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Side Information in PAR Reduced PTS-OFDM Signals

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Abstract— **Orthogonal Frequency Division Multiplexing suffers from large peak-to-average power ratios (PAR). Large PAR causes undesirable effects such as signal distortion and unwanted out of band radiation. Use of partial transmit sequences (PTS) is proposed to improve the PAR statistics of OFDM signals. Phase angles of the modulated symbols are changed in PTS to reduce the PAR. The information about these changes (side information) is critical for the successful operation of PTS. However, a proper way to send side information to the receiver without effecting the optimized PAR statistics has not been reported in the open literature so far. In this paper, we present a novel scheme to insert the side information into PTS-OFDM signals without effecting the improved PAR statistics. Simulation results show that the proposed scheme does not contribute to any peak-regrowth and the bit error rate performance of the system is improved.**

I. Introduction

Orthogonal frequency division multiplexing (OFDM) is successfully used in many wireless digital communication systems [1]. OFDM is also a potential candidate for future broadband wireless communication systems. However, OFDM suffers from high peak-to-average power ratios (PAR). A large PAR leads to disadvantages such as increased complexity of the analog-to-digital converter (A/D) and reduced efficiency of the radio frequency (RF) amplifier. High peaks are clipped by non-linear devices at the transmitter, causing undesirable effects such as high out of band radiation and inband distortion [2]. The PAR reduction techniques are therefore of great importance for OFDM systems. Several solutions to the PAR reduction problem have been reported in the literature [3]–[5].

Partial transmit sequences (PTS) is a distortionless PAR reduction techniques. In PTS, phase angles of the modulated symbols are changed such that the PAR is reduced. However, information about these changes has to be sent to the receiver as the side information (SI). Reliable SI is critical to the correct operation of the PTS-OFDM receiver. One solution is to reserve several subcarriers for SI transmission. However, if the SI is not considered at the optimization process, the PAR statistics would be effected and peak regrowth occurs.

In this paper we propose a novel SI insertion scheme for PTS-OFDM signals. This paper is orgnaized as follows. It starts with an overview of OFDM transmission and PAR of OFDM signals in Section II. Section III describes the fundamentals of PTS and Section IV introduces the issue of side information in PAR reduced PTS-OFDM signals. Section V introduces the novel SI insertion scheme, while some simulation results are presented in Section VI. Section VII concludes the paper.

II. Orthogonal Frequency Division Multiplexing

In OFDM, a block of N symbols, $\{X_n, n = 0, 1, ..., N-1\}$, is formed with each symbol modulating one of a set of N subcarriers, $f_n, n = 0, 1, ..., N-1$. The N subcarriers are chosen to be orthogonal, that is $f_n = n\Delta f$, where $\Delta f = 1/T$ and T is the OFDM symbol duration. The resulting signal can be expressed as

$$x(t) = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} X_n e^{j2\pi f_n t}, 0 \leq t \leq T. \quad (1)$$

A cyclic prefix (called guard interval) is added to the resulting signal in order to avoid the intersymbol interference (ISI) which occurs in multipath channels. At the receiver the guard interval is removed and the time interval $[0, T]$ is evaluated. The guard interval usually a periodic extension of the useful OFDM symbol.

Time-domain samples of OFDM signals in the equivalent complex valued low-pass domain are approximately Gaussian distributed due to the statistical independence of carriers. Resulting high PAR is given by

$$\xi = \frac{\max |x(t)|^2}{E[|x(t)|^2]}. \quad (2)$$

where $E\{.\}$ denotes expectation. This does not depend on the signal set X_n used to modulate the signal. The theoretical maximum of the PAR for N number of sub-carriers is $10\log(N)$dB. Next section introduces the PTS technique. An oversamplinng factor of $L \geq 4$ is often used to estimate the actual PAR from its samples.

III. Partial Transmit Sequences (PTS)

The first step of PTS is to identify a suitable set of sub-blocks. The data frame is defined as a vector, $\mathbf{X} = [X_0, X_1, \ldots, X_{N-1}]^T$, where X_n is the data symbol for subcarrier $n, n = 0, 1, \ldots, N-1$. The problem is to partition

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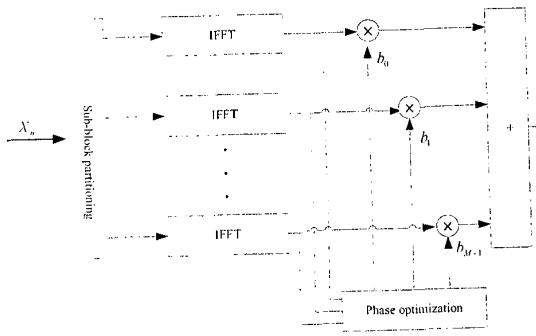

Fig. 1. Block diagram of an PTS scheme.

