Joint Channel Estimation and Data Detection for OFDM Systems via Sphere Decoding

Tao Cui and Chintha Tellambura
Department of Electrical and Computer Engineering
University of Alberta
Edmonton, AB, Canada T6G 2V4
Email: {taocui, chintha}@ece.ualberta.ca

***Abstract*— We develop blind and semi-blind joint estimators of the channel impulse response (CIR) and data symbols for Orthogonal Frequency Division Multiplexing (OFDM) systems over a frequency selective fading channel. Using the maximum likelihood (ML) criterion, we derive two estimators for the transmit data symbols that require minimizing a complex, integer quadratic $\mathbf{x}^T\mathbf{G}\mathbf{x}^*$ where $\mathbf{x}$ is a data vector and the matrix $\mathbf{G}$ characterizes each estimator. Avoiding computationally prohibitive exhaustive search, we use both sphere decoding (SD) and V-BLAST algorithms. We also modify SD algorithm to our complex OFDM system to handle any M-PSK and incorporate a reduced complexity SD into OFDM system. The quadratic for the blind estimator suffers from rank deficiency. We give an efficient solution to the rank deficiency problem. Simulation results confirm good performance of our proposed estimators.**

I. Introduction

Orthogonal Frequency Division Multiplexing (OFDM) is used for high data rate wireless local area network (WLAN) standards, such as Hiperlan and IEEE 802.11a, providing data rates of up to 54Mbit/s, and considered for the fourth-generation (4G) mobile wireless systems and beyond [1]. Pilot-symbol-aided channel estimation for OFDM has been widely investigated [2], [3]. In the wireless standards such as the IEEE 802.16a and IEEE 802.11a, the cyclic prefix (CP) and pilot tones together constitute a significant overhead or bandwidth loss, which has motivated the development of blind techniques for OFDM. They exploit statistical or deterministic properties of the transmit and receive signals.

Blind channel estimators that exploit the redundancy introduced by the CP and pilot subcarriers are developed in [4]. Reference [5] develops a blind subspace approach exploiting the CP-induced cyclostationarity. A blind channel estimator by exploiting the finite alphabet property of modulation symbols is proposed in [6]. Subspace-based blind channel estimator using virtual carriers (VC) has been derived in [7]. In [8], a sufficient condition for blind channel estimation is given. Joint estimation of CIR and data symbols for OFDM has not been investigated extensively. An ML joint blind channel and data estimator [9] exploits the finite alphabet property of modulation symbols and the presence of VCs. In [10] an ML joint estimator is derived which requires pilot symbols for an initial estimate of the channel.

In this paper, we derive two joint estimators for CIR and data symbols making use of the finite alphabet and constant modulus properties of transmit signals. The first one is a blind estimator. We derive the other *(referred to as semi-blind)* assuming that the receiver knows the channel autocorrelation matrix and the noise variance. Both the estimators are obtained by posing the problem of the joint estimation of channel and data as a mixed discrete and continuous LS optimization problem. By eliminating the channel from it, we obtain a discrete integer LS problem (which has the same form for both the estimators) for the data symbols. Since exhaustive search is computationally prohibitive, we use both SD [11] and V-BLAST [12] to solve these quadratic optimization problems.

The rest of the paper is organized as follows. Section II reviews the basic baseband OFDM system model. Section III introduces two joint ML estimators. In Section IV, we apply both V-BLAST and SD to solve the joint estimators. Furthermore, a novel approach to solve the rank deficiency problem in SD is also presented. Section V gives computer simulation results and Section VI concludes the paper.

