# A Novel ICI Cancellation Technique for OFDM Systems Using Adaptive Mapping Signal Constellation

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***Abstract*— We propose a novel Intercarrier Interference (ICI) reduction technique for OFDM systems. By partitioning a $(2M)$-point constellation into two disjoint $M$-point constellations, and adaptively mapping each $\log_2 M$ input data bits into either of these two constellations, this technique allows extra freedom to reduce the ICI. The ICI reduction of the proposed technique is comparable to the rate-half technique while the throughput of the former is higher than that of the latter. In this paper, three suboptimal methods, i.e., the random selection, the modified PTS, and the recursive partial sequence methods, are also proposed to lower the computational complexity. Their performances are evaluated and compared by simulations.**

## I. Introduction

Orthogonal Frequency Division Multiplexing (OFDM) is an important Multicarrier Modulation (MCM) technique widely used in a number of communication systems such as IEEE 802.11a, HIPERLAN/2, and Digital Video Broadcasting (DVB) [1]. By using a set of orthogonal subcarriers, an OFDM system transmits data symbols in parallel to obtain high transmission bit-rate. It can be efficiently implemented by using Inverse Fast Fourier Transform (IFFT) and Fast Fourier Transform (FFT). Moreover, with the use of guard intervals, OFDM system can easily combat the multipath delay spread of a wireless channel.

In wireless communications, OFDM systems suffer from the Intercarrier Interference (ICI) caused by carrier frequency mismatch between the transmitter and receiver, and/or the Doppler shift. This is a significant problem for mobile communication where very accurate frequency estimation may not be possible because of the low-cost requirement [1].

Various techniques have recently been proposed to combat the ICI, such as equalization [2], time-domain windowing [3], and coding [4]–[8].

In [6], an ICI self-cancellation technique was proposed. This technique can significantly reduce the ICI, and is very simple in encoding and decoding. However, it halves the data throughput (referred to as the rate-half repetition coding). By reducing the Peak Interference-to-Carrier Ratio (PICR), [7] proposed the use of Selected Mapping (SLM) and Partial Transmit Sequence (PTS) methods to combat the ICI. However, the use of these methods requires the distortionless transmission of side information.

In this paper, a novel ICI cancellation technique for OFDM systems is proposed. This technique is based on partitioning a $(2M)$-point constellation into two disjoint constellations and adaptively mapping each $\log_2 M$ input bits into either of these two constellations. The consequent extra freedom can be exploited to reduce the PICR. A great advantage of this technique is that no side information is required at the receiver. To reduce the computational complexity for finding the suitable mapping sequence, three methods are also proposed to find suboptimal solutions, which are the random selection, the modified PTS, and recursive partial sequence (RPS) methods. Simulations show that these methods can significantly reduce the PICR.

This paper is organized as follows: Section II reviews the ICI problem and defines the PICR. The adaptive mapping technique is proposed in Section III. In Section IV, three suboptimal methods are proposed for reducing computational complexity. Their simulation results are analyzed and compared in Section V. The conclusion of this paper is given in Section VI.

## II. Characterization of ICI Problem

At the transmitter of an OFDM system, the input bitstream is first encoded to a set of (complex) symbols $X_k$ according to a given $M$-ary signal constellation $\mathcal{Q}_M$ (e.g. MPSK, QAM, etc.). These symbols are then converted to time-domain by using IFFT operation. After the digital-to-analog (D/A) conversion and being modulated to a certain RF carrier frequency $f_c$, the transmitted signal $x_c(t)$ can be represented as

$$x_c(t) = \mathrm{Re}\left\{\frac{1}{\sqrt{N}}\sum_{k=0}^{N-1} X_k e^{j2\pi(f_c+kf_0)t}\right\}, \quad 0 \le t \le T, \quad (1)$$

where $N$ represents the number of subcarriers, $\mathrm{Re}\{\cdot\}$ represents the real part of $(\cdot)$, and $f_0 = 1/T$ represents the subcarrier spacing with $T$ representing the symbol set period.

