Equal Gain Combiner Performance in Equally-Correlated Rayleigh Fading Channels

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***Abstract*—This paper develops a novel approach for performance analysis of multi-branch pre-detection equal gain combining (EGC) in equally-correlated Rayleigh fading channels. We convert a set of equally-correlated channel gains to a set of conditionally independent channel gains. The cumulative distribution function (cdf) of EGC output signal-to-noise ratio (SNR) is therefore derived. The symbol error rate (SER) of different modulation schemes with pre-detection EGC in equally-correlated Rayleigh fading channels is also evaluated. Numerical results that illustrate the effects of equally-correlated fading on the SER performance of EGC are also provided.**

Key words: Diversity, equal-gain combining, equally-correlated Rayleigh fading channels, cumulative distribution function

I. INTRODUCTION

In recent years, diversity reception (by combining L branch signals) and multi-level modulation schemes have received considerable attention for facilitating high-rate data transmission over wireless links. Pre-detection EGC is of practical interest because it offers performance close to the optimal maximal-ratio combining (MRC) technique but with greater simplicity. However, the literature on the performance analysis of EGC is barren compared with that of other diversity combining schemes. This lack stems from the difficulty of finding the pdf of the EGC output SNR (traditionally, the performance is evaluated by averaging the conditional error probability over the EGC output pdf), which depends on the square of a sum of fading amplitudes. Even for independent fading, a closed-form solution to the pdf of this sum has been elusive (dating back to Lord Rayleigh himself [1]) and indeed, even for the case of Rayleigh fading (mathematically simplest distribution) no solution exists for $L > 2$ [2].

Previous studies on independent EGC include the following. Altman and Sichak [3] find the output pdf of dual-branch EGC in Rayleigh fading channels. Due to Beaulieu [2], the pdf of a sum of independent random variables (RV's) can be efficiently approximated. Applying this result, the performances of EGC in Rician and Nakagami fading channels are analyzed [4], [5]. Annamalai *et al.* [6]–[8] use the Parseval theorem to evaluate the average SER of a broad class of coherent, differentially coherent and non-coherent modulation schemes with pre-detection EGC.

However, in real-life applications, statistically independent fading can rarely be achieved. For example, in wireless communication handsets, physical constraints may not allow the use of antenna spacing that is required for independent fading across antennas. Thus quantifying the resultant degradation of the performance of diversity systems is a long standing problem of importance [9]. However, for correlated fading channels, all available results on pre-detection EGC deal with dual branches ($L = 2$) (see [10]–[12]). Mallik *et al.* [10] derive the bit error rate (BER) for dual-branch EGC with coherent detected binary signals in correlated Rayleigh fading channels. Tellambura and Annamalai [11] derive the characteristic function (chf) of EGC output and utilize an infinite series representation for the complementary error function to obtain the BER. In [12], an integral representation for the Gaussian probability integral is used to derive the BER. To the best of our knowledge, no results have ever been reported for the BER/SER performance of multi-branch ($L > 2$) pre-detection EGC in correlated fading channels. In this paper, we develop a novel approach to analyze the performance of multi-branch pre-detection EGC in equally-correlated Rayleigh fading channels.

This paper is organized as follows. Section II develops a new representation for equally-correlated Rayleigh channel gains. Section III derives the output cdf of EGC in equally-correlated Rayleigh fading channels. Section IV evaluates the SER of various digital modulation schemes with pre-detection EGC and also approximate the SER using the Gauss-Laguerre quadrature. Numerical results in Section V illustrate the impact of branch correlation on the performance of pre-detection EGC. Section VI concludes this paper.

II. REPRESENTATION OF CHANNEL GAINS

Rayleigh envelopes are frequently used to model the amplitudes of received signals from urban and suburban areas [13]–[15]. We represent Rayleigh envelopes by using a set of zero-mean complex Gaussian RV's given by

$$G_k = (\sqrt{1-\rho}\,X_k + \sqrt{\rho}\,X_0) + i(\sqrt{1-\rho}\,Y_k + \sqrt{\rho}\,Y_0), \quad (1)$$

for $k = 1, \ldots, L$, where $i = \sqrt{-1}$, $0 \leq \rho \leq 1$, X_k and Y_k, ($k = 0, 1, \cdots, L$), are two sets of independent zero-mean and unit-variance Gaussian RV's. That is, for any $j, k \in$

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$\{0, \ldots, L\}$,

$$E(X_k Y_j) = 0, \tag{2}$$

$$E(X_k X_j) = E(Y_k Y_j) = \begin{cases} 1, & k = j, \\ 0, & k \neq j, \end{cases} \tag{3}$$

where $E(x)$ denotes the statistical expectation of x.

