Unified Error Probability Analysis for Generalized Selection Diversity in Rician Fading Channels

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ABSTRACT — Motivated by practical considerations in the design of low-complexity receiver structures for wideband cellular CDMA, millimeter-wave and ultra-wideband communications, the study on the generalized selection combining receiver that adaptively combines a subset of M "strongest" paths out of L available paths has intensified over the past few years. The study on $\mathrm{GSC}(M, L)$ receiver is also important from a theoretical standpoint because this model encapsulates both the classical selection diversity and maximal-ratio combining (coherent detection) or post-detection equal-gain combining (noncoherent detection) receiver structures as limiting cases. In this paper, we first derive a concise analytical expression for the moment generating function (MGF) of the $\mathrm{GSC}(M, L)$ output signal-to-noise ratio with independent and identically distributed diversity paths over Rician fading channels (in terms of only a single finite range integral whose integrand is composed of tabulated functions). Previous studies have only treated either Rayleigh or Nakagami-m channel models using numerous ad-hoc approaches to simplify an M-dimensional nested integral that arise in the computation of the MGF. The novelty of our mathematical framework for computing the MGF relies on the fact that it allows us to treat all common multipath fading channel models (Rayleigh, Rician, Nakagami-m and Nakagami-q) in a unified sense, it leads to a much more elegant and computationally efficient expression than those available in the literature, and it holds for any combinations of M and L values. Using this newly derived MGFs, we provide a unified error probability analysis for many coherent and noncoherent digital modulation/detection schemes in a myriad of fading environments.

I. INTRODUCTION

Generalized selection combining scheme aims to mitigate the detrimental effects of deep fades experienced in wireless channels by applying an optimal linear combining rule to a subset of the "strongest" available diversity paths, and thereby reducing the receiver complexity and cost (i.e., fewer electronics and lower power consumption). If the branches are coherently combined before signal detection, then we refer the diversity combining scheme as coherent GSC. In noncoherent GSC, noncoherent combining (i.e., post-detection equal-gain combining) of diversity branches is implemented after the signal detection.

GSC receiver merits consideration in a numerous practical applications. For example, in wideband CDMA and ultra wideband communications, the number of available correlators will limit the number of multipaths that can be utilized in a typical rake combiner. It also reduces the complexity of implementation for antenna arrays in millimeter wave communications and improves the throughput performance of packet radio networks employing "selective packet combining". Performance analyses of coherent and noncoherent GSC receivers are also important from a theoretical viewpoint because $\mathrm{GSC}(1, L)$ and $\mathrm{GSC}(L, L)$ are simply the classical selection combining and maximal-ratio combining (coherent detection) or post-detection equal-gain combining (noncoherent detection) receiver, respectively.

Previous studies on GSC have been limited to only Rayleigh [1]-[9] and Nakagami-m channels [10]-[15]. While Rayleigh model may be appropriate for macro-cells, it is customary to model the fading signal amplitudes in micro-cellular and pico-cellular environments as Rician distributed since the propagation paths usually consist of one strong direct line-of-sight component and many random weaker components. The appropriateness of this model has been validated by numerous field measurements. Moreover, Stein [16] and several other researchers have shown that the Nakagami-m approximation for a Rician random variable suggested by Nakagami [17] (i.e., $m = (K+1)^2/(2K+1)$) tends to overestimate the receiver performance particularly at large SNRs owing to the fact that the tails of the Rician and its Nakagami-m approximation[1] distributions do not fit very closely.

Despite the above reasons, performance analyses of both coherent and noncoherent $\mathrm{GSC}(M, L)$ receivers in Rician multipath fading channels are not available in the literature. The primary difficulty stems from the fact that the ordered SNRs $\gamma_{(k)}$ (obtained after rearranging the SNRs of all the L diversity branches, $\gamma_1, \gamma_2, \ldots, \gamma_L$, in descending order such that $\gamma_{(1)} \geq \gamma_{(2)} \geq \ldots \geq \gamma_{(L)}$), because of the inequalities among them, are necessarily dependent. Consequently, finding the moment generating function (MGF) of a linear sum of ordered random variables $\gamma_{gsc} = \sum_{k=1}^{M} \gamma_{(k)}$ (GSC output SNR) is generally much more difficult than for the unordered random

1. It is noted that the real importance of the Nakagami-m fading model lies in the fact that it offers features of analytical convenience in comparison to the Rician distribution. However, the goodness-of-fit tests used by ionospheric physicists to match measured scintillation data to a Nakagami-m distribution do not give special weighting to the deep-fading tail of the distribution [16]. As a result, we sometimes have better fit near the median of the distribution than in the tail region, although the tail behavior is of greater significance to communication systems performance analysis.

