Coding to Reduce Both PAR and PICR of an OFDM Signal

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***Abstract*—The two major drawbacks in orthogonal frequency-division multiplexing (OFDM) system are high peak-to-average power ratio (PAR) and intercarrier interference (ICI) due to frequency offset errors. Several techniques proposed in the literature treat these limitations as two separate problems. We introduce the peak interference-to-carrier ratio (PICR) to measure ICI. This paper shows the existence of codes to reduce both the PAR and PICR simultaneously. The coding scheme is based on selecting only those messages with both low PAR and PICR as valid codewords. We identify those codes by computer search. As an example of an explicit construction for such codes, we construct Golay complementary repetition codes that can reduce both PAR and PICR simultaneously.**

***Index Terms*—Carrier frequency offset (CFO), intercarrier interference (ICI), orthogonal frequency division multiplexing (OFDM), peak interference-to-carrier ratio (PICR), peak-to-average power ratio (PAR).**

I. Introduction

ORTHOGONAL frequency-division multiplexing (OFDM) is an attractive transmission technique for the support of high-data-rate wireless communications systems. However, the OFDM signal has a large dynamic signal range [high peak-to-average power ratio (PAR)] due to large number of subcarriers. These high signal peak can lead to in-band distortion and spectral spreading in the presence of nonlinear devices. It is possible to use block codes for reducing PAR. As few data frames result in large signal peaks, block codes can be designed so that such data frames are never used [1]. Consequently, PAR is reduced. Recently, several authors have further developed block coding to reduce PAR [2]–[4].

Another drawback of OFDM is intercarrier interference (ICI), which is caused by misalignment in carrier frequencies between the transmitter and receiver or by Doppler shift [5]–[9]. Several techniques have been proposed for reducing ICI [5], [6], [8]–[11]. In [5], [6], each data symbol is transmitted on two adjacent subcarriers with opposite polarity in order to cancel ICI. We refer to this scheme in this paper as rate-half repetition coding.

We introduce peak interference-to-carrier ratio (PICR) as an effective measure for ICI effects at the receiver. However, PICR is used at the transmitter to predict ICI effects at the receiver. Thus, the definition of PICR formulates the ICI problem parallel to that of the PAR issue.

Previous coding techniques do not reduce both PAR and ICI simultaneously. For example, rate-half repetition codes [5], [6] reduce ICI but their PAR characteristics are worse than the uncoded case. Golay complementary sequences [3] offer maximum PAR of 3 dB but their PICR characteristics are the same as the uncoded case. To the best of our knowledge, a coding technique that limits PAR and PICR together has not been reported before. This motivates our work in this paper.

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II. OFDM Issues

A. PAR Problem

The complex baseband OFDM signal may be represented as

$$s(t) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} c_k e^{j2\pi k \Delta f t}, \quad 0 \le t \le T \tag{1}$$

where $j^2 = -1$ and N is the total number of subcarriers. The frequency separation between any two adjacent subcarriers is $\Delta f = 1/T$ and T is the OFDM symbol duration. c_k is the data symbol for the kth subcarrier. We shall refer to $\mathbf{c} = (c_0, c_1, \ldots, c_{N-1})$ as a data frame or codeword.

The PAR of the transmitted signal in (1) can be defined as

$$\mathrm{PAR}(\mathbf{c}) = \frac{\max |s(t)|^2}{E[|s(t)|^2]} \tag{2}$$

where $E[\cdot]$ denotes the average.

B. PICR Problem

We assume that $s(t)$ is transmitted on an additive white Gaussian noise channel, and so the received signal sample for the kth subcarrier after discrete Fourier transform (DFT) demodulation can be expressed as

$$y_k = c_k S_0 + \sum_{l=0,\ l \ne k}^{N-1} S_{l-k} c_l + n_k, \quad k = 0, \ldots, N-1 \tag{3}$$

where n_k is a complex Gaussian noise sample (with its real and imaginary components being independent and identically distributed with variance σ^2). The second term in (3) is the ICI term attributable to the CFO. The sequence S_k (the ICI coefficients) depends on the CFO and is expressed as