$\mathbf{X}$ into M sub-blocks. Let I_m be the subcarrier indices for the m-th sub-block, $\mathbf{X}_m$. We now have the following relationships

$$I_n \cap I_l \qquad n \neq l \tag{3}$$

and

$$\bigcup_{m=1}^{M} I_n = \{0, 1, \ldots, N-1\}. \tag{4}$$

Sub-block partitioning can be classified into three categories: adjacent, interleaved and pseudo random sub-block partitioning [6]. In the adjacent scheme, $\frac{N}{M}$ successive subcarriers are sequentially assigned into the same sub-block. That is

$$I_1 = 1, 2, \ldots, \frac{N}{M} - 1, \tag{5}$$

$$I_2 = \frac{N}{M}, \frac{N}{M} + 1, \ldots, \frac{2N}{M} - 1 \tag{6}$$

and so on. For the interleaved sub-block partitioning, every subcarrier spaced M apart is allocated at the same sub-block. That is

$$I_1 = 0, M, 2M, \ldots, \left(\frac{N}{M} - 1\right) M, \tag{7}$$

$$I_1 = 1, M+1, 2M+1, \ldots, \left(\frac{N}{M} - 1\right) M + 1 \tag{8}$$

and so on. Finally, each subcarrier is assigned into any one of the sub-blocks randomly in the pseudo-random scheme. That is, $\frac{M}{N}$ entries in each I_n are selected randomly. In each case, disjoint sub-blocks are generated and optimally combined to reduce the PAR. These three schemes are graphically illustrated in [7]. A general PTS scheme is depicted in Fig. 1.

A. Optimal combining

The IDFT of $\mathbf{X}_m$ s are called partial transmit sequences. Let $y_{k,m}$ for $k = 0, 1, \ldots, LN-1$, $m = 1, 2, \ldots, M$, be the LN point IDFT of $\mathbf{X}_m$ appropriately zero-padded

$$y_{k,m} = \sum_{n \epsilon I_m} X_n e^{\frac{j2\pi nk}{LN}}. \tag{9}$$

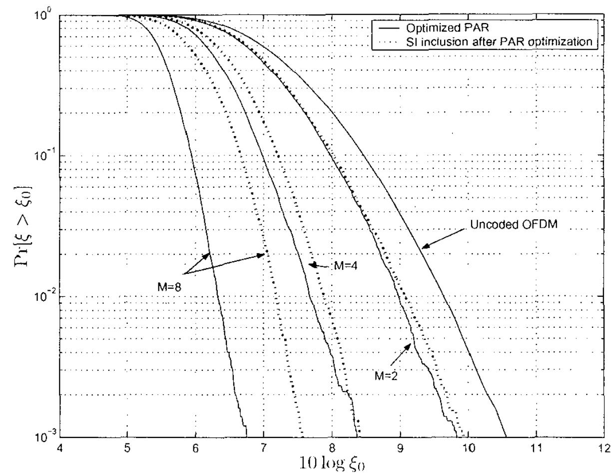

Fig. 2. Performance of PTS OFDM.

Oversampling rate L is used for accurate PAR estimation from the samples of IFFT output. Using the linearity property of IDFT, the time domain samples can be represented as

$$s_k = \sum_{m=1}^{M} b_m y_{k,m} \qquad k = 0, 1, \ldots, LN-1 \tag{10}$$

where $\{b_m, m = 1, 2, \ldots, M\}$ are weighting factors. They are further assumed to have pure rotations (i.e., $b_m = e^{j\phi_m}$). The weighting factors are chosen to minimize the PAR of $\mathbf{X}$.

