A SIMPLIFIED INTERPOLATION EQUALIZATION TECHNIQUE FOR FILTERBANK-BASED DMT SYSTEMS

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ABSTRACT

This paper presents a simplified interpolation equalization technique for filterbank-based DMT systems. In DMT systems, the channel effect in each subchannel can be approximated as a fractional delay, which can be compensated for by using the multi-tap interpolation equalization technique. By using a large number of polyphase components of the analysis filter output, and using the same polyphase index set for all subchannels, a simplified (suboptimal) interpolation equalization technique is proposed. This technique gives rise to a high transmission bit-rate close to the optimal solution of the interpolation equalization technique. It is also suitable for the timing synchronization.

Keywords: *Interpolation, Equalization, DMT, Timing Synchronization*

1. INTRODUCTION

Filterbank-based Discrete Multitone (DMT) systems are a set of multicarrier modulation/demodulation systems used in Digital Subscriber Line (DSL) applications for high-speed transmissions over commonly used twisted-pair copper lines. Examples of DMT systems are the conventional DMT system [1], Filtered Multitone [2], and Discrete Wavelet Multitone [3]. To combat the interferences caused by the the non-ideal communication channel, various channel equalization techniques were proposed for DMT systems, e.g. the Time-domain Equalization (TEQ) [1], the per tone equalization [4], and the output combiner [3]. However, these equalization techniques either involve high computational complexities or cannot fully equalize the channel (therefore, have to combine with other methods such as the cyclic prefix).

In [5], a novel multi-tap interpolation equalization technique was proposed for filterbank-based DMT systems. This technique equalizes the channel fractional delay in each subchannel by combining a set of polyphase components of the analysis filter outputs, leading to a high SNR at a reasonable equalizer length. In this paper, a simplified interpolation equalization technique is proposed to further simplify the equalizer training cost at a minor loss in SNR. This simplification is achieved by using a large number of polyphase components of the analysis filter outputs, and using a same polyphase component index set for all subchannels. Simulation results show that the proposed technique is insensitive to the choice of the optimal polyphase component index set. In addition to the channel equalization, this technique can also be used to combat the sampling timing offset to achieve the timing synchronization between the transmitter and the receiver.

2. FILTERBANK-BASED DMT SYSTEMS

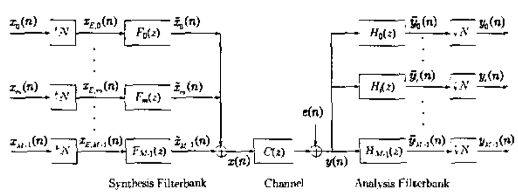

Figure 1: Block diagram of filtebank-based DMT systems

The block diagram of an M-subchannel filterbank-based DMT system is shown in Figure 1, where $F_m(z)$ and $H_i(z)$ $(m, i = 0, \ldots, M-1)$ represent the synthesis and analysis filterbanks, respectively, $C(z)$ represents the channel, and $e(n)$ represents the additive channel noise. In this figure, N represents the expansion/decimation ratio. When $N = M$ $(N > M)$, the analysis and synthesis filterbanks are said to be critically (non-critically) decimated filterbanks [6]. In this paper, only critically decimated filterbanks are considered and M will be used as the expansion/decimation ratio in the following discussion.

By defining $G_{mi}(z)$ and $S_{mi}(z)$ as

$$G_{mi}(z) \; , \; F_m(z)H_i(n)\,, \tag{1}$$

$$S_{mi}(z) \; , \; G_{mi}(z)C(z)\,, \tag{2}$$

the analysis filter output can be expressed as

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$$\tilde{Y}_i(z) = \sum_{m=0}^{M-1} X_m(z^M) S_{mi}(z) + E(z) H_i(z) . \quad (3)$$

Then, the system output can be written as

$$Y_i(z) = \sum_{m=0}^{M-1} X_m(z) \left(S_{mi}(z)\right)_{\downarrow M} + \left(E(z) H_i(z)\right)_{\downarrow M} , \quad (4)$$

where $(\cdot)_{\downarrow M}$ represents decimation by M. Usually the synthesis/analysis filterbanks are biorthogonal [6]

$$\left(G_{mi}(z)\right)_{\downarrow M} = \delta(m-i) . \quad (5)$$

Then, in absence of channel effect ($C(z) = 1$, $e(n) = 0$), the filterbank-based DMT system is a Perfect Reconstruction (PR) system ($y_i(n) = x_i(n)$). However, in practical situations, $S_{mi}(z) \neq G_{mi}(z)$ due to the non-ideal channel. In this case, $y_i(n)$ will be distorted because of ICI and ISI.