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variables. In the past, numerous ad-hoc attempts have been made to compute the MGF of ordered exponential [1]-[3] and Gamma variates [12]-[15], resulting in various complicated formulas. Furthermore, the existing mathematical approaches do not lend themselves to the performance evaluation of GSC receivers in Rician channels easily. As such, one of the objectives of this paper is to report the development of a novel mathematical framework which can facilitate the problem of analyzing the GSC receiver performance with i.i.d diversity paths in a variety of fading environments including the Rician fading model. An attractive feature of our approach is that the MGF of $GSC(M, L)$ output SNR for all common fading channel models as well as for all combinations of M and L values can be simply expressed in terms of only a single finite-range integral whose integrand is composed of tabulated functions. For the special cases of Rayleigh and Nakagami-m fading with positive integer fading severity index, the integral may be further simplified into a closed-form formula.

II. GSC(M, L) COMBINER OUTPUT STATISTICS

From [18], we know that when M strongest diversity branches are selected from a total of L available i.i.d diversity branches the joint PDF is given by

$$p_{\gamma_{(1)}, \ldots, \gamma_{(M)}}(x_1, \ldots, x_M) = M!\binom{L}{M}[F(x_M)]^{L-M}\prod_{k=1}^{M} p(x_k) \qquad (1)$$

where $x_1 \geq \ldots \geq x_M \geq 0$, $p(.)$ and $F(.)$ correspond to the PDF and CDF, respectively, of the SNR for a single channel reception (no-diversity case). Recognizing that the MGF of the combiner's output SNR $\phi_\gamma(.)$ is the key to the unified analysis of many modulation/detection schemes over wireless channels, our immediate intention will be to derive the desired MGF first.

Let $\gamma_{gsc} = \sum_{k=1}^{M} \gamma_{(k)}$ denote the hybrid combiner output SNR. Then, the MGF of γ_{gsc} may be computed as [20]

$$\begin{aligned}\phi_\gamma(s) &= \int_0^\infty \int_0^{x_1} \cdots \int_0^{x_{M-1}} e^{-s\sum_{k=1}^{M} x_k} p_{\gamma_{(1)}, \ldots, \gamma_{(M)}}(x_1, \ldots, x_M)\, dx_M \ldots dx_2 dx_1 \\ &= M!\binom{L}{M}\int_0^\infty e^{-sx_M} p(x_M)[F(x_M)]^{L-M} \\ &\quad \times \underbrace{\int_{x_M}^\infty e^{-sx_{M-1}} p(x_{M-1}) \ldots \int_{x_2}^\infty e^{-s\gamma_1} p(x_1)\, dx_1 \ldots dx_{M-1}}_{(M-1)\text{-fold integral}}\, dx_M \end{aligned} \qquad (2)$$

Eq. (2) can be re-stated very concisely as

$$\phi_\gamma(s) = M\binom{L}{M}\int_0^\infty e^{-sx} p(x)[F(x)]^{L-M}[\phi(s, x)]^{M-1} dx \qquad (3)$$

or equivalently as

$$\begin{aligned}\phi_\gamma(s) &= M\binom{L}{M}\int_0^{\pi/2} e^{-s\tan\theta} p(\tan\theta)[1-\phi(0, \tan\theta)]^{L-M} \\ &\quad \times [\phi(s, \tan\theta)]^{M-1}\sec^2\theta\, d\theta \end{aligned} \qquad (4)$$