To reduce the complexity of this minimization, a suboptimal choice is used by limiting all to +1 or -1. Without any loss of performance we can set $b_1 = 1$ and observe that there are $M-1$ binary variables to be optimized. Finally the optimal PAR (ξ_{optimal}) can be found using

$$\xi_{\text{optimal}} = \frac{\min_{b_1,\ldots,b_M} \left(\max_{0 \le k \le LN} \left| \sum_{m=1}^{M} b_m y_{k,m} \right|^2 \right)}{E[|x(t)|^2]}. \tag{11}$$

In PTS, PAR in (11) is computed for all the binary combinations and hence requires 2^{M-1} iterations. Note that the above development is similar to [6] except for the use of oversampling.

IV. Side information (SI)

To recover the data, the receiver must know the exact weighting factors used to reduce the PAR. Reliable SI is an important factor for PTS to work properly. If the SI is added into the OFDM symbol after the PAR optimization, peak regrowth occur. For an PTS-OFDM system with M sub-blocks and 2^q-ary modulation, $\lceil (M-1)/q \rceil$ symbols to be reserved for the SI. Fig. 2 shows the complementary cumulative density function of the PAR of PTS OFDM signals. Simulation parameters are shown in Table I. Continuous lines show the optimized PAR statistics, while discontinuous lines show the PAR statistics after the inclusion of the SI in to the

PAR reduced PTS-OFDM signals. This figure clearly shows the effect of the SI on the PAR statistics. Clear peak regrowth is observed when the SI is included into the PAR reduced PTS-OFDM signal. When the number of PTS subblocks are increased peak regrowth is also increased. PAR regrowth is about 1 dB in the case of $M = 8$ (number of subblocks is 8) and it is 0.2 dB and 0.1 dB when M is 4 and 2 respectively. This PAR regrowth is directly related to the number of subcarriers reserved for the SI. Subcarriers reserved for the SI are 4, 2 and 1 for M equals to 8, 4 and 2 respectively.

An alternative way to send the SI is proposed in [8]. The basic idea is to insert a marker onto the transmitted data that can uniquely identify the inversion sequence at the receiver.

Marking algorithm [8]: Phase rotation of ± 1 is assumed. If the phase rotation does not occur ($b_m = 1$), do nothing. Otherwise, if $b_m = -1$, then rotate every other tone in the cluster by $\pi/4$. This is equivalent to the use of two constellations for data; one standard constellation and the other rotated by $\pi/4$. This algorithm puts an embedded marker on those clusters that have been rotated.

Detection scheme: With the modulation removed, the data symbols are differentially detected by computing the following test statistics for each cluster

$$Z_m = \sum_{j=1}^{N/M-1} (Y_{j,m} Y^*_{j+1,m})^4 \qquad (12)$$

where $Y_{j,m}$ represents the j-th tone in the m-th sub-block. If the cluster has not been rotated $Z_m = N/M - 1$, where as $Z_m = -(N/M - 1)$ if the cluster has been rotated. Therefore, the optimized binary weighting factors can be recovered. However, this scheme also causes peak regrowth. In the next section we introduce a new scheme to send the SI which will not effect the optimized PAR.

V. Novel Scheme to Send Side Information

We can set $b_1 = 1$ without loss of generality. In our approach, the SI is placed in the first sub-block such that it will not be changed during the optimization process. The idea is to send the SI of the current OFDM symbol in the first sub-block of the next OFDM symbol. The current symbol is optimized with the embedded SI of the previous OFDM symbol. Optimum conditions can be preserved in this manner. The algorithm can be explained as follows.

- μ-th OFDM symbol is partitioned into M-sub-blocks and optimum weighting factors are obtained by performing the required number of iterations, while keeping the first weighting factor unity.
- Identity of the optimal weighting factors are embedded onto the 1-st sub-block of the $\mu + 1$-th OFDM symbol.
- (μ+1) OFDM symbol is optimized and optimal weighting factors are inserted onto the 1-st sub-block of the $\mu + 2$ OFDM symbol.
- This procedure is followed for all the OFDM blocks and very first OFDM symbol is transmitted without any SI, while extra dummy OFDM symbol needs to be transmitted to carry the SI of the last OFDM symbol.

TABLE I

SIMULATION PARAMETERS.