$$S_k = \frac{\sin \pi(k+\varepsilon)}{N \sin \frac{\pi}{N}(k+\varepsilon)} \exp\left[j\pi \left(1 - \frac{1}{N}\right)(k+\varepsilon)\right] \tag{4}$$

where ε is the normalized frequency offset defined as a ratio between the frequency offset (which remains constant over each symbol period) and the subcarrier spacing. For a zero frequency offset, S_k reduces to the unit impulse sequence. The ICI term can be separated from (3) and expressed as

$$I_k = \sum_{l=0,\ l \ne k}^{N-1} S_{l-k} c_l, \text{ for } 0 \le k \le N-1. \tag{5}$$

Note that I_k is a function of both c_k and ε. The admissible sequences $\mathbf{c}$ are called codewords, and the ensamble of all possible codewords is a code $\mathcal{C}$.

We define the Peak Interference-to-Carrier Ratio (PICR) as

$$\mathrm{PICR}(\mathbf{c}, \varepsilon) = \max_{0 \le k \le N-1} \left\{ \frac{|I_k|^2}{|S_0 c_k|^2} \right\}. \tag{6}$$

PICR is the maximum interference-to-signal ratio for any subcarrier. In other words, it specifies the worst-case ICI on any subcarrier. To reduce ICI effects, (6) should be minimized. This definition (6) is similar to PAR [1]–[4]. However, we need to know ε to compute PICR at the transmitter. Although the absolute PICR is a function of ε, we can design codes in which the amount of PICR reduction is independent of ε. This property is crucial as the exact value of ε is not known a priori. For example, the rate-half repetition codes [5], [6] perform independent of ε. Therefore, we can assume a worst-case ε (ε_{wc}) to calculate the PICR.

III. PAR and PICR Reduction by Coding

For a code $\mathcal{C}$, we define

$$\mathrm{PAR}(\mathcal{C}) = \max_{\mathbf{c} \in \mathcal{C}} \mathrm{PAR}(\mathbf{c})$$
$$\mathrm{PICR}(\mathcal{C}, \varepsilon) = \max_{\mathbf{c} \in \mathcal{C}} \mathrm{PICR}(\mathbf{c}, \varepsilon). \tag{7}$$

We next state the main problem arising from these definitions. **For length N, what is the achievable region of quadruple** $(R, d, \mathbf{PAR}(\mathcal{C}), \mathbf{PICR}(\mathcal{C}, \varepsilon))$? where R is the code rate and d is the minimum Euclidean distance of $\mathcal{C}$. So the problem at hand is to find out all the codewords in $\mathcal{C}$ that minimize both PAR and PICR for a fixed ε_{wc} and having error correcting capability (i.e., increased d) with a high code rate.

Since PAR and PICR are functions of $\mathbf{c}$, coding to select only those data frames with low PAR and PICR as valid codewords can reduce both PAR and PICR. This idea was suggested in [1] to reduce PAR.

The achievable code rate (R) of the PAR-PICR-reduction codes for threshold values, $\mathrm{PAR_{th}}$ and $\mathrm{PICR_{th}}$ can be expressed as

$$R = \frac{\log_2 \left[q^N \Pr \left\{ \mathrm{PAR}(\mathbf{c}) < \mathrm{PAR_{th}}, \mathrm{PICR}(\mathbf{c}, \varepsilon) < \mathrm{PICR_{th}} \right\} \right]}{\log_2 q^N} \tag{8}$$

where $\Pr\{x\}$ denotes the probability of x and q is the signal constellation size. R is a function of both $\mathrm{PAR_{th}}$ and $\mathrm{PICR_{th}}$.

To show that PAR-PICR-reduction codes exist, we consider an OFDM system with $N = 4$ and binary phase shift keying (BPSK) modulated subcarriers. The PICR for all data frames is given in Table I in ascending order. Corresponding PAR is also tabulated. Ten data frames have a PICR of -12.49 dB for $\varepsilon = 0.1$. Four data frames have a PAR of 6.02 dB. Some data frames have both moderate PAR and PICR. Clearly, we can choose only these data frames for transmission in order to reduce both PAR and PICR simultaneously.