At the receiver, the received signal is first down-converted to baseband, and (after the analog-to-digital (A/D) conversion) passed through an FFT operator. If the carrier frequency offset between the transmitter and receiver exists, the receiver output becomes

$$Y_k = S_0 X_k + \sum_{l=0, l\neq k}^{N-1} S_{l-k} X_l + n_k,\ k = 0, \ldots, N-1, \quad (2)$$

where $n_k$ represents the additive complex Gaussian noise sample, and $S_k$ represents the ICI coefficient

$$S_k = \frac{\sin \pi(k+\varepsilon)}{N \sin \frac{\pi}{N}(k+\varepsilon)} \exp\left(j\pi\left(1-\frac{1}{N}\right)(k+\varepsilon)\right), \quad (3)$$

with $\varepsilon$ representing the normalized frequency offset defined as the ratio of the frequency offset over the subcarrier spacing $f_0$. The second term in Eqn. (2), denoted as $I_k$

$$I_k \triangleq \sum_{l=0, l\neq k}^{N-1} S_{l-k} X_l + n_k,\ k = 0, \ldots, N-1, \quad (4)$$

represents the ICI term in $Y_k$. If $\varepsilon = 0$, $S_k = \delta_k$ and no ICI exists in $Y_k$.

The ICI problem can also be characterized by the PICR, which is defined as [7]

$$\text{PICR} \triangleq \max_{0\leq k\leq N-1} \left\{ \frac{|I_k|^2}{|S_0 X_k|^2} \right\}. \quad (5)$$

The PICR is directly related to the Bit Error Rate (BER). An upper bound for the BER of the Binary Phase Shift Keying (BPSK) symbol sequences can be found in [7]. Note that the PICR is a function of $\varepsilon$ and the data symbol sequence $\{X_k\}$. Therefore, it is possible to code the input symbols for reducing the PICR, and in turn, improving the BER.

## III. PICR Reduction Using Adaptive Mapping

It has been shown that, for an arbitrary signal constellation, some data sequences lead to low PICR [7]. Based on this fact, one can reduce the PICR by mapping those data sequences having high PICR to some data sequences having low PICR. Moreover, in order to decode the received signal easily and reliably (which is especially important to mobile handset because of its low-cost requirement), the mapping scheme has to be simple and revertible without any side information.

Having above considerations, one can map an $M$-point signal constellation $\mathcal{Q}_M$ to a $(2M)$-point signal constellation $\mathcal{Q}_{2M}$. Then, each point in $\mathcal{Q}_M$ can be represented by either of the two corresponding points in $\mathcal{Q}_{2M}$. That is, each OFDM subcarrier has two modulation choices. The consequent extra freedom allows carefully selecting each subcarrier modulation symbol to reduce the PICR, leading to the adaptive mapping technique.

Various schemes can be used to map $\mathcal{Q}_M$ to $\mathcal{Q}_{2M}$. In this paper, the following scheme that only involves sign-changing is used.

Given an origin-symmetrical signal constellation $\mathcal{Q}_{2M}$ (e.g. 2MPSK or 2M-QAM), one can partition it into two disjoint $M$-point constellations such that

$$\mathcal{Q}_{2M} = \mathcal{Q}_M^{(1)} \cup \mathcal{Q}_M^{(2)}, \quad (6)$$

where for any constellation point $X_k \in \mathcal{Q}_M^{(1)}$, $-X_k$ belongs to $\mathcal{Q}_M^{(2)}$. Therefore, each $\log_2 M$ input data bits can be mapped to either $X_k$ or $-X_k$. Since $X_k$ and $-X_k$ are in different constellations, the decoding does not require any side information. Figure 1 gives an illustration of this mapping scheme. In this figure, the 8PSK constellation is partitioned into two 4-point constellations. Then, the input bits 00 can be mapped to either 1 or $-1$, 01 can be mapped to either $\frac{1}{\sqrt{2}} + j\frac{1}{\sqrt{2}}$ or $-\frac{1}{\sqrt{2}} - j\frac{1}{\sqrt{2}}$, and so on.

