$

Eq. (13) becomes

$$H_{m,n}^S = S_{m,n}(0) [H_{m,n}(0) H_{m,n}(1) \dots H_{m,n}(K-1)]^T. \quad (16)$$

The correlation matrix in (12) has a new form

$$R_{m,n}^S = \bar{S}_0^2 E [H_{m,n} H_{m,n}^\dagger], \quad (17)$$

$$R = R_{m,n}^S = \bar{S}_0^2 R_{m,n}. \quad (18)$$

$$Q^S = \Delta \circ R_{m,n}^S = \sigma_S^2 (\Delta \circ R_{m,n}) \quad (19)$$

where $\bar{S}_0^2 = E [|S_{m,n}(0)|^2]$. Note that the residual NFO is usually small, $\varepsilon \leq 0.2$ [16], the number of subcarriers $K \geq 8$, hence $K \sin(\pi\varepsilon/K) \approx \pi\varepsilon$. \bar{S}_0^2 can be evaluated as

$$\bar{S}_0^2 = E [|S_{m,n}(0)|^2] = [\text{sinc}(\varepsilon_0)]^2. \quad (20)$$

We can easily verify that: if the rank of matrix Q is Γ , then Q^S also has the same rank Γ ; if λ_i is an eigenvalue of Q then $\lambda_i^S = \bar{S}_0^2 \lambda_i$ is an eigenvalue of Q^S .

The first two moments of the term $I_{m,n}(k)$ in (14) by Gaussian approximation are calculated as follows.

$$I_{m,n}(k) = \sum_{p=0}^{K-1} \sum_{p \neq n} c_m(p) H_{m,n}(p) S_{m,n}(p-k). \quad (21)$$

Assume that coded symbols $c_m(p)$ have zero-mean, then $E[I_{m,n}(k)] = 0$. The variance $\sigma_{I_m}^2$ of $I_{m,n}(k)$ is evaluated in Appendix to be $\sigma_{I_m}^2 = 1 - \bar{S}_0^2$. $\sigma_{I_m}^2$ is independent of indices m and n , it is just dependent on the NFO through \bar{S}_0^2 . In (15), let $Z_n(k) = I_n(k) + z_n(k)$ and $I_n(k) = \sum_{m=1}^M I_{m,n}(k)$. With AS2, $I_n(k)$ is a complex Gaussian random variable with zero-mean and variance $M(1 - \bar{S}_0^2)$. Therefore $Z_n(k)$ is also a zero-mean complex Gaussian random variable with variance $\sigma_Z^2 = M(1 - \bar{S}_0^2 + 1/\rho)$. Values of σ_Z^2 is identical for all Rx antennas. Hence equivalent SNR at each Rx antenna with FO is

$$\rho^S = \frac{\bar{S}_0^2}{1 - \bar{S}_0^2 + 1/\rho}. \quad (22)$$

We substitute $\lambda_i^S = \bar{S}_0^2 \lambda_i$ and ρ^S into (10), re-arrange the terms and get the PEP expression in the presence of FO as

$$P(C \rightarrow E) \leq G \left(\frac{2\Gamma N - 1}{\Gamma N} \right) \left(\prod_{i=1}^{\Gamma} \lambda_i \right)^{-N} \rho^{-\Gamma N} \quad (23)$$

where

$$G = \left(\frac{\bar{S}_0^4}{\rho(1 - \bar{S}_0^2) + 1} \right)^{-\Gamma N}. \quad (24)$$

Comparing (10) and (23), we see that G represents the PEP performance loss due to FO. By inspecting (20) and (24), we can draw following theoretical conclusions:

- 1) The design criteria for SF codes in the presence of FO is still valid for small values of FO.
- 2) At the same Tx power, the higher FO, the higher PEP performance loss.
- 3) Higher NFO, the further PEP curve shifts to the right. The shift of lower DO systems is larger than the shift of the system with higher DO.
- 4) If $\rho(1 - \bar{S}_0^2) \gg 1$, at high Tx power

$$P(C \rightarrow E) \leq \left(\frac{2\Gamma N - 1}{\Gamma N} \right) \left(\prod_{m=1}^{\Gamma} \lambda_i \right)^{-N} \times \left(\frac{\bar{S}_0^2}{1 - \bar{S}_0^2} \right)^{-\Gamma N}. \quad (25)$$

So the PEP is no longer inversely proportional to SNR. The PEP reaches its floor level.

To complete this section, we note that one can derive the ML receiver using (7) and (9), the same result as (23) can be obtained.