Notation: Bold symbols denote matrices or vectors. $(\cdot)^T$, $(\cdot)^H$ and $(\cdot)^\dagger$ denote transpose, conjugate transpose and Moore-Penrose pseudo-inverse respectively. For M-ary phase shift keying (MPSK), the signal constellation $\mathcal{Q} = \{e^{j2\pi k/M}, k = 0, 1, \cdots, M-1\}$ and all MPSK $N \times 1$ vectors are denoted by $\mathcal{Q}^N$ and the cardinality of $\mathcal{Q}$ is denoted by $|\mathcal{Q}|$. If x and y are Gaussian random variables (RV) with mean μ_x, μ_y and variance σ^2, $z = x + jy$ (where $j = \sqrt{-1}$) is a complex Gaussian random variable (CGRV) and is denoted by $z \sim \mathcal{CN}(\mu_x + j\mu_y, \sigma^2)$. The discrete Fourier transform (DFT) matrix is given by $\mathbf{F} = 1/\sqrt{N}[e^{-j\frac{2\pi}{N}kl}]$, $k, l \in 0, 1, \cdots, N-1$.

II. OFDM Baseband Model

In an OFDM system, the source data are grouped and/or mapped into the multi-phase symbols from a MPSK constellation $\mathcal{Q}$, which are modulated by inverse DFT (IDFT) on N parallel subcarriers. Note that $X(k)$, $k = 0, 1, \ldots, N-1$ are called modulation symbols or transmit data symbols. The term "OFDM block" is used to denote the entire IDFT output $\{x(0), x(1), \cdots, x(N-1)\}$. The input symbol duration is

T_s and the OFDM block duration is NT_s. These samples are appropriately pulse shaped to construct the time domain signal $x(t)$ for transmission. We assume that the composite CIR which includes transmit and receive pulse shaping and the physical channel response between the transmitter and receiver may be modelled as [13, p.802]

$$h(t) = \sum_{l=0}^{L-1} h_l \delta(t - \tau_l) \tag{1}$$

where $h_n \sim \mathcal{CN}(0, E[h_n^2])$ and τ_l is the delay of the lth tap. Typically, it is assumed that $\tau_l = lT_s$ and this results in a finite impulse response filter with an effective length L. Assuming that the channel remains constant during each OFDM block, but it varies between OFDM blocks, and the cyclic prefix is sufficiently long ($N_g > L$), the post-DFT received samples $Y(k)$ are given as follows:

$$Y(k) = H(k)X(k) + W(k), \quad 0 \le k \le N-1 \tag{2}$$

where $H(k) = H(j2\pi k/N)$ is the complex channel frequency response at subcarrier k, $H(j\omega)$ is the Fourier transform of the CIR and $W(k)$ $k = 0, 1, \cdots, N-1$ are independent and identically distributed (i.i.d) CGRV's, each of which has zero mean and variance σ_n^2. Assuming $\tau_l = lT_s$, we find $\mathbf{H} = \mathbf{F}_L\mathbf{h}$, where $\mathbf{H} = [H(0), H(1), \cdots, H(N-1)]^T$, $\mathbf{h} \in \mathcal{C}^L$ is the CIR and $\mathbf{F}_L$ is a $N \times L$ submatrix of DFT matrix $\mathbf{F}$, which corresponds to each channel path. We can vectorize (2) as

$$\mathbf{Y} = \mathbf{X}_D\mathbf{F}_L\mathbf{h} + \mathbf{W} \tag{3}$$

where $\mathbf{X}_D = \text{diag}\{X(0), X(1), \cdots, X(N-1)\}$ is a diagonal matrix. Note that (3) is the basis of our joint channel and data estimators.

III. ML JOINT CHANNEL ESTIMATION AND DATA DETECTION

Using ML principles, we next derive blind and semi-blind joint estimators of CIR and data symbols. The semi-blind estimator is derived by assuming the availability of the exact knowledge of channel correlation matrix and noise variance.

A. Blind estimator

Since the noise vector $\mathbf{W}$ in (3) is i.i.d Gaussian, the ML estimator of the CSI ($\mathbf{h}$) and transmitted symbols ($\mathbf{X}_D$) is given by

$$(\hat{\mathbf{h}}, \hat{\mathbf{X}}_D) = \underset{\tilde{\mathbf{h}} \in \mathcal{C}^L, \tilde{\mathbf{X}}_D \in \mathcal{Q}^N}{\arg\min} \|\mathbf{Y} - \tilde{\mathbf{X}}_D\mathbf{F}_L\tilde{\mathbf{h}}\|^2. \tag{4}$$

