Crest Factors of Shapiro-Rudin Sequence Based Multi-Code MC-CDMA Signals

Byoung-Jo Choi, Chintha Tellambura and Lajos Hanzo[1]

Dept. of ECS, University of Southampton, SO17 1BJ, UK.
Tel: +44-23-8059-3125, Fax: +44-23-8059-4508
Email: lh[1]@ecs.soton.ac.uk, http://www-mobile.ecs.soton.ac.uk

Abstract – **The envelope power of MC-CDMA signals usually exhibits wide fluctuations, leading to a high value of the associated crest factor. However, when Shapiro-Rudin based sequences, namely a Shapiro-Rudin pair, a Shapiro-Rudin based sub-complementary pair and a Shapiro-Rudin based Sivaswamy's complementary set are used for two- and four-code BPSK modulated MC-CDMA spreading sequences, the crest factor is found to be bounded by 3dB. These sequences belong to the so-called orthogonal complementary set and Walsh-Hadamard sequences constitute a special case of these sets, when the sequence length is equal to the number of codes. When a typical nonlinear power amplifier is used, the low crest factor of the Shapiro-Rudin sequence based multi-code MC-CDMA scheme results in reduced amplifier-induced signal clipping. Hence a reduced power loss is imposed due to clipping in comparison to OFDM and to multi-code MC-CDMA schemes based on other spreading sequences, such as Walsh codes, orthogonal Gold codes, Frank codes and Zadoff-Chu codes.**

1. INTRODUCTION

Multi-Carrier CDMA (MC-CDMA) [1, 2] combines the techniques of Orthogonal Frequency Division Multiplexing (OFDM) and Spread Spectrum (SS) arrangements, capitalising on the joint benefits of both schemes. While DS-CDMA exploits frequency diversity using Rake receivers, its achievable diversity order is limited to the number of fingers in the Rake receiver. The achievable diversity order of MC-CDMA is limited by the number of subcarriers and by the duration of the cyclic prefix [3]. Since the duration of the cyclic prefix is usually determined by the maximum delay spread and provided that the number of subcarriers is sufficiently high for ensuring frequency flat fading over each subcarrier, MC-CDMA has the potential of exploiting the frequency diversity provided by the wide-band channel, more efficiently than DS-CDMA. One of the main drawbacks of OFDM is that the envelope power of the transmitted signal fluctuates widely, requiring a highly linear RF power amplifier. Since our advocated MC-CDMA system spreads a message symbol across the frequency domain and uses an OFDM transmitter for conveying each chip of the spread bit, its transmitted signal also exhibits a high crest factor (CF), which is defined as the ratio of the signal's peak amplitude to its root mean square (rms) value [4].

The financial support of LGE, Korea; The CEC, Brussels; EPSRC, UK; and that of the Mobile VCE, UK is gratefully ackowledged.

VTC'2002, Birmingham, Alabama, USA

The fundamental problem of reducing the dynamic range of the sum of trigonometric series has been studied in numerous fields, such as in mathematics [5], radar engineering, measurement science and in OFDM [6]. As a result, various binary and poly-phase codes have been developed [7–9]. *Jones, Wilkinson* and *Barton* [6] observed that not many message codes yield high CFs and proposed a block-coding scheme, in order to eliminate those messages. Since then, various methods have been proposed for reducing the CF in OFDM. The envelope power of the multi-code MC-CDMA signal was also analysed [10] and it was shown that the combined aperiodic correlation properties of the spreading sequences employed characterise the envelope power. A 'good' set of spreading sequences designed for multi-code MC-CDMA should exhibit low values of aperiodic correlations, while satisfying the orthogonality between the sequences.

In this contribution we characterise the CF properties of three Shapiro-Rudin sequence [5] based spreading sequences, namely those of a Shapiro-Rudin pair, a Shapiro-Rudin code based sub-complementary pair and those of a Shapiro-Rudin based Sivaswamy's complementary set, in the context of two- and four-code BPSK modulated MC-CDMA schemes in order for transmitting two and four bits simultaneously. The next section introduces the multi-code MC-CDMA system model considered and reviews the envelope power expression of the multi-code MC-CDMA signal. Section 3 presents the crest factor properties of three Shapiro-Rudin code based spreading sequences. The effects of the signal clipping imposed by the nonlinear amplifier are discussed in terms of power loss and the spurious out-of-band power emissions in Section 4. