Relationship between Hamming weight and Peak-to-mean envelope power ratio of orthogonal frequency division multiplexing

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Abstract — **We show that for certain binary codewords, the PMEPR is exactly determined by their Hamming weight or the Hamming distance from the worst-case PMEPR sequences. Consequently, it is relatively easy to devise selection metrics and construct codes that achieve initial PMEPR reductions of 3 to 6** ***dB*** **without the need to perform high complexity computations. The technique used in this paper applies the metric of Hamming Distance of a sequence from a sequence with highest PMEPR. More generally, we consider the metric of Hamming Distance from any sequence with known PMEPR. In particular we build code 'clusters' around low PMEPR complementary sequences so as to construct codes with reasonable rate and low PMEPR.**

The complex baseband OFDM signal is represented as

$$s_b(t) = \frac{1}{\sqrt{n}} \sum_{r=0}^{n-1} \xi^{b_r} e^{j2\pi r \Delta f t} \quad 0 \le t \le T \tag{1}$$

where $\xi = e^{j2\pi/M}$, $j = \sqrt{-1}$, $b_r \in \{0, 1, \ldots, M-1\}$ $\forall r$, $r\Delta f$ is the frequency of the r-th subcarrier. The peak-to-mean envelope power ratio (PMEPR) of codeword $\mathbf{b} = (b_0, \ldots, b_{n-1})$ is defined as PMEPR$(\mathbf{b}) = \max_t |s_b(t)|^2$. For all possible M^n codewords, $1 <$ PMEPR$(\mathbf{b}) \le n$. The following notation will also be used where necessary. An (n, k, d, η)-code is a code of length n, which contains k information symbols and has minimum distance d and maximum PMEPR η.

Theorem 1. *For a binary codeword* $\mathbf{b}$ *of Hamming weight* k, *the PMEPR is exactly*

$$PMEPR(\mathbf{b}) = \frac{(n-2k)^2}{n}$$

provided $k \le \frac{n}{2(2+\sqrt{2})}$ *and the leading principal minors of the Hermitian matrix* $[\beta_{|l-m|}]$ *are non-negative. The* β_r*'s are defined as*

$$\beta_r = \begin{cases} \frac{(n-2k)^2}{n} - 1 & r = 0 \\ \frac{-2}{n} \sum_{i=0}^{n-1-r} (-1)^{b_i + b_{i+r}} & r = 1, 2, \ldots, n-1. \end{cases} \tag{2}$$

Let $R(t) = \sharp\{\, \mathbf{a} \mid \text{PMEPR}(\mathbf{a}) \ge t\}$ for all $t \ge 0$. For example $R(1) = 2^n$ and $R(n) = 4$ for BPSK OFDM. The exact expression of $R(t)$ is unknown and is considered an extremely complicated problem. We can show that

$$R(n/2) \ge 4 \sum_{K=0}^{n/4} \binom{n}{k}. \tag{3}$$

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The factor 4 arises by considering the codewords which are at Hamming distance k from the 4 worst-case sequences: $000\ldots$, $111\ldots$, $1010\ldots$ and $0101\ldots$. We also strongly conjecture that

$$\sharp\left\{ \mathbf{a} \mid \text{PMEPR}(\mathbf{a}) = \frac{(n-2k)^2}{n} \right\} = 4\binom{n}{k} \tag{4}$$

for small values of k.

The results suggest that a 3 dB PMEPR reducing code can easily be constructed for any large n without computing any FFTs at all. We observe that Paterson and Tarokh [1] proposed the existence of codes with PMEPR growth of $\log n$.

Theorem 2. *There exists a*

$$\left(2^m, \log_2\Big(\sum_{r=0}^{k} \binom{2^m}{k}\Big) + m + \log_2(m!), 1, \Big(\sqrt{2} + \frac{k}{2^{\frac{m-2}{2}}}\Big)^2\right)$$

code for $m \ge 1$ *and* $0 \le k \le 2^{m-3}$.

Theorem 2 can also be generalized to $M = 2^h$, $h > 1$, in a straightforward manner.

Theorem 3. *There exists a*

$$\left(2^m, m + \log_2(m!) + \log_2(A(2^m, d, k)), d, \Big(\sqrt{2} + \frac{k}{2^{\frac{m-2}{2}}}\Big)^2\right)$$

code, where $A(n, d, k)$ *is the upper bound on the code size of a binary constant weight code of length* n*, distance* d*, and weight* k*. It is known that*

$$A(2^m, d, k) \ge \frac{1}{2^{m(\frac{d}{2}-1)}} \binom{2^m}{k}$$

where the bound on A *can be found in the literature.*

For example, we find that there is a constant weight code with parameters $A(64, 4, 4) = 10416$. In this case, $m = 6$, $d = 4$, and $k = 4$ satisfies $k \le 6$. From Theorem 3 we can therefore construct a $(64, 6+\log_2(720)+\log_2(10416), 4, 5.83) = (64, 28.84, 4, 5.83)$ binary code. For the same length, if we assign $d = 4$ then k can be a maximum of 6. From the literature there is a constant weight code such that $A(64, 4, 6) \ge 1166592$. From Theorem 3 we can therefore construct a $(64, 6+\log_2(720)+\log_2(1166592), 4, 8.49) = (64, 35.64, 4, 8.49)$ binary code.

References

[1] K. G. Paterson and V. Tarokh, "On the existence and construction of good codes with low peak-to-average power ratio," *IEEE Trans. Inform. Theory.*, vol. 46, pp. 1974–1987, Sept. 2000.

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