3. MULTI-TAP INTERPOLATION EQUALIZATION

It is well known that when M is large, the frequency response of each subchannel can be approximated as a piecewise flat magnitude and piecewise linear phase in accordance with

$$C(e^{j\omega}) = |C(e^{j\omega})| e^{-j\phi(\omega)} \approx C_i e^{-j d_i \omega}, \omega_1 \leq \omega \leq \omega_2 , \quad (6)$$

where $|C(e^{j\omega})|$ represents the channel magnitude response (approximated as a constant number C_i), and $\phi(\omega)$ represents the channel phase response (approximated as linear phase $d_i \omega$). Note that d_i is not necessarily an integer, it was shown in [5, 7] that the channel fractional delay d_i plays a dominant role the the received signal distortion. Specifically, only when $d_i = JM$ (J is an integer), the correct samples can be picked out by the decimator. Otherwise, the samples picked out by the decimator are indeed those which should have been discarded, while the samples discarded include the correct samples. Therefore, by modifying the DMT receiver, it should be possible to pick out the correct samples for equalization. Moreover, by using a bilinear interpolation of the correct samples, it should be possible to compensate for the fractional delay in each subchannel.

Figure 2 illustrates the block diagram of the modified DMT system, where at the receiver, each decimator is replaced by a multi-tap interpolation equalizer EQ_i, which is shown in Figure 3. In Figure 3, a Serial-to-Parallel (S/P) convertor is used to parse $\tilde{y}_i(n)$ into its polyphase components with respect to M. Then, L successive polyphase components $\tilde{y}_i^{(l_i+k)}(n)$ ($k = 0, \ldots, L-1$, and $l_i \geq 0$ is a design parameter) of $\tilde{y}_i(n)$ are selected by an M-input-L-output selector, and are subsequently passed through a bank of multi-tap interpolation equalizers $A_{i,k}(z)$ ($k = 0, \ldots, L-1$). The recovered signal is obtained by summing the outputs of $A_{i,k}(z)$ together.

A detailed discussion can be found in [7]. By assuming that, a) all $x_i(n)$ are mutually uncorrelated, b) $e(n)$ is AWGN with a constant Power Spectral Density (PSD) σ_e^2, and c) each $x_i(n)$ is uncorrelated with $e(n)$, the optimal

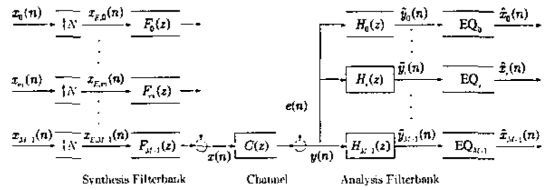

Figure 2: The modified DMT system

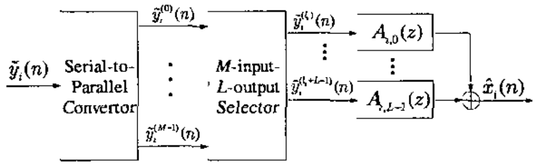

Figure 3: The i-th multi-tap interpolation equalizer

equalizer coefficients can be found by minimizing the Mean Squared Error (MSE) between $x_i(n)$ and $\hat{x}_i(n)$, in accordance with

$$\mathbf{A}_{i,\mathrm{opt.}} = \left(\sum_{m=0}^{M-1} \mathbf{S}_{mi} \mathbf{R}_{xx}^{(m)} \mathbf{S}'_{mi} + \sigma_e^2 \sum_{m=0}^{M-1} \mathbf{H}_{mi} \mathbf{H}'_{mi} \right)^{-1} \cdot \mathbf{S}_{ii} \mathbf{R}_{xx}^{(i)} \mathbf{u}_{k_i} , \quad (7)$$

with the following notations: L_s, L_a, and ML_h represent the lengths of $s_{mi}^{(l_i+k)}(n)$, $a_{i,k}(n)$, and $h_i(n)$, respectively, $(\cdot)^T$ and $(\cdot)'$ represent matrix transpose and conjugate transpose, respectively, $\mathbf{R}_{xx}^{(m)}$ represents the autocorrelation matrix of $\mathbf{x}_m(n)$ ($= [x_m(n), \ldots, x_m(n - L_s - L_a + 2)]^T$), $\mathbf{u}_{k_i} = \left[\mathbf{0}_{1\times k_i} \quad 1 \quad \mathbf{0}_{1\times(L_s-k_i-1)}\right]'$ (k_i is the relative delay between $x_i(n)$ and $\hat{x}_i(n)$), and

$$\mathbf{A}_i = \begin{bmatrix} \mathbf{A}_{i0} \\ \vdots \\ \mathbf{A}_{i(L-1)} \end{bmatrix}, \mathbf{S}_{mi} = \begin{bmatrix} \mathbf{S}_{mi}^{(0)} \\ \vdots \\ \mathbf{S}_{mi}^{(L-1)} \end{bmatrix}, \mathbf{H}_{mi} = \begin{bmatrix} \mathbf{H}_{i0}^{(m)} \\ \vdots \\ \mathbf{H}_{i(L-1)}^{(m)} \end{bmatrix},$$

where

$$\mathbf{A}_{ik} = \begin{bmatrix} a_{i,k}(0) & \cdots & a_{i,k}(L_a-1) \end{bmatrix}^T , k = 0, \ldots L-1,$$

$\mathbf{H}_{ik}^{(m)}$ and $\mathbf{S}_{mi}^{(k)}$ ($k = 0, \ldots L-1$) are defined in Eqns (8) and (9), respectively.