V. ICI SELF-CANCELLATION WITH SF CODES

Su *et al.* [6] show that if each row of the ST codeword is repeated r times, the resulting SF code achieves at least DO $d = 2MN$. We consider $r = 2$, a SF codeword is

$$C_1 = \begin{bmatrix} c_1(0) & c_2(0) & \dots & c_M(0) \\ c_1(0) & c_2(0) & \dots & c_M(0) \\ \vdots & \vdots & \dots & \vdots \\ c_1(K-1) & c_2(K-1) & \dots & c_M(K-1) \\ c_1(K-1) & c_2(K-1) & \dots & c_M(K-1) \end{bmatrix}. \quad (26)$$

We develop the simple but effective idea of ICI self-cancellation of authors in [13] for MIMO-OFDM so that each row of ST code matrix is repeated once and the repeated row is sign reversed to form a new SF codeword.

$$C_2 = \begin{bmatrix} c_1(0) & c_2(0) & \dots & c_M(0) \\ -c_1(0) & -c_2(0) & \dots & -c_M(0) \\ \vdots & \vdots & \dots & \vdots \\ c_1(K-1) & c_2(K-1) & \dots & c_M(K-1) \\ -c_1(K-1) & -c_2(K-1) & \dots & -c_M(K-1) \end{bmatrix}. \quad (27)$$

The number of OFDM subcarriers is $2K$ in (26) and (27). The codeword formulation is different with the approach of [6], where each row is simply repeated r times ($r = 2$ here). We can prove that the new mapping method yields the same coding gain and diversity gain (at least $d = 2MN$) compared with the approach in [6] for $r = 2$, but our modified scheme integrates ICI self-cancellation capability. To prove that SF codes described in (26) and have the same coding gain and diversity gain, the reader can verify that if vector $X_1 = [x_1 \ x_2 \ x_3 \ x_4 \ \dots \ x_{2K-1} \ x_{2K}]$ is an eigenvector of $Q_1 = \Delta_1 \circ R$, then $X_2 = [x_1 \ -x_2 \ x_3 \ -x_4 \ \dots \ x_{2K-1} \ -x_{2K}]$ is an eigenvector of $Q_2 = \Delta_2 \circ R$, where Q_1, Δ_1 and Q_2, Δ_2 are quantities associated with SF codewords defined in (10) and (11) respectively. Then if λ is an eigenvalue associated with X_1 , it is also an eigenvalue associated with X_2 . Note that (10) provides an upper bound for PEP, the actual performance of different codes can be evaluated via simulations.

VI. SIMULATION RESULTS AND DISCUSSION

The simulations are to verify the theoretical analysis in Section IV and V. We use a simple two-path channel with uniform power delay profile (delay time between the two paths is $5\mu s$ and COST207 six-path typical urban channel model [14]. In [6], each row of the space time (ST) code is repeated r times ($1 \leq r \leq L$) to form SF codewords. This approach reduces the spectrum efficiency. However the resulting SF codes are guaranteed to achieve diversity order of at least $d = rMN$. Note that coding gain and spectral efficiency are out of our interests. Thus we use algebraic ST convolutional codes [5], [19], codes rate $1/2$ with generator polynomial $(5, 7)$ [20] without channel interleaver. The Viterbi decoder [19], [20] is used.

We compare the performance of SF codes for 1%, 10% and 20% NFO. Figs. 1 and 2 illustrate PEP curves of two OFDM systems with 64 subcarriers and two Tx antennas. System

1 is equipped with one Rx antenna and System 2 has two Rx antennas, so that the DO of the two systems is at least $d = 2$ and $d = 4$, respectively. Using (20), for 1% NFO, $S_0 = 0.9997, L_0 \approx 1$ in the SNR region of interest (≤ 30 dB). Therefore, theoretically the performance loss is not significant. Figs. 1 and 2 confirm this conclusion. If the NFO is small, 1%, the PEP curves almost overlap the PEP of the systems with no FO. In case NFO is 10%, the PEP curves of all systems are shifted to the right, this shift is larger for the system with smaller DO. The SNR needed to compensate the effect of FO increases with NFO. However, when NFO is 10% the PEP curves are less steeper than the curves of PEP with 1% NFO. When NFO is 20%, the PEP reduces slightly even SNR increases largely. The PEP performance reaches a floor at SNR is about 22 dB. This symptom is more serious for low DO systems, where the floor level is higher than that of higher DO systems.

Fig. 3 presents the performance of the SF coding scheme proposed in [6] (SC1) and our modified scheme (SC2) with the same DO, $r = 2, K = 64, M = 2, N = 1$. SC2 with integrated ICI self-cancellation mechanism performs very well when $E_0 = 10\%$. Performance of SC2 is always better than SC1. At $PEP = 10^{-3}$, when $E_0 = 0$ and 10%, SC2 provides 0.5 dB and 1.5 dB improvement respectively. More important, SC2 lowers the error floor level extensively even at high FO $E_0 = 20$.

