Analysis of Equal-Gain Diversity Receiver in Correlated Fading Channels

A. Annamalai, V. Ramanathan and C. Tellambura

Mobile and Portable Radio Research Group
Bradley Department of Electrical and Computer Engineering
Virginia Tech
445 Durham Hall, Blacksburg, VA 24061, USA
Phone: (540)231-2129, Fax: (540)231-2968, E-mail: annamalai@vt.edu

ABSTRACT — Performance evaluation of equal-gain combining (EGC) diversity receivers are known to be a much more difficult task in comparison with other diversity combining techniques such as the selection diversity or the maximal-ratio combining method. The difficulty of the above mathematical problem is compounded when the diversity branches are correlated. This paper presents a novel mathematical framework for analyzing a dual-branch EGC receiver performance over nonidentical Rayleigh and Nakagami-m fading channels when the diversity branches are correlated. It is also shown that the average bit error rate (ABER) formula for coherent BPSK and BFSK schemes reduces to the familiar expressions in the literature for the limiting case of independent diversity paths by setting the correlation coefficient to zero. Selected numerical plots that illustrate the effect of branch correction on the ABER performance are also provided.

I. INTRODUCTION

One of the fundamental limitations of achieving reliable communications over wireless links is the time-varying nature of the wireless channels (due to user mobility and multipath fading). The detrimental effects of deep fades in radio channels can be mitigated through use of diversity mechanisms. EGC is of practical interest because it provides performance comparable to the optimal maximal ratio combining receiver but with greater simplicity. Surprisingly perhaps is that the literature on EGC receiver performance in fading channels is barren compared to that of other popular diversity combining methods. This lack may have stemmed from the difficulty of finding the pdf of the EGC output SNR (conventionally, the average bit error probability is evaluated by averaging the conditional error probability over the pdf of the EGC combiner output). Obviously, the above problem will be compounded when the diversity branches are correlated. To the best of the authors' knowledge, neither analytical nor simulation results for the coherent EGC receiver performance over Nakagami-m channels has been reported thus far when the fading statistics of the diversity branches are correlated. In this paper, we attempt to provide a partial solution to the problem at-hand by deriving simple-to-evaluate average bit error rate (ABER) expressions for two different binary digital modulation schemes in conjunction with a dual-diversity coherent EGC receiver.

In the following, we will summarize the important developments and previous related studies on the EGC diversity systems. In [1], Altman and Sichak have found the pdf of the sum of two independent Rayleigh fading amplitudes. For higher order of diversity, Jakes [2] has made use of a small argument approximation for the pdf suggested by Schwartz et. al [3]. In [4], Beaulieu devised an approximate infinite series technique to compute the pdf for the sum of independent Rayleigh random variables. Applying this technique, [5] and [6] analyze the performance of EGC receiver for coherent and differential binary signaling schemes in Nakagami-m and Rician fading channels, while [7] examines various six 16-ary signal constellations. While the convergent infinite series technique suggested in [5]-[7] is accurate, the solution is still an approximation. A simple method for bounding the aliasing and truncation errors resulting from the use of the convergent infinite series [4] is discussed in [8]. In [9]-[11], a powerful frequency-domain technique is developed for the EGC receiver analysis in a variety of fading environments and for different modulation schemes. The final "exact" expression requires an evaluation of a finite-range integral whose integrand is composed of a product of the characteristic function of each of the fading amplitudes and also the Fourier transform of the conditional error probability. Some closed-form expressions for the exact average bit error rate of a variety of binary modulation schemes in Nakagami-m fading channels (restricted to second-order diversity for noncoherent detection and third-order diversity for coherent detection) are also derived in [10]. For the special case of Rayleigh fading, these results are in agreement with those provided in [12]. Building on our previous work, [13] derived exact average error probability expressions for a broad class of modulation schemes with EGC diversity in different multipath fading channel models without imposing any restriction on the diversity order. The key to these results is the reformulation of the error probability expression in the frequency domain [11] and the solution to a Laplace transform integral involving the product of Kummer functions [10]. Finally, some approximate but simple closed-form expressions for the EGC receiver performance in Nakagami-m fading channel were obtained in [14].