where $\phi(s, x) = \int_x^\infty e^{-st} p(t)\, dt$ denotes the marginal MGF of SNR of a single diversity branch and $F(x) = 1 - \phi(0, x)$. If $\phi(s, x)$ can be evaluated in closed-form, then it is apparent from (3) and (4) that the computational complexity of $\phi_\gamma(.)$ involves only one-fold integration (instead of an M-fold integration as in the case of (2)) because the integrand can be evaluated term by term for different x. Fortunately, closed-form solutions for $\phi(s, x)$ are available for all the statistical channel models that are typically used to characterize the variations in the received signal power over wireless channels. The net result is a considerable reduction in computational complexity of $\phi_\gamma(.)$ illustrated by (2). Both $p(x)$ and $\phi(s, x)$ needed in (4) for Rician, Nakagami-m and Rayleigh fading environments are summarized below.

A. Rician Fading

$$p(x) = \frac{1+K}{\Omega}\exp\left[-K - \frac{(1+K)x}{\Omega}\right] I_0\left[2\sqrt{\frac{K(K+1)x}{\Omega}}\right], x \geq 0 \qquad (5)$$

$$\begin{aligned}\phi(s, y) &= \frac{1+K}{s\Omega + K + 1} Q\left(\sqrt{\frac{2K(K+1)}{s\Omega + K + 1}}, \sqrt{\frac{2(s\Omega + K + 1)y}{\Omega}}\right) \\ &\quad \times \exp\left[\frac{-sK\Omega}{s\Omega + K + 1}\right]\end{aligned} \qquad (6)$$

where $\Omega = E[x]$ denotes the average SNR per branch, $K \geq 0$ is the Rice factor, $Q(\sqrt{2a}, \sqrt{2b}) = \int_b^\infty \exp(-t-a) I_0(2\sqrt{at})\, dt$ is the first-order Marcum Q-function, $I_0(.)$ denotes the modified Bessel function of the first kind.

B. Nakagami-m Fading

$$p(x) = \frac{1}{\Gamma(m)}\left(\frac{m}{\Omega}\right)^m x^{m-1}\exp\left(\frac{-mx}{\Omega}\right), x \geq 0 \qquad (7)$$

$$\phi(s, y) = \frac{1}{\Gamma(m)}\left(\frac{m}{m + s\Omega}\right)^m \Gamma[m, y(s + (m/\Omega))] \qquad (8)$$

where $\Gamma(a, x) = \int_x^\infty \exp(-t) t^{a-1}\, dt$ denotes the complementary incomplete Gamma function and $m \geq 0.5$ is the fading severity index. If the fading index m assumes a positive integer value, (8) may be simplified as

$$\phi(s, y) = \left(\frac{m}{m + s\Omega}\right)^m \exp\left(-sy - \frac{my}{\Omega}\right)\sum_{k=0}^{m-1}\frac{[y(s + m/\Omega)]^k}{k!} \qquad (9)$$

The $p(x)$ and $\phi(s, x)$ for Rayleigh fading may be obtained as a special case from the above expressions by setting $K = 0$ or $m = 1$.

Substituting (7) and (8) into (3), we obtain

$$\begin{aligned}\phi_\gamma(s) &= \frac{M m^{(L-M)(m-1)+m}}{[\Gamma(m)]^{L-M+1}}\binom{L}{M}\left(\frac{m}{m + s\Omega}\right)^{m(M-1)}\sum_{k=0}^{M-1}\binom{M-1}{k}\left[\frac{-1}{\Gamma(m)}\right]^k \\ &\quad \times \frac{\Gamma[m(L-M+k+1)](s\Omega + m)^{mk}}{m^k[(s\Omega + m)(k+1) + (L-M)m]^{m(L-M+k+1)}} \\ &\quad \times F_A[m(L-M+k+1); \underbrace{1, \ldots, 1}_{L-M+k}; \underbrace{1+m, \ldots, 1+m}_{L-M+k\text{ terms}}; \\ &\quad ; \underbrace{X, \ldots, X}_{L-M\text{ terms}}, \underbrace{Y, \ldots, Y}_{k\text{ terms}}]\end{aligned} \qquad (10)$$