Parameters	Value
Modulation	QPSK-OFDM 16QAM-OFDM
FFT size (N)	64
Total number of subcarriers	52
Number of pilot subcarriers	4
OFDM symbol duration	4 μs
Guard interval	0.8 μs
Signal bandwidth	16.6 MHz

At the receiver, the current OFDM symbol is recovered using the SI received through the next OFDM symbol. There is a delay of one OFDM symbol at the receiver to recover the data completely. The phase angles of the modulated symbols in the first sub-block is not changed during the optimization process. Therefore, the SI corresponding to the previous OFDM symbol is preserved and it can be recovered without decoding the OFDM symbol completely.

The process can be explained mathematically as follows. Let the complete OFDM symbol be given by $[S_{\mu-1,0}, \ldots, S_{\mu-1,c-1}, X_{\mu,c}, \ldots, X_{\mu,N-1}]$, where μ refers to the OFDM symbol number and $S_{\mu,n}$ is the n-th subcarrier reserved for the SI of $(\mu - 1)$-th OFDM symbol. c is the total number of subcarriers reserved for the SI. Then the PAR optimization algorithm can be expressed using the linearity property of DFT as before

$$\xi_{\text{optimal}} = \min_{b_1,\ldots,b_M} \left[\max_k \left(|s_{\mu-1,k}|^2 + |y^1_{\mu,k}|^2 + \left| \sum_{m=2}^{M} b_m y^m_{\mu,k} \right|^2 \right) \right] \times \frac{1}{E[|x(t)|^2]} \qquad (13)$$

where

$$s_{\mu-1,k} = \sum_{n=0}^{c-1} S_{\mu-1,n} e^{j2\pi nk/N},$$

$$y^1_{\mu-1,k} = \sum_{n=c}^{N/M-1} X_{\mu,n} e^{j2\pi nk/N},$$

and

$$y^m_{\mu,k} = \sum_{n=N/M}^{N-1} X^m_{\mu,n} e^{j2\pi nk/N}$$

where $X^m_{\mu,n}$ is the n-th subcarrier of the m-th sub-block of μ-th OFDM symbol. (13) shows that the SI of $\mu - 1$-th OFDM symbol can be recovered easily at the receiver which can be described as follows. The transmitted signal is given by,

$$y^{optimal}_{\mu,k} = s_{\mu-1,k} + y^1_{\mu,k} + \sum_{m=2}^{M} b^{optimal}_m y^m_{\mu,k}, \quad k = 0, 1, \ldots, N-1. \qquad (14)$$

At the receiver a DFT operation is performed first

$$R_n = \sum_{k=0}^{N-1} y_{\mu,k}^{optimal} e^{-j2\pi nk/N} \quad (15)$$

giving out

$$\begin{aligned} R_n =& [S_{\mu-1,0}, \ldots, S_{\mu-1,c-1}, X_{\mu,c}, \ldots, \\ & X_{\mu,N/M-1}, b_{\mu,2}^{optimal} X_{\mu,N/M}, \ldots, \\ & b_{\mu,M}^{optimal} X_{N-1}]. \end{aligned} \quad (16)$$

Thus, the SI of the $\mu-1$-th OFDM symbol, $\mathbf{S}_{\mu-1}$ is recovered. It is suggested to use a simple forward error correcting code such as a simple Hamming code to protect the SI against errors.

A. Can we send the SI in the same symbol?

We have shown that the insertion of the SI after the PAR optimization process causes peak regrowth. However, we know that the first subblock will be transmitted without modifying it's phase angles. Although it is possible to place SI in the first subblock of the same OFDM symbol without incurring peak regrowth, the complexity of the PAR reduction will be very high. In this case we have to take additional IFFTs of the first subblock equal to the number of iterations performed. In this case, we have to consider the first sublock with corresponding SI of the iteration at each iteration performed. For example, if $b_m = \pm 1$ and number of subblocks is M, the number of additional IFFTs to be performed is 2^M.

VI. Results

Simulation parameters were selected to represent the performance of the physical layer of the IEEE 802.11a wireless local area networks. Table I shows the simulation parameters. Fig. 3 depicts the CCDF of PAR of PTS-OFDM signals with embedded side information. Continuous lines show the PAR statistics after PAR reduction without embedded side information. Dotted lines show the PAR statistics after the inclusion of side information in to the PAR reduced OFDM symbols. Clear peak regrowth is observed when the SI is placed in the PAR reduced OFDM signals. This regrowth is higher when the number of subblocks is increased. PAR regrwoth is more than 1 dB when the number of subblocks are eight ($M = 8$). PAR statistics of the PTS-OFDM signals with embedded the SI using the proposed scheme is shown in discontinuous lines. No peak significant regrowth is observed with the proposed scheme. Optimized PAR statistics remain unchanged for all the values of M with this scheme. Suitability of the proposed scheme can also be examined by the bit-error rate (BER) performance of the system.