It should be emphasized, however, that in all of the above studies, the fading statistics across the diversity branches are assumed to be independent. This assumption is violated in a number of real-life scenarios. For instance, if there is insufficient antenna separation in small mobile units equipped with either space or polarization diversity, the

0-7803-7484-3/02/$17.00 ©2002 IEEE.

fading statistic in the distinct diversity branches will still be highly correlated. Experimental results carried out by Turin [15] and Bajwa [16] also reveal that the correlation coefficient between adjacent multipaths in frequency-selective channels may be as high as 0.6. In real base-stations, correlation coefficients usually vary between 0.3 to 0.7. As a result, the maximum theoretical diversity gain that can be realized via multipath or spatial diversity cannot be achieved. Hence, any performance analysis must be revamped to account for the effect of correlation between the combined signals.

However, only [17] and [18] have studied the performance of correlated dual-diversity EGC for BPSK in Rayleigh fading. In [17], the authors exploited the joint characteristic function (chf) given in [19, pp. 409] along with the hypothesis testing technique suggested by [12] for computing the ABER. In [18], the authors derived the chf of EGC output SNR and utilized an infinite series representation for the complementary error function to obtain the ABER. Different from the above two approaches, in this paper we utilize an integral representation for the Gaussian probability integral [20][10, Eq. (18)] to derive the ABER. Even for the special case of Rayleigh fading, our derivation is very concise in comparison with [17]. A by-product of this study is the derivation of closed-form formulas for the Appell's hypergeometric function of the form

$$F_2\left(\frac{1}{2};\frac{1}{2}-k,1-k;\frac{1}{2},\frac{3}{2};X,Y\right)$$

when k is a non-negative integer or an half-odd positive integer (i.e., $k = 0, \frac{1}{2}, 1, \frac{3}{2}, \ldots$).

II. ASER ANALYSIS

In EGC combiner, the output of different diversity branches are first co-phased and weighted equally before being summed to give the resultant output. The instantaneous SNR at the output of a dual-branch EGC combiner is

$$\gamma_{egc} = \frac{1}{2}(\gamma_1+\gamma_2+2\sqrt{\gamma_1\gamma_2}) = \frac{1}{2}(R_1+R_2)^2 \qquad (1)$$

where γ_1 and γ_2 are the SNRs on individual diversity branches, while R_1 and R_2 denote the signal amplitudes normalized by the noise voltage.

Next, we show our derivation for the ABER of coherently detected binary signals in a Rayleigh multipath fading channel that employs a dual-branch EGC receiver, so as to highlight our methodology.

The average bit error rate (ABER) of coherent BPSK and coherent BFSK is given by

$$\overline{P_b} = E\{Q(\sqrt{2g\gamma_{egc}})\} = E\{Q(\sqrt{g}(R_1+R_2))\} \qquad (2)$$

where $g = 1$ for BPSK and $g = 1/2$ for BFSK.

From [11], we know that the chf of $R_1+R_2 \equiv \alpha$ is needed for the exact analysis of EGC. It is not very difficult to show that the chf of a dual-branch EGC output in correlated Rayleigh fading is given by [18]

$$\phi_\alpha(\omega) = E\{e^{j\omega(R_1+R_2)}\} = (1-\rho)e^{-\omega^2(1-\rho)\Omega/4}$$
$$\times\sum_{k=1}^{\infty}\rho^{k-1}\left[\frac{(2k-1)!}{2^{k-1}(k-1)!}D_{-2k}(-j\omega\sqrt{(1-\rho)\Omega/2})\right]^2 \qquad (3)$$

where $\Omega = E\{R_1^2\} = E\{R_2^2\}$, ρ is the correlation coefficient, and $D_{-\nu}(.)$ denotes the parabolic cylinder function.

Using an alternative exponential integral representation for the Gaussian probability integral [10, Eq. (18)], (2) may be conveniently expressed as

$$\overline{P_b} = \frac{1}{2}-\frac{1}{2\pi}\int_0^\infty\frac{e^{-t}}{t}\mathrm{Imag}\{\phi_\alpha(\sqrt{2gt})\}dt \qquad (4)$$

Taking the trapezoidal sum approximation of (4), we obtain a convergent infinite series for the ABER, viz.,

$$\overline{P_b} \cong \frac{1}{2}-\frac{2}{\pi}\sum_{\substack{n=1\\ n\ \mathrm{odd}}}^{\infty}\frac{e^{-n^2\omega_0^2/2}}{n}\mathrm{Imag}\{\phi_\alpha(n\omega_0\sqrt{g})\} \qquad (5)$$

where ω_0 is a suitably small constant.