We may write G_k in terms of polar coordinates,

$$G_k = R_k \exp(j\theta_k), \quad k = 1, 2, \cdots, L, \tag{4}$$

where $R_k = |G_k|$ is the amplitude of the G_k. Since G_k is a set of complex Gaussian RV's with zero means, R_k is a set of Rayleigh envelopes with mean square

$$E(R_k^2) = 2. \tag{5}$$

The cross-correlation coefficient between any G_k and G_j ($k \neq j$) equals to ρ,

$$\frac{E(G_k G_j^*)}{\sqrt{E(|G_k|^2) E(|G_j|^2)}} = \rho, \quad k \neq j. \tag{6}$$

The relationship between the power correlation (i.e. the correlation between G_k and G_j) and envelope correlation (i.e. the correlation between the R_k and R_j) is [1, Eq. (1.5-26)],

$$\rho_e = \frac{(1+\rho) E_i\left(\frac{2\sqrt{\rho}}{1+\rho}\right) - \frac{\pi}{2}}{2 - \frac{\pi}{2}}, \tag{7}$$

where $E_i(\eta)$ denotes the complete elliptic integral of the second kind with modulus η. For a given ρ_e, solving (7) yields ρ. The solution methods are discussed in [16], [17]. Thus, using (1) and (7), we can readily represent a set of equally-correlated Rayleigh envelopes with a specified value of envelope correlation.

Next, we introduce a 'trick' that will be used in the performance analysis. When $X_0 = x_0$ and $Y_0 = y_0$ are fixed, G_k is a set of complex Gaussian RV's with means $\sqrt{\rho}(x_0 + iy_0)$. Consequently, R_k is a set of identically and independent distributed (iid) Rician RV's with Rician factor $\rho(x_0^2+y_0^2)/2/(1-\rho)$. Performance analysis can now be carried out in two steps. First, we evaluate the conditional performance of EGC for a given $x_0^2+y_0^2$. Second, we average the conditional results over the distribution of $X_0^2 + Y_0^2$.

III. Derivation of EGC Output CDF

Using the new representation for the channel gains (1), we derive the expression for the cdf of balanced branch EGC output SNR in equally-correlated Rayleigh fading channel. We assume the receive signals at different branches to be identically distributed and equally-correlated with each other. The noise components at different branches are assumed to be independent of the signal components and uncorrelated with each other.

In pre-detection EGC, received signals are first co-phased and then summed to form the resultant output. The instantaneous SNR at the output can be written as

$$\gamma_{egc} = \frac{(R_1 + R_2 + \cdots + R_L)^2 E_s}{L N_0} \tag{8}$$

where L denotes the diversity order, N_0 is the noise power spectral density per branch and E_s is the energy of transmitted signal.

We consider X_0 and Y_0 to be fixed ($X_0^2 + Y_0^2 = U$). Then $R = R_1 + R_2 + \cdots + R_L$ is a sum of L iid Rician RV's with Rice factor and average power given by

$$K = \frac{\rho u}{2(1-\rho)}, \tag{9}$$

$$\Omega = 2(1-\rho) + \rho u. \tag{10}$$

Evaluating the output cdf for a fixed $x_0^2 + y_0^2 = u$, we obtain the conditional output cdf,

$$\begin{aligned} F_{\gamma_{egc}}(y|_{x_0^2+y_0^2=u}) &= \Pr\left(\frac{r^2 E_s}{L N_0} \leq y \bigg|_{x_0^2+y_0^2=u}\right) \\ &= \Pr\left(r \leq \sqrt{\frac{L N_0 y}{E_s}} \bigg|_{x_0^2+y_0^2=u}\right) \\ &= F_R\left(\sqrt{\frac{2Ly}{\overline{\gamma}}} \bigg|_{x_0^2+y_0^2=u}\right), \end{aligned} \tag{11}$$

where $\overline{\gamma} = \frac{E_s}{N_0} E(R_k^2) = \frac{2E_s}{N_0}$ is the average branch SNR, and $F_R(x)$ is the cdf of R which can be computed using a convergent infinite series [2]. The computation for $F_R(x)$ has been discussed in detail [5, Eq.(9,14,15)]. For brevity, we omitted those formulas here.

Notice that $X_0^2 + Y_0^2$ is chi-square distributed with two degrees of freedom and its pdf can be found [18]

$$p(u) = \frac{1}{2} e^{-u/2}, \quad u \geq 0. \tag{12}$$

Averaging the conditional cdf (11) over the distribution of U, we finally obtain the cdf for multi-branch EGC in equally correlated Rayleigh fading channel,

$$F_{\gamma_{egc}}(y) = \frac{1}{2} \int_0^\infty F_R\left(\sqrt{\frac{2Ly}{\overline{\gamma}}} \bigg|_{x_0^2+y_0^2=u}\right) e^{-u/2} du. \tag{13}$$

This is a novel result which enables the evaluation of the outage of multi-branch EGC systems in equally-correlated Rayleigh fading channels.