In Table I, codewords 0011, 0110, 1001, and 1100 have a PAR of 3.73 dB and a PICR of -14.76 dB. Thus, by transmitting only these data frames, PAR can be reduced by 2.3 dB and PICR can be reduced by 2.3 dB for $\varepsilon = 0.1$. This can be done by block coding the data such that 2 bits of data are mapped on to 4 best data frames. Thus, this code halves the data throughput. Note also that the complete rank-ordering of all codewords in terms of PICR is independent of the frequency offset. That suggests that this block code's performance is independent of ε.

TABLE I
PICR and PAR for $N = 4$ and BPSK Modulation

d_1	d_2	d_3	d_4	PICR (dB) $\varepsilon = 0.05$	PICR (dB) $\varepsilon = 0.1$	PAR (dB)
0	1	0	1	-28.06	-21.87	6.02
1	0	1	0	-28.06	-21.87	6.02
0	0	1	1	-20.97	-14.76	3.73
0	1	1	0	-20.97	-14.76	3.73
1	0	0	1	-20.97	-14.76	3.73
1	1	0	0	-20.97	-14.76	3.73
0	0	0	0	-18.56	-12.49	6.02
0	0	0	1	-18.56	-12.49	2.32
0	0	1	0	-18.56	-12.49	2.32
0	1	0	0	-18.56	-12.49	2.32
0	1	1	1	-18.56	-12.49	2.32
1	0	0	0	-18.56	-12.49	2.32
1	0	1	1	-18.56	-12.49	2.32
1	1	0	1	-18.56	-12.49	2.32
1	1	1	0	-18.56	-12.49	2.32
1	1	1	1	-18.56	-12.49	6.02

A. Golay Complementary Repetition Codes

A simple approach to construct codes to reduce both PAR and PICR simultaneously is to use repetitive coding of Golay complementary sequence of length $N/2$. Davis and Jedwab [3] explicitly determine a large class of Golay complementary sequences over Z_{2^h} of length 2^m. The ith element of a Golay sequence of length 2^m for any b, $b_k \in Z_{2^h}$ can be expressed as

$$f_i = 2^{h-1} \sum_{k=1}^{m-1} x_{\pi(k)} x_{\pi(k+1)} + \sum_{k=1}^{m} b_k x_k + b \tag{9}$$

where $x = (x_1, x_2, \ldots, x_m)$ is the binary representation of the integer $i = \sum_{j=1}^{m} x_j 2^{m-j}$, h is the number of bits per symbol and π is the any permutation of the symbols $\{1, 2, \ldots, m\}$. Moreover, (9) determines $2^{h(m+1)} m!/2$ number of Golay sequences over Z_{2^h}.

Now, consider the 2^h-PSK codeword given by

$$c_{gr} = \left(\alpha^{f_0}, -\alpha^{f_0}, \alpha^{f_1}, -\alpha^{f_1}, \ldots, \alpha^{f_{N/2-1}}, -\alpha^{f_{N/2-1}} \right) \tag{10}$$

where $\alpha = e^{(j2\pi/2^h)}$. It can be easily proved that the PAR of c_{gr} is no greater than 6 dB while its PICR is equal to that of repetition codes. The code rate of a binary Golay complementary repetition code can be expressed as

$$R = \frac{\log_2 \left(\frac{2^{(m+1)} m!}{2} \right)}{N} \tag{11}$$

where $m = \log_2(N/2)$. For large N, the code rate decreases rapidly.

IV. Simulation Results

The simulation results were obtained for an OFDM system with $N = 16$. Note that we choose small N and BPSK in this study so that all possible (2^N) sequences can be investigated.

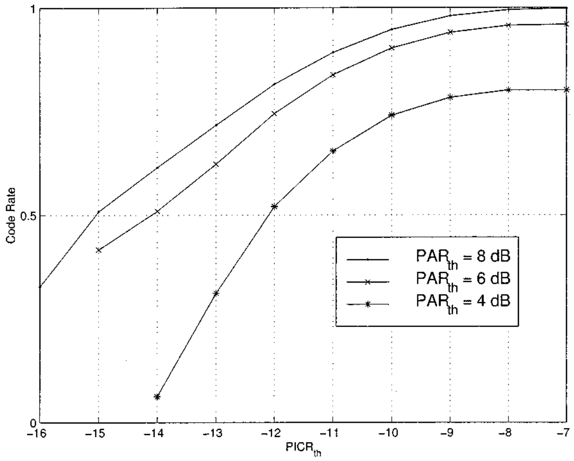

Fig. 1. Variation of code rate with PAR and PICR for $N = 16$ and $\varepsilon = 0.1$.