Substituting the $D_{-2k}(.)$ terms in (3) with its equivalent Kummer functions and then utilizing identity [10, Eq. (C.1)], (4) may be simplified as

$$\overline{P_b} = \frac{1}{2}-2(1-\rho)\sqrt{\pi\chi}\sum_{k=1}^{\infty}\rho^{k-1}\left[\frac{(2k-1)!}{2^{2k-1}(k-1)!}\right]^2\Big/\left[\Gamma(k)\Gamma\left(k+\frac{1}{2}\right)\right]$$
$$\times F_2\left(\frac{1}{2};\frac{1}{2}-k,1-k;\frac{1}{2},\frac{3}{2};\chi,\chi\right) \qquad (6)$$

where $\chi = \dfrac{g(1-\rho)\Omega}{2[1+g(1-\rho)\Omega]}$.

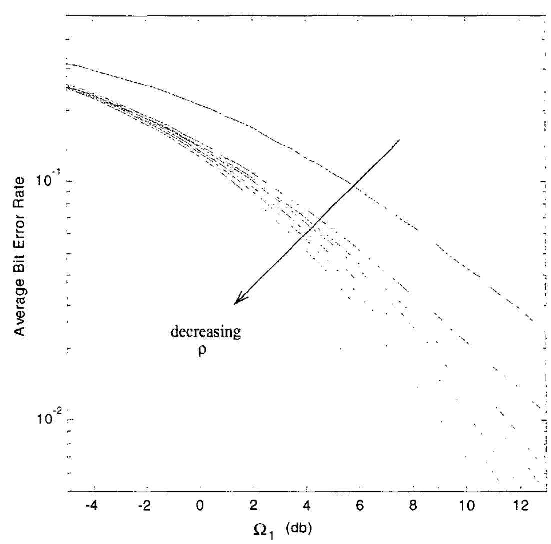

Fig. 1. ABER performance of coherently detected BFSK in conjunction with dual-diversity EGC over Rayleigh fading for different correlation coefficients $\rho \in \{0, 0.3, 0.5, 0.7, 0.8, 0.9, 1\}$. It is assumed that $\Omega_1 = \Omega_2$.

It should be further noted that the Appell's hypergeometric function in (6) may be replaced by a finite polynomial, viz.,

$$F_2\left(\frac{1}{2};\frac{1}{2}-k,\ 1-k;\frac{1}{2},\ \frac{3}{2};\chi,\ \chi\right) = \sum_{n=0}^{k-1}\binom{k-1}{n}\frac{(-\chi)^n}{(1+2n)}(1-\chi)^{k-n-\frac{1}{2}} \times \sum_{v=0}^{n}\binom{n}{v}\frac{(k+v-1)!2^v}{(k-1)!(2v-1)!!}(-\chi)^v \quad (7)$$

For the special case of $\rho = 0$ (i.e., independent fading), (6) reduces into

$$\overline{P}_e = \frac{1}{2} - \frac{\sqrt{g\Omega(g\Omega+2)}}{2(1+g\Omega)} \quad (8)$$

which agrees with [12, Eq. (23)] and [10, Eq. (32)].

A generalization of (6) which takes into account of the effect of dissimilar signal strengths across the diversity branches is also quite straight-forward [21]. In this case, we need to slightly modify (3)as

$$\phi_\alpha(\omega) = (1-\rho)e^{-\omega^2\left(\frac{1-\rho}{8}\right)(\Omega_1+\Omega_2)}\sum_{k=1}^{\infty}\rho^{k-1}\left[\frac{\Gamma(2k)}{2^{k-1}\Gamma(k)}\right]^2 \times D_{-2k}\left(-j\omega\sqrt{\frac{\Omega_2}{2}(1-\rho)}\right)D_{-2k}\left(-j\omega\sqrt{\frac{\Omega_1}{2}(1-\rho)}\right) \quad (9)$$