VII. CONCLUSION

We analyzed PEP performance of SF codes with FO. Through the PEP upper bound expression of SF codes with FO, we showed that the code design criteria remained valid. The effect of FO is more severe for low DO SF codes. To minimize the effects of FO, we proposed an enhanced ICI self-cancellation scheme using SF codes. Finally we conclude that diversity not only improves the performance of OFDM systems in the dispersive channels, but also makes the system robust to the effects of FO.

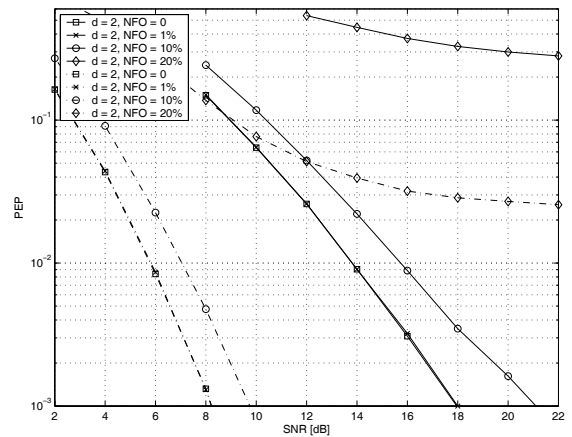


Fig. 1. Performance SF codes with $K = 64$, constant FO for the two-path channel model.

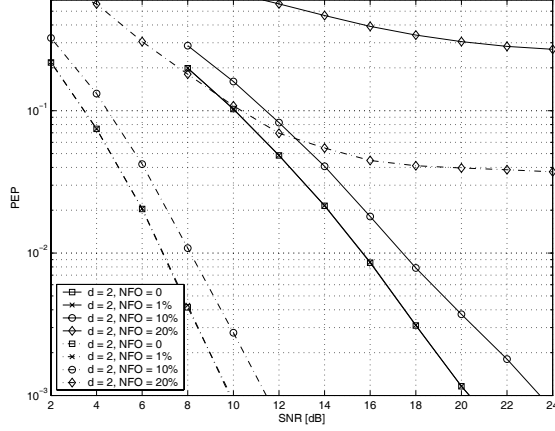


Fig. 2. Performance of SF codes with $K = 64$, constant FO for the six-path COST207 typical urban channel model.

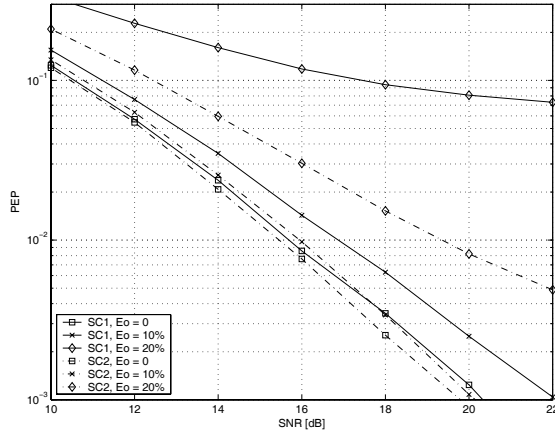


Fig. 3. Performance SF codes with $K = 128$, $r = 2$, two-path channel model with and without ICI self-cancellation

APPENDIX

We calculate $\sigma_{I_m}^2$ as follows:

$$\begin{aligned}
 \sigma_{I_m}^2 &= E \left[|I_m(k)|^2 \right] \\
 &= E \left[\left| \left(\sum_{p=0}^{K-1} c_m(p) H_{m,n}(p) S_{m,n}(p-k) \right) \right|^2 \right] \\
 &\quad - E \left[|c_m(k) H_{m,n}(k) S_{m,n}(0)|^2 \right] \\
 &= \sum_{p=0}^{K-1} E \left[|c_m(p)|^2 \right] E \left[|H_{m,n}(p)|^2 \right] E \left[|S_{m,n}(p-n)|^2 \right] \\
 &\quad - E \left[|c_m(k)|^2 \right] E \left[|H_{m,n}(k)|^2 \right] E \left[|S_{m,n}(0)|^2 \right]
 \end{aligned} \tag{28}$$

In (28), the term $E \left[|c_m(k)|^2 \right]$ is the signal power, it is normalized to 1. The second term is the average of the channel power, and it is also normalized to 1, (27) becomes

$$\sigma_{I_m}^2 = \sum_{p=0}^{K-1} E \left[|S(p-k)|^2 \right] - \overline{S_0}^2 \tag{29}$$

It is found in [8] that the sum $\sum_{p=0}^{K-1} E \left[|S(p-k)|^2 \right] = 1$. Substitute the sum into (28) completes the proof.

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