4. SIMPLIFIED INTERPOLATION EQUALIZATION

The resulting optimal equalizer coefficient matrix $\mathbf{A}_{i,opt.}$ depends on the choice of l_i. Generally, a search for optimal l_i is required for each subchannel in equalizer training stage. However, because only a small subset of the M polyphase components of the decimator output signals are critical for the equalization of each subchannel, it may be argued that some of the polyphase components are suitable ("relevant") for subchannel equalization while others are not suitable ("irrelevant"). Once the "relevant" polyphase components are included in the polyphase component set used for multi-tap interpolation equalization, the output SNR will be high

$$
\mathbf{H}_{ik}^{(m)} = \begin{bmatrix} h_{ik}^{(m)}(0) & \cdots & h_{ik}^{(m)}(L_h-1) & 0 & \cdots & 0 \\ 0 & h_{ik}^{(m)}(0) & \cdots & h_{ik}^{(m)}(L_h-1) & 0 & \cdots \\ \vdots & & \vdots & & & \vdots \\ 0 & \cdots & 0 & h_{ik}^{(m)}(0) & \cdots & h_{ik}^{(m)}(L_h-1) \end{bmatrix}, \quad (8)
$$

$$
\mathbf{S}_{mi}^{(k)} = \begin{bmatrix} s_{mi}^{(l_i+k)}(0) & \cdots & s_{mi}^{(l_i+k)}(L_s-1) & 0 & \cdots & 0 \\ 0 & s_{mi}^{(l_i+k)}(0) & \cdots & s_{mi}^{(l_i+k)}(L_s-1) & 0 & \cdots \\ \vdots & & \vdots & & & \vdots \\ 0 & \cdots & 0 & s_{mi}^{(l_i+k)}(0) & \cdots & s_{mi}^{(l_i+k)}(L_s-1) \end{bmatrix}. \quad (9)
$$

(not necessarily the highest) regardless of how many and which "irrelevant" polyphase components are also included in the set. In this way, by combining more polyphase components, the index set of the optimal polyphase components will have a larger range, and the output SNR will be less sensitive to the choice of the used polyphase component set. Moreover, experiments show that the difference of the optimal polyphase component index set between most subchannels is relatively small. Therefore, by combing a large number of polyphase components in equalization, one can expect that the index sets of the optimal polyphase components for all subchannels are the same (or similar).

Based on the above discussion, a simplified interpolation equalization technique can be developed by using a large L and using the same polyphase component index set for all subchannel equalizations in accordance with $l_i \equiv l$ $(0 \leq i \leq M-1)$. It can be shown that this technique is also the generalization of the per tone equalization technique.

5. SIMULATION RESULTS

A computational investigation has been carried out to confirm the validity of the simplified interpolation equalization technique by comparing the transmission bit-rates

$$
b = \frac{f_s}{M} \sum_{m=0}^{M-1} \log_2(1 + 10^{\mathrm{SNR}_{\mathrm{marg.},i}}), \qquad (10)
$$

for various choices of l, where $\mathrm{SNR}_{\mathrm{marg.},i}$ is conventionally defined in dB as

$$
\mathrm{SNR}_{\mathrm{marg.},i}(\mathrm{dB}) \triangleq \mathrm{SNR}_i(\mathrm{dB}) + \gamma_c(\mathrm{dB}) - \gamma_n(\mathrm{dB}) - \Gamma_g(\mathrm{dB}), \quad (11)
$$

with the coding gain $\gamma_c(\mathrm{dB}) = 3\mathrm{dB}$, noise margin $\gamma_n(\mathrm{dB}) = 6\mathrm{dB}$, and SNR gap $\Gamma_g(\mathrm{dB}) = 9.8\mathrm{dB}$. The channel used in the simulation is CSA loop #1 and its magnitude response can be found in Figure 4. The filterbank used in the simulation is the Cosine-Modulated Filterbank (CMFB). The parameters used in the simulation are as listed in Table 1. Figure 5 illustrates the transmission bit-rate as a function of the optimal polyphase component index set l for CSA loop #1. The simulation parameters are the same as those in Table 1 and with $L_a = 4$. For the sake of comparison, the corresponding transmission bit-rates for the multi-tap interpolation equalization technique using the optimal polyphase component index set are tabulated in Table 2.