Then, the ABER of BPSK and BFSK can be shown to be

$$\overline{P}_b = \frac{1}{2} - \frac{\sqrt{2g}}{2}\sum_{k=1}^{\infty}\rho^{k-1}(1-\rho)^{3/2}\left[\frac{\Gamma(2k)}{2^{2k-1}\Gamma(k)}\right]^2 \Big/ \left[\Gamma(k)\Gamma\left(k+\frac{1}{2}\right)\right] \times \sum_{x=1, y\neq x}^{2}\sqrt{\frac{\Omega_y\pi}{\Upsilon}}F_2\left(\frac{1}{2};\frac{1}{2}-k,\ 1-k;\frac{1}{2},\ \frac{3}{2};\frac{A\Omega_x}{\Upsilon},\ \frac{A\Omega_y}{\Upsilon}\right) \quad (10)$$

where $A = g(1-\rho)/2$ and $\Upsilon = 1 + A(\Omega_1+\Omega_2)$.

It is further pointed out that the hypergeometric function of two variables can be evaluated in closed-form as

$$F_2\left(\frac{1}{2};\frac{1}{2}-k,\ 1-k;\frac{1}{2},\ \frac{3}{2};X,\ Y\right) = \sum_{n=0}^{k-1}\frac{(1/2)_n(1-k)_n}{(3/2)_n n!}(1-X)^{k-n-\frac{1}{2}} \times (Y)^n\sum_{v=0}^{n}\frac{(-n)_v(k)_v}{(1/2)_v v!}(X)^v \quad (11)$$

where $(a)_n = \Gamma(a+n)/\Gamma(a)$ is the Pochhammer symbol.

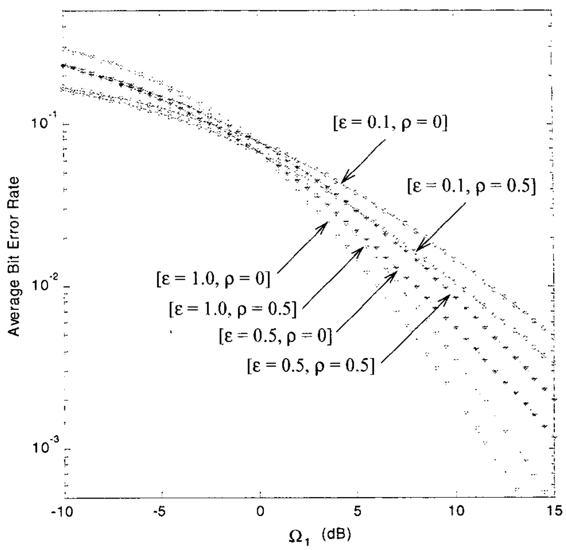

Fig. 3. ABER performance of BPSK with dual-diversity EGC over Rayleigh fading. It is assumed that $\Omega_2 = \varepsilon\Omega_1$.

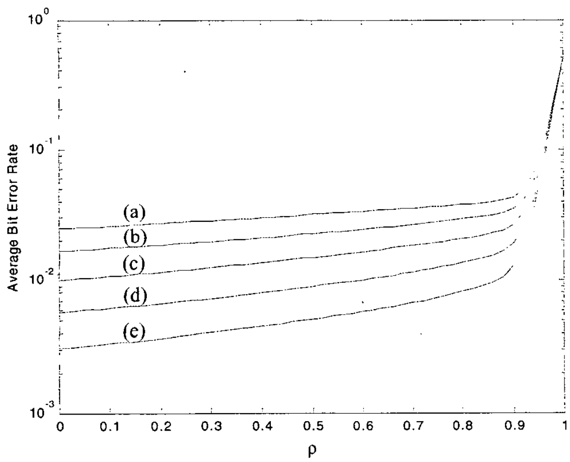

Fig. 2. Investigation on the effects of dissimilar signal strengths and branch correlation on ABER performance of coherently detected BFSK with dual-diversity EGC receiver over Rayleigh fading:

(a) $\Omega_1 = 2$dB, $\Omega_2 = 10$dB (b) $\Omega_1 = 5$dB, $\Omega_2 = 10$dB
(c) $\Omega_1 = 8$dB, $\Omega_2 = 10$dB (d) $\Omega_1 = 11$dB, $\Omega_2 = 10$dB
(e) $\Omega_1 = 14$dB, $\Omega_2 = 10$dB