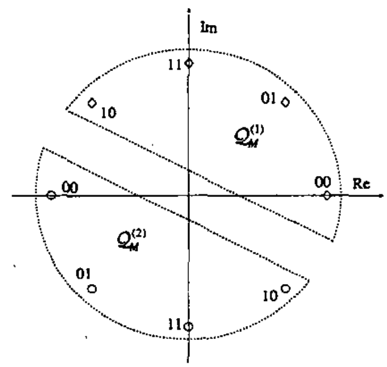


Fig. 1. Partitioning 8PSK into two disjoint constellations

With this mapping scheme, suppose the input data bits of an OFDM system are first encoded into $\mathcal{Q}_M^{(1)}$. Then, the input sequence of the IFFT operator becomes

$$\mathbf{X} = [X_0, \ldots, X_{N-1}],\ X_k \in \mathcal{Q}_M^{(1)}, \quad \text{and } k = 0, \ldots, N-1. \quad (7)$$

Then one has the freedom to change the signs of some $X_k$ (i.e., mapping them to $-X_k \in \mathcal{Q}_M^{(2)}$) and use the consequent sequence as the input of the OFDM system to reduce the PICR. Therefore, by defining a mapping sequence $\mathbf{c} = [c_0, \ldots, c_{N-1}]$, the PICR reduction problem becomes

$$\min_{\mathbf{c}} (\text{PICR of } \mathbf{c} \odot \mathbf{X}), \quad \text{subject to } \mathbf{c} \in \{1, -1\}^N, \quad (8)$$

where $\odot$ represents element-wise multiplication, and $\{\cdot\}^N$ represents the $N$-dimensional solution space.

This technique introduces redundancy in OFDM systems. Since $\mathcal{Q}_{2M}$ can represent $\log_2(2M) = \log_2 M + 1$ input bits while with this scheme, it only represents $\log_2 M$ bits, the coding rate is

$$R = \frac{\log_2 M}{\log_2 M + 1}, \quad (9)$$

or 1 bit redundancy in each ($\log_2 M + 1$) encoded bits. However, when a large constellation is used, the coding rate can be high (e.g. $R = 85.71\%$ for $M = 64$).

With the redundancy introduced, this technique can significantly reduce the PICR. As an example, the PICR Complementary Cumulative Distribution Functions (CCDF) for the proposed technique and the rate-half technique are compared in Figure 2. As a reference, the PICR CCDF for uncoded symbol sequences is also shown in this figure. Here, the bit-rates of the three cases are same. Specifically, the symbol sequences used for the uncoded case are BPSK ($M = 2$) symbol sequences with the length $N = 16$. For coded cases, the proposed technique maps BPSK symbol sequences (with the length $N = 16$) to QPSK symbol sequences ($M = 4$). On the other hand, the rate-half technique extends QPSK symbol sequences from the length $N = 8$ to $N = 16$. From this figure, one can see that the proposed technique leads to 7dB PICR reduction and its performance is slightly better than that of the rate-half technique.

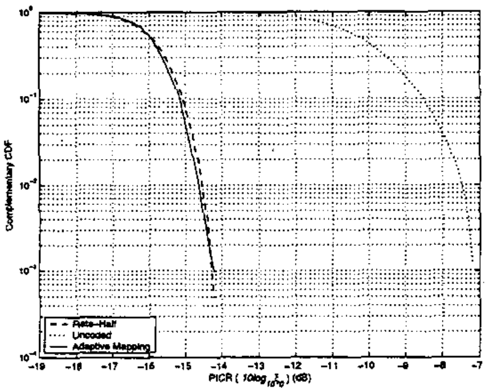


Fig. 2. PICR comparison between uncoded, rate-half coded, and adaptive-mapping coded symbol sequences

A disadvantage of the adaptive mapping technique is its large search space. Note that for each input sequence, any mapping sequence pair $\mathbf{c}$ and $-\mathbf{c}$ lead to the same PICR. Therefore, one can always let $c_0 \equiv 1$, and the search space is $2^{N-1}$ possible mapping sequences for each input sequence. Obviously the complete search becomes impractical when $N$ is large. Hence, suboptimal methods have to be used to reduce the computational complexity.

## IV. Suboptimal Methods

### A. Random Selection Method

This method is based on the random selection of $\mathbf{c}$. For each input sequence, $K$ mapping sequences are randomly selected, and the corresponding PICRs are calculated. The sequence leading to the minimum PICR is then used as the optimum $\mathbf{c}$.