where $X = \frac{m}{Z}$, $Y = \frac{s\Omega + m}{Z}$, $Z = (s\Omega + m)(k+1) + (L-M)m$ and real $m \geq 0.5$. If the fading severity index m assumes a positive integer value, then (3) reduces into

$$\phi_\gamma(s) = \frac{M}{\Gamma(m)}\binom{L}{M}\sum_{k=0}^{L-M}(-1)^k\binom{L-M}{k}\sum_{n=0}^{(M-1)(m-1)}\beta(n, M-1, m)$$
$$\times \sum_{z=0}^{k(m-1)}\frac{\beta(z,k,m)\,\Gamma(z+n+m)m^{mM+z}}{(m+s\Omega)^{m(M-1)-n}[sM\Omega+m(M+k)]^{z+n+m}} \tag{11}$$

and the coefficients of multinomial expansion are given by

$$\beta(z, k, c) = \sum_{i=z-c+1}^{z}\frac{\beta(i, k-1, c)}{(z-i)!}I_{[0,(k-1)(c-1)]}(i) \tag{12}$$

where $I_{[a,b]}(i)=\begin{cases}1 & a \le i \le b\\ 0 & \text{otherwise}\end{cases}$, $\beta(1, k, c) = k$, $\beta(z, 1, c) = \frac{1}{z!}$ and $\beta(0, 0, c) = \beta(0, k, c) = 1$.

Utilizing the Laplace inversion method suggested in [19], the CDF of γ_{gsc} is given by

$$F_\gamma(x) \cong \sum_{c=0}^{C}\binom{C}{c}\sum_{b=0}^{c+B}\frac{\alpha_b(-1)^b}{2^{C-1}e^{-A/2}}\mathrm{Re}\left[\phi_\gamma\left(\frac{A+j2\pi b}{2x}\right)/(A+j2\pi b)\right] \tag{13}$$

where $\alpha_0 = 0.5$, $\alpha_b = 1$ for any $b \ge 1$ and the constants A, B and C are arbitrarily chosen to be 30, 18 and 24 respectively, to yield an accuracy of at least 10^{-13}.

III. ASER ANALYSIS

The average bit or symbol error rate performance of a variety of digital modulation/detection schemes can be investigated using (4). Several examples are described below.

A. Coherent GSC Receiver

Using the MGF method [8][15], the ASER of M_c-ary PSK with GSC can be shown to be

$$\overline{P}_s = \frac{1}{\pi}\int_0^{\pi-\pi/M_c}\phi_\gamma\left(\frac{\sin^2(\pi/M_c)}{\sin^2\theta}\right)d\theta \tag{14}$$

where M_c denotes the alphabet size.

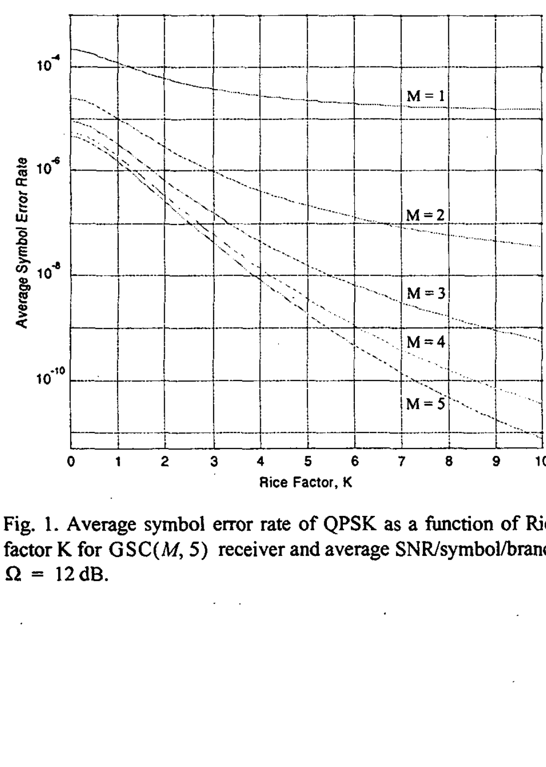

Fig. 1. Average symbol error rate of QPSK as a function of Rice factor K for GSC(M, 5) receiver and average SNR/symbol/branch $\Omega = 12$ dB.