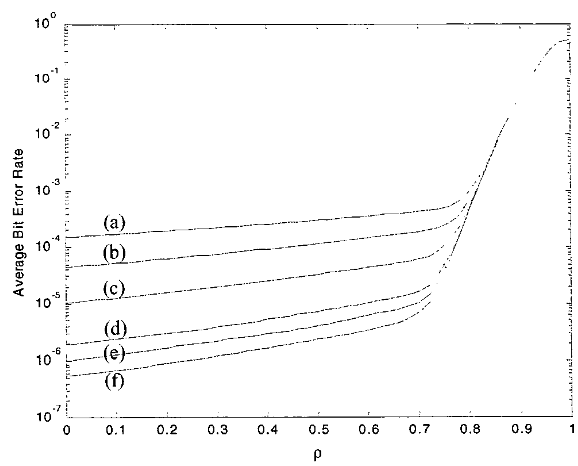

Fig. 4. Investigation on the effects of dissimilar signal strengths and branch correlation on the ABER performance of coherently detected BFSK with dual-diversity EGC receiver over Nakagami-m fading (fading severity index $m = 3$):

(a) $\Omega_1 = 2$dB, $\Omega_2 = 10$dB (b) $\Omega_1 = 5$dB, $\Omega_2 = 10$dB
(c) $\Omega_1 = 8$dB, $\Omega_2 = 10$dB (d) $\Omega_1 = 11$dB, $\Omega_2 = 10$dB
(e) $\Omega_1 = 12$dB, $\Omega_2 = 10$dB (f) $\Omega_1 = 13$dB, $\Omega_2 = 10$dB

Using the above approach, it is not very difficult to show that the ABER performance of BPSK and BFSK in a Nakagami-m fading environment is given by

$$\overline{P_b} = \frac{1}{2} - \frac{\sqrt{2g}}{2}\sum_{k=1}^{\infty} XY\sqrt{1-\rho}\Big/\left[\Gamma\left(m+k-\frac{1}{2}\right)\Gamma(k+m-1)\right]$$
$$\times \sum_{x=1, y\neq x}^{2} \sqrt{\frac{\Omega_y \pi}{\Upsilon}} F_2\left(\frac{1}{2};\frac{3}{2}-k-m, 2-k-m;\frac{1}{2},\frac{3}{2};\frac{A\Omega_x}{\Upsilon},\frac{A\Omega_y}{\Upsilon}\right) \quad (12)$$

where $X = \left(\frac{1-\rho}{4}\right)^{k+m-1}\Gamma[2(k+m-1)]$, $\Upsilon = 1 + A(\Omega_1+\Omega_2)$, $A = g(1-\rho)/2$ and $Y = \frac{4\rho^{k-1}(1-\rho)^{2-2k-m}}{\Gamma(m)\Gamma(k)\Gamma(m+k-1)}$.

To the best of our knowledge, the above result is new.

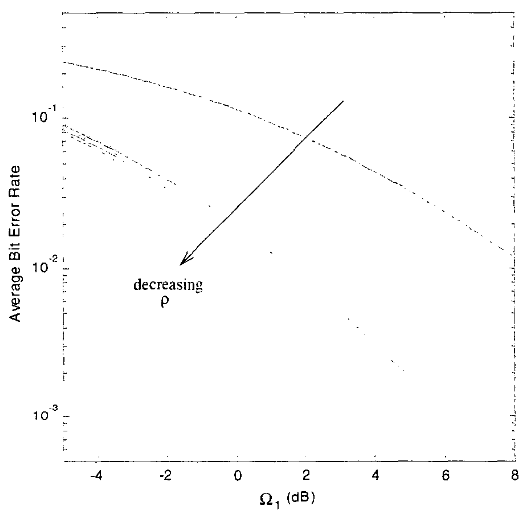

Fig. 5. ABER performance of BPSK in conjunction with dual-diversity EGC receiver over Nakagami-m fading (m = 2) for different correlation coefficients $\rho \in \{0, 0.3, 0.5, 0.7, 0.8, 1\}$. It is assumed that $\Omega_2 = \Omega_1$.

It is also possible to extend the above results to include different digital modulation schemes, including M-ary PSK. Details are provided in [21].

REFERENCES

[1] F. J. Altman and W. Sichak, "A Simplified Diversity Communication System for Beyond the Horizon Links," *IRE Trans. on Commun. Systems, Vol. 4*, pp. 50-55, March 1956.