The minimization in (4) is a complex LS problem for $\hat{\mathbf{h}}$ and an integer LS problem for $\hat{\mathbf{X}}_D$. Given $\hat{\mathbf{X}}_D$ (we assume that $\hat{\mathbf{X}}_D = \mathbf{X}_D$) the channel response $\hat{\mathbf{h}}$ that minimizes (4) is given by the LS estimate

$$\hat{\mathbf{h}} = \left[(\hat{\mathbf{X}}_D\mathbf{F}_L)^H(\hat{\mathbf{X}}_D\mathbf{F}_L)\right]^{-1}(\hat{\mathbf{X}}_D\mathbf{F}_L)^H\mathbf{Y}. \tag{5}$$

Substituting (5) into (4), we obtain

$$\hat{\mathbf{X}}_D = \underset{\mathbf{X}_D}{\arg\min} \left\| \left[\mathbf{I}_N - \mathbf{X}_D\mathbf{F}_L\left(\mathbf{F}_L^H\mathbf{F}_L\right)^{-1}\mathbf{F}_L^H\mathbf{X}_D^H\right]\mathbf{Y}\right\|^2 \tag{6a}$$

$$= \underset{\mathbf{X}_D}{\arg\min}\, \mathbf{Y}^H\mathbf{X}_D\left[\mathbf{I}_N - \mathbf{F}_L\left(\mathbf{F}_L^H\mathbf{F}_L\right)^{-1}\mathbf{F}_L^H\right]\mathbf{X}_D^H\mathbf{Y} \tag{6b}$$

$$= \underset{\mathbf{x}}{\arg\min}\, \mathbf{x}^T\mathbf{Y}_D^H\left[\mathbf{I}_N - \mathbf{F}_L\left(\mathbf{F}_L^H\mathbf{F}_L\right)^{-1}\mathbf{F}_L^H\right]\mathbf{Y}_D\mathbf{x}^* \tag{6c}$$

where $\mathbf{Y}_D = \text{diag}\{Y(0), Y(1), \cdots, Y(N-1)\}$ and $\mathbf{x} \in \mathcal{Q}^N$ is the vector whose elements are the diagonal elements of matrix $\mathbf{X}_D$. Eq. (6a) is due to the use of the constant modulus constellation MPSK; Eq. (6a) follows from the fact that the matrix $\mathbf{I}_N - \mathbf{X}_D\mathbf{F}_L\left(\mathbf{F}_L^H\mathbf{F}_L\right)^{-1}\mathbf{F}_L^H\mathbf{X}_D^H$ is an orthogonal projection matrix onto $null(\mathbf{X}_D\mathbf{F}_L)$ and the projection matrix has the property $P_\perp^2 = P_\perp$ and $P_\perp^H = P_\perp$.

The rank of the matrix $\mathbf{B} = \mathbf{Y}_D^H(\mathbf{I}_N - \mathbf{F}_L\mathbf{F}_L^H)\mathbf{Y}_D$ is only $N-L$. Note that $\mathbf{B}$ can be QR factorized as $\mathbf{B} = \mathbf{QR}$, where $\mathbf{Q}$ and $\mathbf{R}$ are unitary and upper-triangular, respectively. Since the last L rows of $\mathbf{R}$ are zero, both the standard V-BLAST and SD algorithms fail in this case. We next modify (6c) so that both SD and V-BLAST can be applied.

Using the constant modulus property (e.g., $|X(k)|^2 = 1$ for $X(k) \in \mathcal{Q}$), $\mathbf{x}^T\mathbf{Y}_D^H\mathbf{Y}_D\mathbf{x}^* = \sum_{k=0}^{N-1}|Y(k)|^2$ is a constant. Therefore the optimization problem (6c) is equivalent to

$$\begin{aligned}\hat{\mathbf{X}}_D &= \underset{\mathbf{x}\in\mathcal{Q}^N}{\arg\min}\, \eta\mathbf{x}^T\mathbf{Y}_D^H\mathbf{Y}_D\mathbf{x}^* + \mathbf{x}^T\mathbf{B}\mathbf{x}^* \\ &= \underset{\mathbf{x}\in\mathcal{Q}^N}{\arg\min}\, \mathbf{x}^T\mathbf{Y}_D^H[(\eta+1)\mathbf{I}_N - \mathbf{F}_L\mathbf{F}_L^H]\mathbf{Y}_D\mathbf{x}^*.\end{aligned} \tag{7}$$

Since $\mathbf{F}_L\mathbf{F}_L^H$ is positive semi-definite matrix with non-zero eigenvalue 1, $(\eta+1)\mathbf{I}_N - \mathbf{F}_L\mathbf{F}_L^H$ is a positive definite matrix if $\eta > 0$. For simplicity, we let $\eta = 1$.