Using variable substitution $x = u/2$ and then applying Gauss-Laguerre quadrature, we obtain a series expression for (13)

$$F_{\gamma_{egc}}(y) \approx \sum_{m=1}^{M} w_m F_R\left(\sqrt{\frac{2Ly}{\overline{\gamma}}} \bigg|_{x_0^2+y_0^2=2x_m}\right) \tag{14}$$

where x_m is the m-th zero of Laguerre polynomials $L_M(x)$ and w_m is the corresponding weight factor given by [19]

$$\omega_m = \frac{(M!)^2 x_m}{(M+1)^2 [L_{M+1}(x_m)]^2}. \tag{15}$$

Tables for x_m and ω_m can be found in [19], [20].

IV. Performance Analysis

Following the same approach as the above, we may evaluate the average SER of various digital modulation schemes with pre-detection EGC in equally-correlated Rayleigh fading channels. We assume that channel fading is nonselective and changing slowly enough so that the channel parameters remain constant for the duration of signaling interval.

Recall that when X_0 and Y_0 are fixed ($X_0^2 + Y_0^2 = U$), R_k is a set of iid Rician RV's with Rician factor given by (9). Due to Annamalai *et al.* [7], we can readily obtain the conditional SER for a fixed $x_0^2 + y_0^2 = u$

$$P_s(\overline{\gamma}|_{x_0^2+y_0^2=u}) = \frac{2}{\pi} \int_0^{\pi/2} \frac{\Psi(\tan\zeta|_{x_0^2+y_0^2=u})}{\sin(2\zeta)} d\zeta \tag{16}$$

where $\Psi(\omega) = \mathrm{Real}[\omega G(\omega)\phi_r^*(\omega)]$ and where $\mathrm{Real}(x)$ denotes the real part of x, $*$ denotes conjugate operation. $G(\omega)$ is the Fourier transform (FT) of generic conditional error probability which can be found in [7] and $\phi_r(\omega)$ is the chf of the square root of output SNR ($\sqrt{\gamma_{egc}}$) given by [7]

$$\begin{aligned}
&\phi_r(\omega|_{x_0^2+y_0^2=u}) \\
&= \left[\frac{e^{-\frac{\rho u}{2(1-\rho)}}}{\pi} \int_0^{\pi} \exp\left(\left(j\frac{\omega\xi}{2} + \sqrt{\frac{\rho u}{1-\rho}} \frac{\cos\theta}{2} \right)^2 \right) \right. \\
&\qquad \left. \times D_{-2}\left(-j\omega\xi - \sqrt{\frac{\rho u}{1-\rho}} \cos\theta \right) d\theta \right]^L
\end{aligned} \tag{17}$$

where $j = \sqrt{-1}$, $\xi = \sqrt{\frac{(1-\rho)\overline{\gamma}}{2L}}$ and $D_{-2}(x)$ is parabolic cylinder function which is defined as [21, Eq. (9.240)].

Averaging the conditional SER (16) over the distribution of U (12), we obtain the average SER as

$$P_s(\overline{\gamma}) = \frac{1}{\pi} \int_0^{\infty} \int_0^{\pi/2} \frac{\Psi(\tan\zeta|_{x_0^2+y_0^2=u})}{\sin(2\zeta)} d\zeta e^{-u/2} du. \tag{18}$$

Therefore, we may evaluate the average SER of various modulation schemes with pre-detection EGC using [7, Eq. (11,13,16,17,20)] and (17) with (18). For example, the SER of differential binary phase-shift-keying (DPSK) can be obtained as

$$\begin{aligned}
P_s(\overline{\gamma}) = \frac{1}{\pi} \int_0^{\infty} \int_0^{\pi/2} \mathrm{Real} &\left\{ \frac{\tan\zeta}{2} \left[\frac{\sqrt{\pi}}{2} \exp\left(-\frac{\tan^2\zeta}{4} \right) \right.\right. \\
&\left.\left. + jF\left(\frac{\tan\zeta}{2}\right) \right] \phi_r^*(\tan\zeta|_{x_0^2+y_0^2=u}) \right\} \frac{e^{-u/2}}{\sin(2\zeta)} d\zeta \, du
\end{aligned} \tag{19}$$

where $F(x)$ denotes the Dawson integral [22].