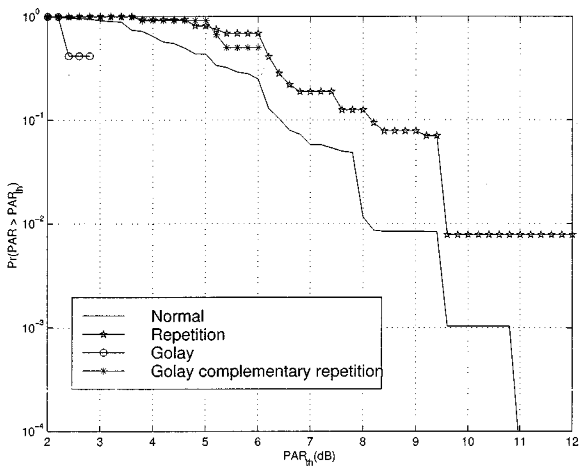

Fig. 2. CCDF of PAR of an OFDM system.

For large N and higher order modulation formats, only a small subset of all possible sequences can be investigated. Further, we assume an AWGN channel and a worst-case ε of 0.1.

Fig. 1 shows the code rate as a function of $\mathrm{PAR_{th}}$ and $\mathrm{PICR_{th}}$. All codewords (2^{16}) of length 16 are generated to find the code rate. The block coding principle offers the flexibility to choose the amount of both PAR and PICR reduction and the code rate. For example, for a code rate of 1/2, PAR and PICR can be limited to 4 and -12 dB, respectively. Thus, an 8 dB reduction in maximum PAR and 5 dB reduction in maximum PICR. In fact, the code rate reduces rapidly for large PAR and PICR reductions.

Figs. 2 and 3 shows the complementary cumulative density function (CCDF) of PAR and PICR, respectively. Note that the PAR characteristics of the rate-half repetition codes are worse than normal OFDM while the PICR characteristics of Golay complementary sequences are almost same as normal OFDM. The Golay complementary repetition codes show better PAR and PICR characteristics due to their code-structure. Its maximum PAR is limited to 6 dB while its PICR is almost same as rate-half repetition codes. However, the price for these benefits is low data throughput of 0.35.

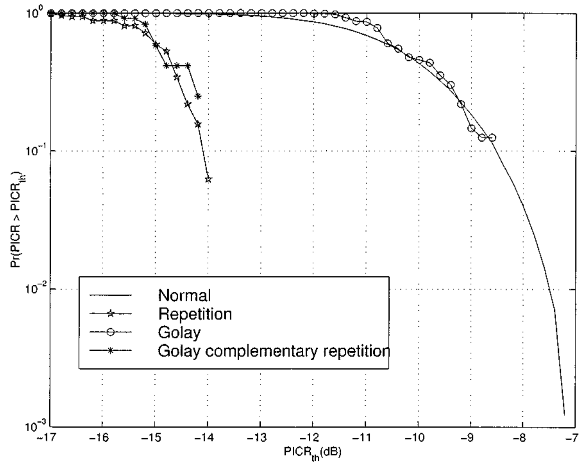

Fig. 3. CCDF of PICR of an OFDM system with $\varepsilon = 0.1$.

V. Conclusion

This letter has shown that PAR-PICR-reduction codes exist. The proposed block coding scheme offers the flexibility to trade code rate against both PAR and PICR reductions. As an example, we constructed Golay complementary repetition codes. PAR of these codes are limited to 6 dB while their PICR is almost the same as that of rate-half repetition codes. However, data throughput of these codes are low, particulary for large N. Algebraic constructions for high-rate PAR-PICR reduction codes would therefore be very useful. This study paves the way for further research on this direction.

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