Table 1: Simulation parameters

M (Number of subchannels)		64
Length of the prototype filter		768
Sampling frequency f_s		2.208M Hz
$x_i(n)$	Mean	0
$(0 \leq i \leq M-1)$	PSD (dBm/Hz)	-40
Channel noise	Mean	0
	PSD (dBm/Hz)	-140

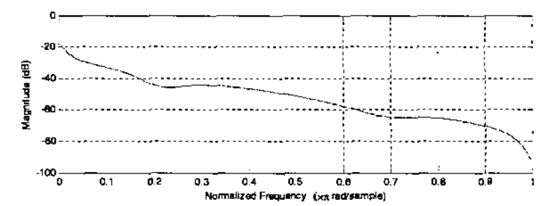

Figure 4: Magnitude response of CSA loop #1

Table 2: Transmission bit-rate for multi-tap interpolation equalization using optimal polyphase component index set

Equalizer settings	Bit-rate (Mb/s)
$L=16, L_a=4$	11.84
$L=8, L_a=4$	11.26
$L=3, L_a=4$	11.02

By comparing the curves in Figure 5 and the entries in Table 2, it becomes clear that the transmission bit-rate for the suboptimal interpolation equalization becomes close to the bit-rate for the optimal multi-tap interpolation equalization for large L. Moreover, the transmission bit-rate becomes less sensitive to the specific choice of the polyphase component index set l with increasing L. Therefore, a further simplification of the interpolation equalization can be made by selecting the optimal l from a pre-selected index range instead of an exhaustive search. The pre-selected index range can be found by using some methods similar to the intuitive search for optimal decision delay in the conventional DMT TEQ techniques.

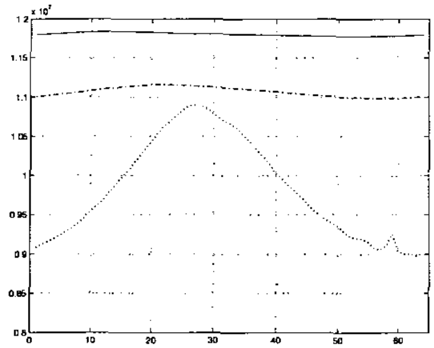

Figure 5: Bit-rate as a function of polyphase index set l for CSA loop #1, with $L_a = 4$; a) for $L = 16$ (—); b) for $L = 8$ (— ·); and c) for $L = 3$ (· · ·)

6. TIMING SYNCHRONIZATION USING SIMPLIFIED INTERPOLATION EQUALIZATION

In DMT systems, the phase difference between the sampling clock at the transmitter and that at the receiver will introduce interferences in the received signal. This timing synchronization problem is less critical in the conventional DMT systems because the phase error caused by the timing offset can be treated as an additional complex gain, which can be compensated for by using the Frequency Domain Equalizer (FEQ). Moreover, if the timing offset is small so that the channel impulse response is shorter than the length of cyclic prefix, the phase error will not destroy the subcarrier orthogonality and no interference will be introduced [8]. However, in other filterbank-based DMT systems, the timing offset has to be carefully compensated for since no cyclic prefix is used.

In hitherto literature, usually the timing offset is first estimated and a timing recovery loop is used to achieve the timing synchronization, which will increase the implementation cost of the DMT systems. On the other hand, the timing offset problem can be incorporated into the channel effect, and then the simplified interpolation equalization technique can be used to achieve the timing synchronization and equalize the channel at the same time without any additional implementation cost.

Let $y_c(t)$ represents the received continuous-time signal. Then after Nyquist sampling, the corresponding discrete-time signal $y(n)$ can be written as

$$y(n) = y(nT) = y_c(t_0 + nT) = y_c(T(n + \frac{t_0}{T})), \quad (12)$$

where T represents the sampling period, and t_0 is the timing offset. Therefore, the timing offset indeed introduces a (usually fractional) delay $d \triangleq \frac{t_0}{T}$, which is constant for all subchannels. Based on this fact, one can incorporate the timing offset into the channel effect, and use the simplified interpolation equalization to combat both the timing offset and the channel distortion. In this case, the equivalent subchannel delay that the interpolation equalizer has to compensate for becomes $d_i + d$. As discussed in [7], the exact value of d_i is not necessary for the channel equalization, as long as the channel impulse response is known. Also, by using pilot symbols in the training stage, the exact value of d is also unnecessary for the timing synchronization using the simplified interpolation equalization.

7. CONCLUSION

In this paper, a simplified interpolation equalization technique is proposed. This technique gives rise to a high transmission bit-rate close to the optimal solution of the multitap interpolation equalization technique. In addition to the channel equalization, this technique is also suitable for timing synchronization.

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