After $\hat{\mathbf{X}}_D$ is estimated from (7) using V-BLAST (suboptimal) or the SD (optimal), the LS estimate $\hat{\mathbf{h}}$ can be obtained by substituting $\hat{\mathbf{X}}_D$ into (5). Both $\mathbf{x}$ and $\mathbf{x}e^{j\phi}$ $\phi \in (0, 2\pi)$ satisfy (7), which shows that the blind algorithm exhibits a phase ambiguity. This can be solved by using a pilot tone.

B. Semi-blind estimator

This requires the knowledge of the autocorrelation matrix $\mathbf{R}_h$ of the CIR $\mathbf{h}$ and the noise variance σ_n^2. We classify it as a semi-blind estimator. From (3), $\mathbf{h}$ and $\mathbf{W}$ are zero-mean complex Gaussian random vectors. The received samples $Y(k)$ can be modelled as i.i.d. zero-mean CGRV's via the central limit theorem when N is large. The autocorrelation matrix of the received signal is given by

$$\begin{aligned}\mathbf{R}_Y = E\{\mathbf{Y}\mathbf{Y}^H\} &= \mathbf{X}_D\mathbf{F}_L\mathbf{R}_h\mathbf{F}_L^H\mathbf{X}_D^H + \sigma_n^2\mathbf{I}_N \\ &= \mathbf{X}_D(\mathbf{F}_L\mathbf{R}_h\mathbf{F}_L^H + \sigma_n^2\mathbf{I}_N)\mathbf{X}_D^H.\end{aligned} \tag{8}$$

The determinant of $\mathbf{R}_Y$ can be expressed as

$$\begin{aligned}\det(\mathbf{R}_Y) &= \det(\mathbf{X}_D)\det(\mathbf{F}_L\mathbf{R}_h\mathbf{F}_L^H + \sigma_n^2\mathbf{I}_N)\det(\mathbf{X}_D^H) \\ &= \det(\mathbf{F}_L\mathbf{R}_h\mathbf{F}_L^H + \sigma_n^2\mathbf{I}_N).\end{aligned} \tag{9}$$

Note that the determinant of $\mathbf{R}_Y$ is independent of $\mathbf{X}_D$. Ignoring terms that are independent of $\mathbf{X}_D$, the log-likelihood function is given by

$$\Lambda(\mathbf{Y}|\mathbf{X}_D) = -\mathbf{Y}^H\mathbf{R}_Y^{-1}\mathbf{Y}. \quad (10)$$

As with (6c), maximizing the log likelihood function is equivalent to solving

$$\hat{\mathbf{X}}_D = \arg\min_{\mathbf{x}\in\mathcal{Q}^N} \mathbf{x}^T\mathbf{Y}_D^H(\mathbf{F}_L\mathbf{R}_h\mathbf{F}_L^H + \sigma_n^2\mathbf{I}_N)^{-1}\mathbf{Y}_D\mathbf{x}^*. \quad (11)$$

As the matrix $(\mathbf{F}_L\mathbf{R}_h\mathbf{F}_L^H + \sigma_n^2\mathbf{I}_N)^{-1}$ is positive definite, V-BLAST [12] and the sphere decoding algorithm [11] can be used to solve (11). This is developed in detail in Section V.

Remark:

- The proposed blind and semi-blind estimators can be generalized to OFDM systems with virtual carriers and pilot symbols. Both the estimators can be improved via iterative decision feedback.
- The semi-blind estimator (11) needs the knowledge of channel covariance matrix $\mathbf{R}_h$ and the noise variance σ_n^2. The semi-blind estimator design robust to mismatch will be given in the full journal version of this paper.