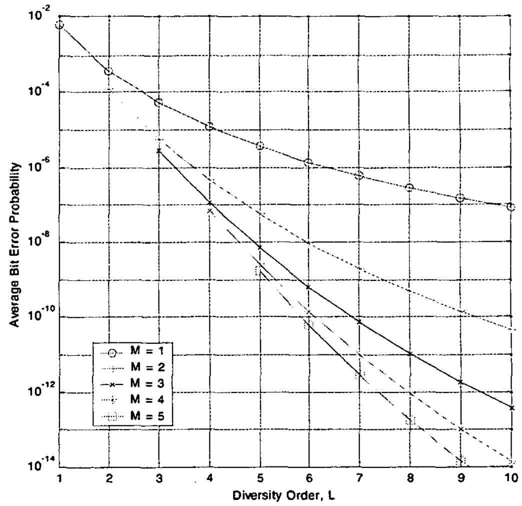

Fig. 2. Average bit error rate of BPSK versus diversity order L for different M values in a Rician channel with $K = 3.5$ and mean SNR/bit/branch $\Omega = 10$ dB.

Although the coherent GSC receiver is typically employed for coherent modulation/detection schemes (i.e., channel estimates are required for coherent diversity combining implementation), the analyses of noncoherent modulation schemes with coherent GSC receivers are also of interest because they provide a lower bound on the error rates with noncoherent GSC receivers. This is particularly useful if the CEP for the multichannel quadratic receivers are not known or available in a complicated form. For instance, the ASER of M-ary DPSK with coherent GSC receiver is given by

$$\overline{P}_s = \frac{1}{\pi}\int_0^{\pi-\frac{\pi}{M_c}}\phi_\gamma\left[\frac{\sin^2(\pi/M_c)}{1+\cos(\pi/M_c)\cos\theta}\right]d\theta \tag{15}$$

For the special case of binary DPSK ($M_c = 2$), (15) simplifies to $\overline{P}_b = \phi_\gamma(1)/2$.

Using the procedure above, it is also possible to write down ASER formulas for other modulation schemes. They are omitted here for brevity.

B. Noncoherent GSC Receiver

Since square-law detection (also known as post-detection equal-gain combining) circumvents the need to co-phase and weight the diversity branches, the multichannel quadratic receiver has a simple implementation and suitable for use in noncoherent and differentially coherent communication systems. The ABER of DPSK, BFSK and π/4-DQPSK with noncoherent GSC(M, L) receiver may be computed using

$$\overline{P}_b = \frac{1}{2\pi}\int_0^{2\pi}\frac{g(\theta)}{1-2\beta\cos\theta+\beta^2}\phi_\gamma\left[\frac{b^2}{2}(1-2\beta\cos\theta+\beta^2)\right]d\theta \tag{16}$$

where $0^+ < \beta = a/b < 1$,

$$g(\theta) = \frac{1}{(1+\eta)^{2M-1}}\sum_{k=0}^{2M-1}\binom{2M-1}{k}\beta^{k+1-M}\eta^k$$
$$\times\{\cos[(k-M+1)\theta]-\beta\cos[(k-M)\theta]\} \tag{17}$$

and the values for constants a, b and η for the three different modulation schemes are summarized in Table 1 below.

TABLE 1. Modulation related parameters for several noncoherent and differentially coherent communication systems.

Modulation	a	b	η
DPSK	0	$\sqrt{2}$	1
BFSK	0	1	1
π/4-DQPSK	$\sqrt{2-\sqrt{2}}$	$\sqrt{2+\sqrt{2}}$	1