Effect of the SI on the bit-error rate (BER) is clearly visible in higher order modulation schemes compared to QPSK. Therefore, we have selected 16-ary quadrature amplitude modulations (16-QAM) in our BER simulations. BER performance is observed without using the convolutional encoder in the IEEE 802.11a PHY layer signals. Fig. 4 depicts the BER performance of PTS-OFDM signals after passing through a

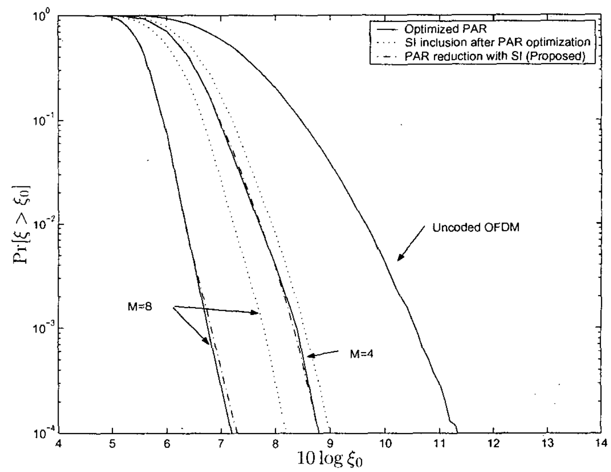

Fig. 3. PAR statistics with the proposed SI inclusion scheme.

SSPA non-linear amplifier [9] in an additive white Gaussian (AWGN) channel. The back off of the non-linear amplifier is set at 7 dB. OFDM signal without PAR reduction shows a higher error floor above 10^{-2} when passing through a SSPA non-linearity even in an AWGN channel. BER is clearly improved with the PAR reduction. BER improvement is not significant when the PAR is reduced using a PTS scheme with 4 sublocks. Clear BER improvement is observed when the PAR is reduced by a PTS scheme with 8 subblocks. However, we can observe a clear effect of the SI on BER performance. Perfect SI without any errors at the receiver is assumed in the case of perfect SI.

Degradation of BER is observed due to errors in the SI in other two cases. However, worst BER performance is observed when the PAR is reduced without the SI and the SI is inserted later. This scheme has a higher error floor compared to the proposed scheme. The proposed scheme shows an improved BER performance. The number of peaks clipped at the SSPA non-linearity is much less in the proposed scheme due to the improvement in the PAR statistics. This improvement in BER increases with the number of subblocks used for the PTS optimization process. BER degradation in the proposed scheme due to the errors in the SI can be further reduced using a simple forward error correction code such a Hamming code to protect the SI [10] .

VII. Conclusions

Side information is critical for data recovery in PAR reduced PTS-OFDM signals. This paper proposes a simple side information inclusion scheme. The proposed scheme does not contribute to any PAR regrowth. Both PAR and BER of IEEE 802.11a PHY layer signals are improved with the proposed scheme. Further BER improvement is expected using a simple forward error correction code to protect the side information.

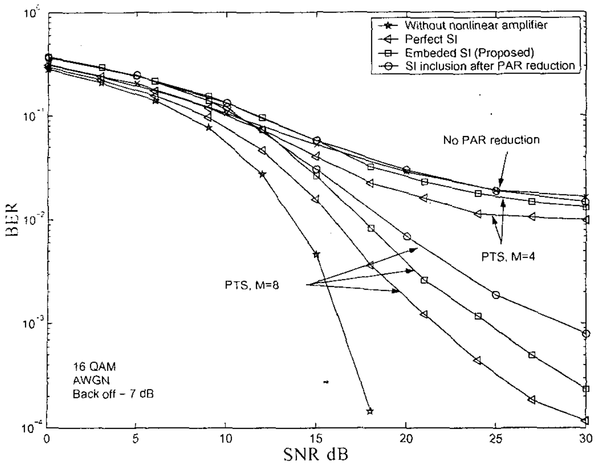

Fig. 4. BER performance in an AWGN channel.

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