Note that in Eqn (7), $\mathbf{X}$ comes from half of the $(2M)$-ary constellation. By using the randomly selected mapping sequence $\mathbf{c}$, the mapped sequence becomes a sequence coming from the $(2M)$-ary constellation $\mathcal{Q}_{2M}$. Therefore, this method is equivalent to randomly selecting a sequence from $\mathcal{Q}_{2M}$. A tradeoff between $K$ and the PICR requirement can be made.

### B. Modified Partial Transmit Sequence (PTS) Method

PTS technique [7] partitions the input sequence into $G$ disjoint subblocks. A mapping sequence, where each element corresponds to the sign for each subblock, is used to obtain a suboptimal solution. With some modifications, PTS technique can also be used here.

As mentioned above, $\mathbf{X}$ comes from $\mathcal{Q}_M^{(1)}$, which is a non-origin-symmetrical constellation. This fact implies that its maximum PICR is much higher than that of the origin-symmetrical constellation $\mathcal{Q}_{2M}$ (c.f. Figure 3). Therefore, using the PTS method without any modification would not achieve significant PICR reduction compared to using uncoded $\mathcal{Q}_M$ symbol sequences.

Since the PTS method works well for origin-symmetrical constellation symbol sequences, the modification is simple. First, randomly generate an initial mapping sequence, denoted as $\mathbf{c}_{\text{ini}}$ such that $\mathbf{X}$ is mapped to a $\mathcal{Q}_{2M}$ symbol sequence $\mathbf{X}_{\text{ini}} = \mathbf{c}_{\text{ini}} \odot \mathbf{X}$. Then, PTS technique can be used on $\mathbf{X}_{\text{ini}}$ to reduce its PICR.

As an example, Figure 3 illustrates the PICR CCDFs for uncoded $\mathcal{Q}_M^{(1)}$ symbol sequences, uncoded $\mathcal{Q}_{2M}$ symbol sequences, PTS coded $\mathcal{Q}_M^{(1)}$ symbol sequences, and PTS coded $\mathcal{Q}_{2M}$ symbol sequences. Here, $N = 64$ and $M = 8$ are used. When using PTS coding, each symbol sequence is partitioned into 4 subblocks ($G = 4$). From this figure one can see that, the PICR for $\mathcal{Q}_M^{(1)}$ symbol sequences is quite high no matter if the PTS technique is used or not. On the other hand, with the use of initial mapping sequence $\mathbf{c}_{\text{ini}}$, the PICR can be significantly reduced.

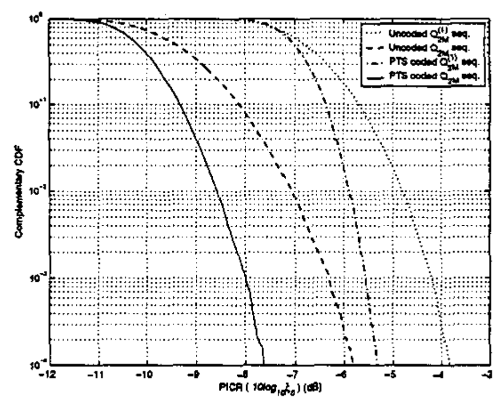


Fig. 3. PICR comparison between $\mathcal{Q}_M^{(1)}$ and $\mathcal{Q}_{2M}$ symbol sequences

### C. Recursive Partial Sequence (RPS) Method

This method is based on the recursive optimization of the partial symbol sequence. Given a symbol sequence $\mathbf{X}$ defined in Eqn. (7), let us form a partial symbol sequence $\tilde{\mathbf{X}}$ and a corresponding mapping sequence $\tilde{\mathbf{c}}$, respectively, as

$$\tilde{\mathbf{X}} = [X_0,\ X_1], \tag{10}$$

$$\tilde{\mathbf{c}} = [c_0,\ c_1], \tag{11}$$

where $c_0 \equiv 1$. Then, $c_1$ can be found by minimizing the PICR of $\tilde{\mathbf{c}} \odot \tilde{\mathbf{X}}$. After finding $c_1$, $\tilde{\mathbf{X}}$ and $\tilde{\mathbf{c}}$ are then updated, respectively, to accommodate anther symbol $X_2$ and the corresponding symbol sign $c_2$