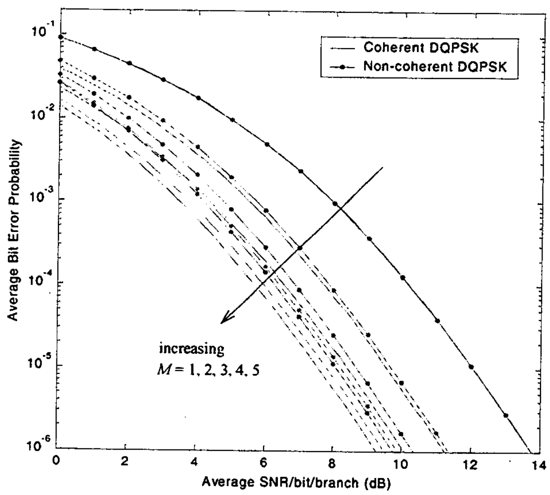

Fig. 3. Average bit error rate performance of π/4-DQPSK with coherent and noncoherent GSC$(M, 5)$ receiver structures in a Rician fading environment ($K = 2.5$).

Note that as $\beta \to 0$, (16) assumes an indeterminate form but its limit converge smoothly to the exact ABER. Thus, the ABER of DPSK and BFSK can be computed using (16) with good accuracy by setting $a = 10^{-3}$ instead of zero. Alternatively, one may utilize the characteristic function method [20] to yield an infinite series expression for the ABER with M-order post-detection equal-gain combining, viz.,

$$\overline{P_b} \cong \frac{4}{T}\sum_{\substack{n=1\\ n\ \mathrm{odd}}}^{\infty} \mathrm{Real}\left[\phi_\gamma\left(\frac{-j2\pi n}{T}\right)G_\gamma\left(\frac{2\pi n}{T}\right)\right] \qquad (18)$$

where $G_\gamma(\omega) = \mathrm{FT}[P_b(\gamma)] = \int_0^\infty P_b(\gamma)e^{-j\omega\gamma}d\gamma$ and constant T is selected to be sufficiently large such that $Pr(x > T) \le \varepsilon$ and ε can be set to a very small value.

Recognizing that the exact CEP of an M-order post-detection EGC for DPSK and BFSK is simply

$$P_b(\gamma) = \frac{1}{2^{2M-1}}\sum_{k=1}^{M}\binom{2M-1}{M-k}\frac{\Gamma(k, g\gamma)}{\Gamma(k)} \qquad (19)$$

where $g = 1$ for DPSK and $g = 1/2$ for BFSK, we obtain

$$G_\gamma(\omega) = \frac{1}{2^{2M-1}(j\omega)}\sum_{k=1}^{M}\binom{2M-1}{M-k}\left[1-\left(\frac{g}{g+j\omega}\right)^k\right] \qquad (20)$$

This approach may be extended to other digital modulation schemes. For instance, it is not very difficult to show that $G_\gamma(\omega)$ for M-ary orthogonal FSK with noncoherent GSC(M, L) receiver is given by

$$G_\gamma(\omega) = \sum_{n=1}^{M_c-1}\binom{M_c-1}{n}\sum_{k=0}^{n(M-1)}\frac{\beta(k,n,M)}{2(M_c-1)}\sum_{i=0}^{k}\binom{M+k-1}{k-i}\frac{(-1)^{n+1}k!M_c}{(1+n)^{M+k+i}}$$
$$\times\left[\left(\frac{n}{1+n}\right)^2+\omega^2\right]^{-(i+1)/2}\exp\left[-j(i+1)\tan^{-1}\left(\frac{\omega}{n}(n+1)\right)\right] \qquad (21)$$

IV. MEAN COMBINED SNR AND OUTAGE PROBABILITY

The mean combined SNR $\bar{\gamma}_{gsc}$ is another useful performance measure of diversity systems.

Since $\bar{\gamma}_{gsc} = \sum_{k=1}^{M}\bar{\gamma}_{(k)}$, we can utilize the density function of the the k-th strongest branch SNR $\gamma_{(k)}$ from a population of L i.i.d random variables $\gamma_1, \gamma_2, \ldots, \gamma_L$ given in [13] or [18] to get

$$\bar{\gamma}_{gsc} = \sum_{k=1}^{M}k\binom{L}{k}\int_0^\infty x[F(x)]^{L-k}[1-F(x)]^{k-1}p(x)dx \qquad (22)$$

Eq. (22) can be computed very efficiently via Gauss-Legendre quadrature method. For the special case of Nakagami-m fading, (22) may be replaced by a closed-form formula. This development is omitted here for brevity.