Annamalai *et al.* [7] derive an approximation for $P_s(\overline{\gamma}|_{x_0^2+y_0^2=u})$ using Gauss-Chebychev quadrature. Based on their results and applying Gauss-Laguerre quadrature, we approximate the average SER as

$$P_s(\overline{\gamma}) \approx \frac{1}{N_1} \sum_{m=1}^{M} w_m \sum_{n=1}^{N_1} \frac{\Psi\left[\frac{1}{\sqrt{2}} \tan\left(\frac{(2n-1)\pi}{4N_1} \right) \Big|_{x_0^2+y_0^2=2x_m} \right]}{\sin\left(\frac{(2n-1)\pi}{2N_1} \right)} \tag{20}$$

where x_m and w_m are defined as (15) and $\phi_r(\omega|_{x_0^2+y_0^2=2x_m})$ can be approximated as [23]

$$\begin{aligned}
&\phi_r(\omega|_{x_0^2+y_0^2=2x_m}) \\
&\approx \left[\frac{e^{-\frac{\rho x_m}{(1-\rho)}}}{N_2} \sum_{n=1}^{N_2} \exp\left(\left(j\frac{\omega\xi}{2} + \sqrt{\frac{2\rho x_m}{1-\rho}} \frac{\cos\theta_n}{2} \right)^2 \right) \right. \\
&\qquad \left. \times D_{-2}\left(-j\omega\xi - \sqrt{\frac{2\rho x_m}{1-\rho}} \cos\theta_n \right) \right]^L
\end{aligned} \tag{21}$$

where $\theta_n = \frac{(2n-1)\pi}{2N_2}$. There is a tradeoff involved in the choice of N_1, N_2 and M. A greater accuracy may be obtained using larger values, but at the expense of increasing computation complexity.

V. Numerical Results

Numerical results are next given to illustrate the effect of correlation on the performance of pre-detection EGC in equally-correlated Rayleigh fading channels. In all the figures, ρ is the power correlation coefficient and $\overline{\gamma}$ is the average branch SNR.

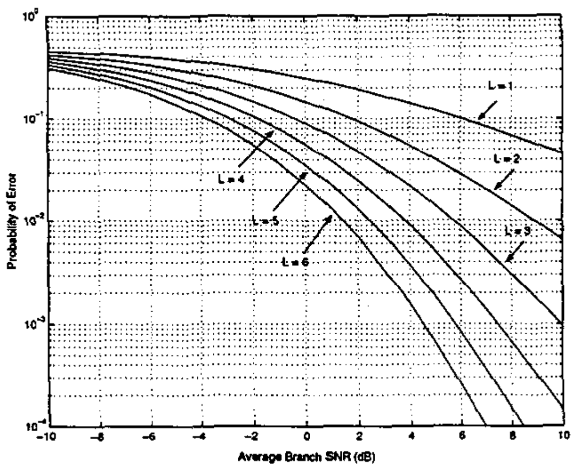

Fig. 1. Average error rate of DPSK with pre-detection EGC in equally-correlated Rayleigh fading channels;($\rho = 0.5$).

Fig. 1 shows the effect of diversity order L on the performance of DPSK with pre-detection EGC in equally-correlated Rayleigh fading channels with $\rho = 0.5$. The case of $L = 1$ represents a situation with no diversity. As expected, diversity gain can still be achieved even with correlated fading. The maximum diversity gain is achieved with dual-branch diversity. With increasing of L, diversity gain diminishes. This is more pronounced at a higher SNR level.

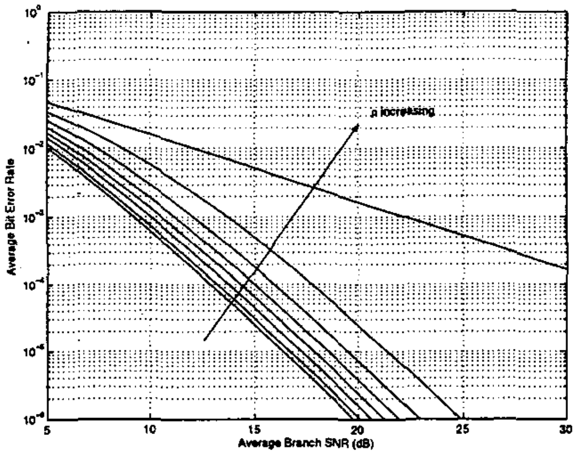

Fig. 2. Average error rate of DPSK with triple-branch pre-detection EGC in equally-correlated Rayleigh fading channels; ($L = 3$, $\rho = 0, 0.3, 0.5, 0.6, 0.7, 0.8, 0.9, 1$). Circles denote simulation results

Figs. 2 shows the effect of ρ on the performance of DPSK with triple-branch pre-detection EGC in equally-correlated Rayleigh fading channels. The case of $\rho = 0$ represents independent fading. Observe that the correlation between the branch signals results in large loss in performance. The diversity gain decreases as ρ increases.

VI. Conclusion

We have developed a new representation for equally-correlated Rayleigh channel gains. We showed that this representation enables the performance analysis of pre-detection EGC in such channels. Numerical results show that diversity gains can still be achieved in correlated Rayleigh fading channels.

The representation developed in this paper can also be used to analyze the performance of multi-branch SC and generalized SC schemes in equally-correlated Rayleigh fading channels. These results will be reported in a forthcoming paper.

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