IV. Data Detection Algorithms

A. V-BLAST detection

The blind estimator (7) and the semi-blind estimator (11) can be solved via V-BLAST detection algorithm [12]. These estimators can be written in a general form as

$$\hat{\mathbf{X}}_D = \arg\min_{\mathbf{x}\in\mathcal{Q}^N} \mathbf{x}^T\mathbf{G}\mathbf{x}^* = \|\mathbf{M}\mathbf{x}^*\|^2 \quad (12)$$

where $\mathbf{G}$ is a positive definite matrix, which can be Cholesky factored as $\mathbf{G} = \mathbf{M}^H\mathbf{M}$. The V-BLAST ordering is to find the permutation matrix Π such that the QR decomposition of $\mathbf{M}' = \mathbf{M\Pi}$ has the property that $\min_{1\le i\le N} r_{ii}$ is maximized over all column permutations. For $k = N, N-1, \ldots, 1$, the algorithm chooses $\pi(k)$ such that

$$\pi(k) = \arg\min_{j\notin\{\pi(1),\ldots,\pi(k-1)\}} \|(\mathbf{G}_k)_j\|^2 \quad (13)$$

where $(\mathbf{G}_k)_j$ is the jth row of $\mathbf{G}_k$, $\mathbf{G}_k$ is the pseudo inverse of $\mathbf{M}_k$ and $\mathbf{M}_k$ denotes the matrix obtained by zeroing columns $\pi(1),\ldots,\pi(k-1)$ of $\mathbf{M}$. The QR factorization of $\mathbf{M}' = \mathbf{M\Pi}$ is denoted by $\mathbf{QR} = \mathbf{M}'$. Eq. (12) can be expressed as

$$\hat{\mathbf{X}}_D = \arg\min_{\mathbf{x}\in\mathcal{Q}^N} \|\mathbf{R}\mathbf{x}^*\|^2. \quad (14)$$

Since $\mathbf{R}$ is upper triangular, the k-th element of (14) is given by

$$\hat{x}_k = \arg\min_{x_k,\ldots,x_N}(r_{kk}x_k^* + \sum_{i=k+1}^{N} r_{ki}x_i^*)^2 \quad (15)$$

where the estimate is free of interference from subcarriers $1, 2, \ldots, k-1$. Thus, x_k can be estimated by minimizing (15). Proceeding in the order $x_N, x_{N-1}, \ldots, x_1$ and assuming correct previous decisions, $\mathbf{x}$ can be estimated and the interference between subcarriers can be cancelled in each step. This sequential detection suffers from error propagation even with V-BLAST optimal ordering. More accurate detection algorithm needs to be derived.

B. Sphere decoding algorithm

The sphere decoding [11], [14] is an efficient method to find an optimal solution to an integer least-squares problems. This is the problem of finding the closest lattice point in N-dimensions to a given point $x \in \mathcal{C}^N$. In our case, the search space has M^N lattice points so the exhaustive search is only possible for small N only. To the best of the authors' knowledge, the SD has not been applied for joint channel estimation and data detection in OFDM systems.

The original SD can only handle real systems. Complex systems can be decoupled to formulate real systems. The complex problem (12) can be transformed into the real matrix equation

$$\tilde{\mathbf{X}}_D = \arg\min_{\tilde{\mathbf{x}}\in\mathcal{A}^{2N}} \tilde{\mathbf{x}}^T\tilde{\mathbf{M}}^T\tilde{\mathbf{M}}\mathbf{x} \quad (16)$$

where

$$\begin{aligned}\tilde{\mathbf{x}} &= \begin{bmatrix} \mathrm{Re}\{\mathbf{x}^T\} & -\mathrm{Im}\{\mathbf{x}^T\} \end{bmatrix}\\ \tilde{\mathbf{M}} &= \begin{bmatrix} \mathrm{Re}\{\mathbf{M}^T\} & -\mathrm{Im}\{\mathbf{M}^T\} \\ \mathrm{Im}\{\mathbf{M}^T\} & \mathrm{Re}\{\mathbf{M}^T\} \end{bmatrix}\end{aligned} \quad (17)$$

and $\mathcal{A} = \{\mathrm{Re}(\mathcal{Q}), \mathrm{Im}(\mathcal{Q})\}$. Thus (16) can be solved via SD. For example, if $\mathbf{x}$ belongs to 4QAM, each entry of $\tilde{\mathbf{x}}$ belongs to BPSK. Any M-QAM can be decoupled similarly. Thus, (12) can be reduced to a real system (16).