$$\tilde{\mathbf{X}} = [X_0, X_1, X_2], \quad (12)$$

$$\tilde{\mathbf{c}} = [c_0, c_1, c_2]. \quad (13)$$

Similarly, $c_2$ can be found by minimizing the PICR of the updated $\tilde{\mathbf{c}} \odot \tilde{\mathbf{X}}$. This procedure is repeated until $c_{N-1}$ is found. Generally, this method can be written as

$$\begin{aligned} &\text{FOR} \quad k=1 \quad \text{TO} \quad N-1 \\ &\quad \tilde{\mathbf{X}} \triangleq [X_0, ..., X_k], \quad \tilde{\mathbf{c}} \triangleq [c_0, ..., c_k] \\ &\quad c_k = \arg\min_{c_k} \left(\text{PICR of } \tilde{\mathbf{c}} \odot \tilde{\mathbf{X}}\right) \\ &\text{END FOR} \end{aligned} \quad (14)$$

By using this method, the search space consists of only $2(N-1)$ possible symbol sign choices. Furthermore, the $k$-th search requires $(k+1)$ multiplications and $(k+1)$ additions $(1 \leq k \leq N-1)$. Compared to the complete search ($2^{N-1}$ possible choices, each search requiring $N$ multiplications and $N$ additions), the proposed method significantly reduces the computational complexity.

## V. Simulation Results

In this section, numerical simulations are performed to demonstrate the effect of the adaptive mapping technique using suboptimal methods. The simulation parameters are listed in Table I. These parameters are selected such that the search space is roughly the same for all three cases using adaptive mapping technique, and the bit-rate of the transmitted signal (including redundancy) is the same ($= 256/T$) for all cases. However, if the redundancy is not taken into account, the bit-rate of the rate-half technique is the lowest. In the following simulations, the weighting factor for each subblock is either 1 or -1, and $\varepsilon = 0.1$ is used.

TABLE I

SIMULATION PARAMETERS

| Techniques | | $N$ | $M$ | Bit-rate (bps) | Other parameters |
|---|---|---|---|---|---|
| Uncoded | | 64 | 16 | $256/T$ | |
| Adaptive Mapping | Modified PTS | 64 | 8 | $192/T$ | $G = 8$ |
| | Random Selection | 64 | 8 | $192/T$ | $K = 126$ |
| | RPS | 64 | 8 | $192/T$ | |
| Rate-Half | | 32 | 16 | $128/T$ | |

Figure 4 illustrates the PICR CCDFs for the techniques listed in Table I. It is clear that all these methods can significantly reduce the PICR. At the CCDF level of $10^{-4}$, the RPS method can obtain 5.5dB PICR reduction, which is about 0.8dB better than the random selection method, and 2.3dB better than the modified PTS method. On the other hand, since the RPS method is just a suboptimal solution of the adaptive mapping technique, its PICR reduction is about 3.5dB lower than that of the rate-half technique. However, the bit-rate of RPS method is higher than that of the rate-half technique, and can be increased when using larger signal constellations.

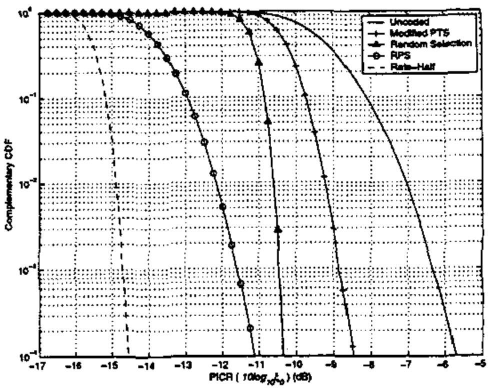


Fig. 4. PICR CCDFs for (1) the uncoded input sequences, (2) coded sequences using the modified PTS method, (3) coded sequences using the random selection method, (4) coded sequences using the RPS method, and (5) coded sequences using the rate-half technique

## VI. Conclusion

In this paper, a novel ICI reduction technique for OFDM systems has been developed. By adaptively mapping an $M$-point constellation to a $(2M)$-point constellation, extra freedom can be obtained to reduce the PICR. A great advantage of this technique is that no side information is required at the receiver side. The random selection, the modified PTS, and the recursive partial sequence methods have also been proposed in this paper to find suboptimal mapping sequences with significantly lowered computational complexity. Simulations show that the proposed technique can significantly reduce the PICR.

## VII. Acknowledgement


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