[2] W. C. Jakes, *Microwave Mobile Communications*, New York: Wiley, 1974.

[3] M. Schwartz, W. R. Bennett and S. Stein, *Communication Systems and Techniques*, New York: McGraw-Hill, 1966.

[4] N. C. Beaulieu, "An Infinite Series for the Computation of the Complementary Probability Distribution Function of a Sum of Independent Random Variables and Its Application to the Sum of Rayleigh Random Variables," *IEEE Trans. on Commun., Vol. 38*, pp. 1463-1474, Sept. 1990.

[5] N. C. Beaulieu and A. Abu-Dayya, "Analysis of Equal Gain Diversity on Nakagami Fading Channels," *IEEE Trans. on Commun.*, Vol. 39, pp. 225-234, Feb. 1991.

[6] A. Abu-Dayya and N. C. Beaulieu, "Microdiversity on Rician Fading Channels," *IEEE Trans. on Communications*, Vol. 42, June 1994, pp. 2258-2267.

[7] X. Dong, N. C. Beaulieu and P. H. Wittke, "Two Dimensional Signal Constellations for Fading Channels," *Proc. 1998 IEEE GLOBECOM* (Communication Theory), pp. 22-27.

[8] C. Tellambura and A. Annamalai, "Further Results on the Beaulieu Series," *IEEE Trans. on Commun.*, Vol. 48, Nov. 2000, pp. 1774-1777.

[9] A. Annamalai, C. Tellambura and V. K. Bhargava, "Exact Evaluation of Maximal-Ratio and Equal-Gain Diversity Receivers for M-ary QAM on Nakagami Fading Channels," *IEEE Trans. on Commun.*, Vol. 47, Sept. 1999, pp. 1335-1344.

[10] A. Annamalai, C. Tellambura and V. Bhargava, "Equal-Gain Diversity Receiver Performance in Wireless Channels," *IEEE Trans. on Commun.*, Vol. 48, Oct. 2000, pp. 1732-1745.

[11] A. Annamalai, C. Tellambura and V. K. Bhargava, "Unified Analysis of Equal-Gain Diversity on Rician and Nakagami Fading Channels," *Proc. 1999 IEEE WCNC*, pp. 10-14.

[12] Q. T. Zhang, "Probability of Error for Equal-Gain Combiners over Rayleigh Channels: Some Closed-Form Solutions," *IEEE Trans. on Commun.*, Vol. 45, March 1997, pp. 270-273.

[13] A. Annamalai, Jing Su and C. Tellambura, "Exact Analysis of Equal Gain Diversity Systems over Fading Channels," *Proc. IEEE VTC (Spring)*, May 2000, pp. 1031-1034.

[14] A. Annamalai and C. Tellambura, "Error Rates for Nakagami-m Fading Multichannel Reception of Binary and M-ary Signals," *IEEE Trans. on Commun.*, Jan. 2001, pp. 58-68.

[15] G. Turin, F. Clapp, T. Johnston, S. Fine and D. Lavry, "A Statistical Model of Urban Multipath Propagation," *IEEE Trans. on Vehic. Technology*, Vol. 21, Feb. 1972, pp. 1-9.

[16] A. Bajwa, "UHF Wideband Statistical Model and Simulation of Mobile Radio Multipath Propagation Effects," *IEE Proceedings*, Vol. 132, Aug. 1985, pp. 327-333.

[17] R. Mallik, M. Win and J. Winters, "Performance of Predetection Dual Diversity in Correlated Rayleigh Fading: EGC and SD" *Proc. IEEE GLOBECOM'00*, pp. 932-936.

[18] C. Tellambura and A. Annamalai, "Wireless Communications Systems Design" in *Encyclopedia of Telecommunications*, Wiley, 2002.

[19] D. Middleton, *An Introduction to Statistical Communications Theory*, McGraw-Hill, 1960.

[20] C. Tellambura and A. Annamalai, "Derivation of Craig's Formula for Gaussian Probability Function," *Electronics Letters*, Vol. 35, No. 17, Aug. 1999, pp. 1424-1425.

[21] A. Annamalai, Ramanathan and C. Tellambura, "Equal-Gain Diversity Receiver Performance in Correlated Nakagami-m Channels," manuscript in preparation.