Since the matrix $\tilde{\mathbf{M}}^T\tilde{\mathbf{M}}$ is positive definite, Cholesky factorization of it yields $\tilde{\mathbf{M}}^T\tilde{\mathbf{M}} = \mathbf{R}^T\mathbf{R}$, where $\mathbf{R}$ is an upper triangular matrix with r_{ii} real and positive. We then have

$$\mathbf{s}^T\mathbf{R}^T\mathbf{R}\mathbf{s} = \sum_{i=1}^{N}\left| r_{ii}s_i + \sum_{j=i+1}^{N} r_{ij}s_j \right|^2 \le r^2. \quad (18)$$

where $\mathbf{s} = [\mathrm{Re}\{\mathbf{x}^T\}, -\mathrm{Im}\{\mathbf{x}^T\}]^T$. The SD algorithm searches the lattice inside a hypersphere of radius r instead of searching the whole lattice. Substituting $q_{ii} = r_{ii}^2$ for $i = 1, \cdots, N$ and $q_{ij} = r_{ij}/r_{ii}$ for $j = i+1, \cdots, N$, starting from s_N and working backwards, the lower and upper bounds of s_i can be obtained as $\mathrm{L}_i = \lceil -A - B \rceil$, $\mathrm{U}_i = \lfloor A - B \rfloor$, where

$$\begin{aligned} A &= \sqrt{\frac{r^2 - \sum_{l=i+1}^{N} q_{ll}\left| s_l + \sum_{j=l+1}^{N} q_{lj}s_j \right|^2}{q_{ii}}} \\ B &= \sum_{j=i+1}^{N} q_{ij}s_j. \end{aligned} \quad (19)$$

$\lceil\cdot\rceil$ denotes the smallest integer greater than or equal to its argument. $\lfloor\cdot\rfloor$ denotes the largest integer less than or equal to its argument. Therefore s_i is selected from

$$s_i \in [\mathrm{L}_i, \mathrm{U}_i] \bigcap \mathcal{Z} \tag{20}$$

The SD continues with the same process for s_{i-1} and so on. If there is no lattice point within the region say s_m, the SD comes back to s_{m+1} and chooses another candidate value within the bounds. When SD reaches s_1, it generates a candidate lattice point say, $\mathbf{s}'$, inside the hypersphere of radius r. The SD computes the value of $C = \mathbf{s}'^T \mathbf{R}^T \mathbf{R}\mathbf{s}'$. If this value is less than r^2, the radius r is updated to $\sqrt{C}$. This process continues until no further lattice points is found within the hypersphere. The lattice point that achieves the smallest value of (18) within the hypersphere is the ML solution.

However not all M-PSK constellations can be decoupled into real systems (e.g., 8-PSK). Hochwald and Brink [15] show a modified SD to handle complex constellations. But this involves the computationally inefficient $\cos^{-1}$ operation, which will slow down the SD. We address here the decouple algorithm can still be used to handle M-PSK. Reorder the elements of $\tilde{x}$ to formulate $\mathbf{s}$ as

$$\mathbf{s} = [\mathrm{Re}(x_1), \mathrm{Im}(x_1), \ldots, \mathrm{Re}(x_N), \mathrm{Im}(x_N)]^T. \tag{21}$$