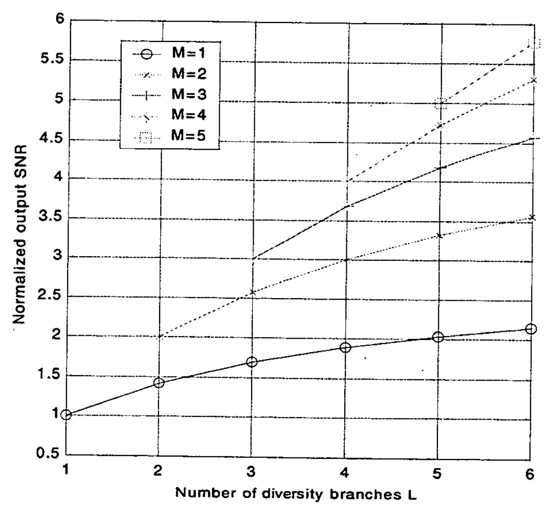

Fig. 4. The normalized mean GSC output SNR $\bar{\gamma}_{gsc}/\Omega$ versus the total number of diversity branches L for various M values in a Rician channel ($K = 1.5$).

The outage probability P_{out} is defined as the probability that the instantaneous bit or symbol error rate will exceed a specified value (say P_e*). Thus, (13) can be used to predict the efficacy of a GSC diversity receiver on the outage probability metric, viz., $P_{out} = F_\gamma(\gamma^*)$ where γ^* is the threshold SNR. Given a digital modulation scheme with conditional error probability $P_s(\gamma)$, γ^* is obtained by solving $P_s(\gamma^*) = P_e*$. For example, if $P_e* = 10^{-3}$ is specified, $P_{out} = F_\gamma(4.77)$ because $\gamma^* = [\mathrm{erfc}^{-1}(2\times10^{-3})]^2 = 4.77$ for

BPSK modulation scheme. Similarly, for QPSK or square 4-QAM, $P_{out} = F_{\gamma}(10.83)$.

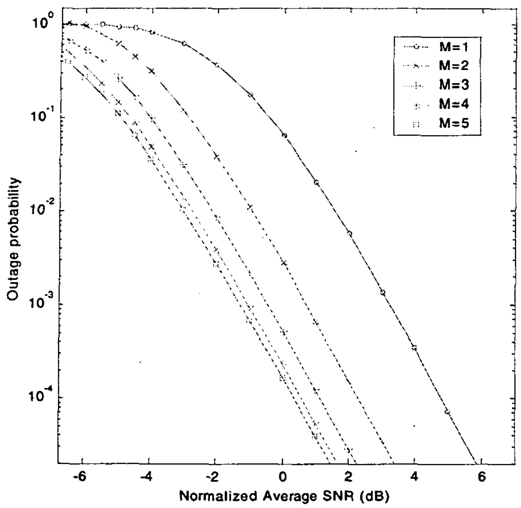

Fig. 5. Outage probability $F_{\gamma}(\gamma^*)$ versus normalized average SNR Ω/γ^* for $GSC(M, 5)$ receiver on a Rician channel ($K = 3$).

V. CONCLUDING REMARKS

We have derived simple-to-evaluate ASER formulas for both coherent and noncoherent $GSC(M, L)$ receiver with i.i.d diversity paths in Rician fading which heretofore had resisted solution in a simple form. Unified expressions for computing the MGF and CDF of GSC output SNR in a myriad of fading environments are also derived. The MGF of γ_{gsc} is used to facilitate an unified ASER analysis for different modulation/detection schemes while the outage rate of error probability performance is predicted from the CDF expression. Our mathematical framework for computing the MGF of γ_{gsc} has several advantages: (a) it treats all common fading channel models in a unified sense; (b) it leads to a much more elegant and computationally efficient expression for $\phi_{\gamma}(.)$ than those available in the literature; (c) it also holds for any combinations of M and L values and arbitrary fading parameters. A new closed-form expression for the MGF of GSC output SNR in i.i.d Nakagami-m fading is also derived.

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