And $\tilde{\mathbf{M}}$ is rearranged as $\tilde{\mathbf{M}}'$ accordingly. If $\mathrm{Re}(x_i)$ is selected, the candidates for $\mathrm{Im}(x_i)$ are limited by the constellation. Take 8-PSK for example, the constellation set $Q_8 = \{0.9239 + 0.3827j, 0.3827 + 0.9239j, -0.3827 + 0.9239j, -0.9239 + 0.3827j, -0.9239 - 0.3827j, -0.3827 - 0.9239j, 0.3827 - 0.9239j, 0.9239 - 0.3827j\}$. The candidate set for $\mathrm{Re}(x_i)$ is $\mathcal{Q}_R^i = \{0.9239, 0.3827, -0.3827, -0.9239\}$. If $\mathrm{Re}(x_i)$ chooses one element of $\mathcal{Q}_R$ within the bounds (19), for example $\mathrm{Re}(x_i) = 0.3827$, $\mathrm{Im}(x_i)$ can only be chosen from $Q_I^i(\mathrm{Re}(x_i)) = \{0.9239, -0.9239\}$ due to the constraint on constellation, where $Q_I^i(\mathrm{Re}(x_i))$ means the candidate set for $\mathrm{Im}(x_i)$ depends on $\mathrm{Re}(x_i)$. $\mathcal{Q}_R^i$ is the same for all x_i and we omit superscript i. Therefore s_i is selected from

$$s_i \in \begin{cases} [\mathrm{L}_i, \mathrm{U}_i] \cap \mathcal{Q}_R \\ [\mathrm{L}_i, \mathrm{U}_i] \cap \mathcal{Q}_I(s_{i-1}) \end{cases}. \tag{22}$$

Since $Q_I(s)$ can be pre-computed for each s from $\mathcal{Q}_R$ and be stored in memory, additional computational complexity is avoided and the former SD algorithm is not changed much. This idea can be used to handle any constellations. The decoupling for M-QAM can be viewed as a special case of the new generalization in that $Q_I(s)$'s are the same for any s from $\mathcal{Q}_R$ and $\mathcal{Q}_R = \mathcal{Q}_I$.

V. SIMULATION RESULTS

Simulation results are given for the proposed blind and semi-blind estimators. We consider a frequency-selective slow Rayleigh fading channel with $L = 3$ Gaussian complex coefficients h_l with mean power of $\sigma_l^2 = E[|h_l|^2] = \sigma_0^2 e^{-l/5}$ for $l = 1, \cdots, L$. An OFDM system with N=32 subcarriers and binary phase shift keying (BPSK) are simulated. The carrier frequency is 5GHz and the data rate is 3MHz. A training symbol is transmitted at the Nth subcarrier to solve the scaling ambiguity.

Both estimators are tested on OFDM systems with the above simulation parameters under different SNR. Fig. 1 shows the normalized mean square error (NMSE) of channel estimation which is defined as

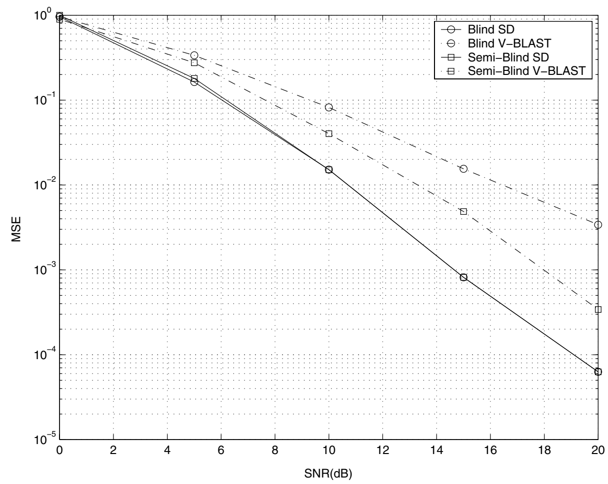

Fig. 1. NMSE of the joint ML CIR estimation versus SNR

$$\mathrm{NMSE} = \frac{E\{\sum_{l=1}^{L} |h_l - \hat{h}_l|^2\}}{\sum_{l=1}^{L} E\{|h_l|^2\}}. \tag{23}$$

Both estimators with SD have almost the same NMSE performance in all SNR region. The performance of semi-blind with V-BLAST is 2.5dB better than that of blind with V-BLAST but is 2dB worse than the two estimators with SD at NMSE=10^{-2}. The BER performance of the OFDM system is compared with that of the ideal case, where the channel parameters are exactly known at the receiver (Fig. 2). The performance of one-tap equalization is used as benchmark. Both estimators with SD are within 0.2dB of the ideal case in the high SNR region. The performance of V-BLAST detection for semi-blind estimator is comparable to that of SD in high SNR region. Even in low SNR region, the gap between SD and V-BLAST is within 1dB. This result seems to be contradict the results given in [16], where great performance improvement is achieved by using SD in MIMO systems. The reason for such contradiction may be that the order of the constellation is not very high. The computational complexity as a function of the SNR is given in Fig. 3. Note that the complexity of the exhaustive search is 1.81×10^{13} flops while even for an SNR of 0dB all estimators' complexity are within 10^6 flops by using SD. The complexity of both the estimators increase with the increase of SNR. At 0dB, the semi-blind estimator is 15 times faster than the blind estimator while at 20dB it is more than 31 times faster. The blind estimator has higher complexity than the semi-blind estimator. This is possibly due to the inherent rank-deficiency in (6c) while the complexity is

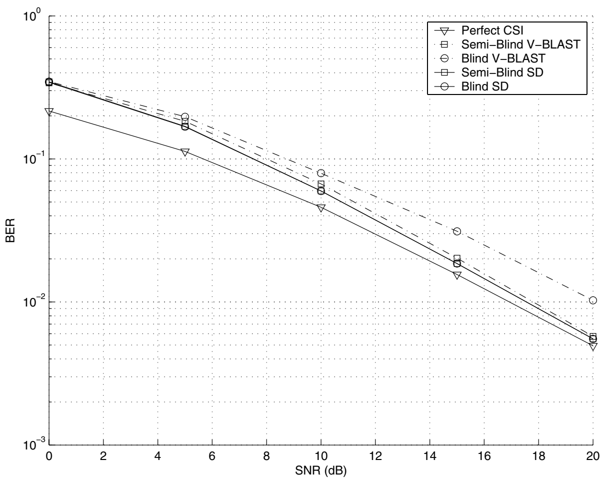

Fig. 2. Bit error rate of the joint ML estimation algorithm versus SNR

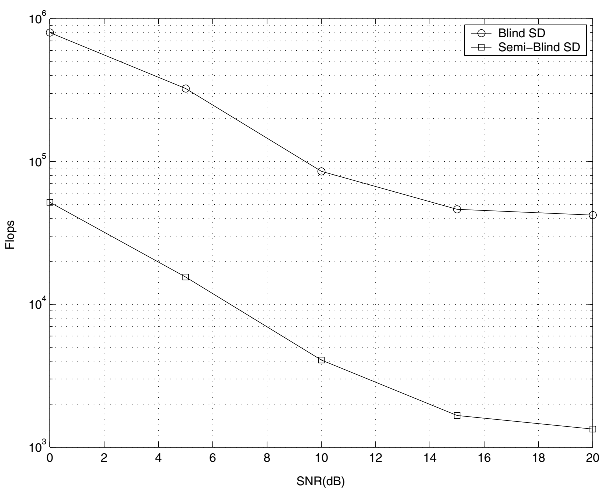

Fig. 3. The computational complexity versus SNR

significantly reduced compared with the exhaustive search in the first $N - L$ variables in semi-blind estimator. The semi-blind estimator is preferable when the channel statistics are known at the receiver.

VI. CONCLUSION

We have developed novel blind and semi-blind joint estimators for channel and data in OFDM systems. Two detection algorithms based on V-BLAST detection and sphere decoding have been developed to efficiently solve the resulting integer LS optimization problems. We generalize the original sphere decoder to handle any M-PSK constellations and to achieve reduced complexity. Further, the minimization problem for the blind estimator suffers from rank deficiency. We derive an efficient solution to the rank deficiency problem. Simulation results show that our proposed estimators perform fairly close to the ideal case. The estimators proposed in this paper may also be extended to MIMO-OFDM systems and OFDM over fast fading channels. These applications are currently being investigated.

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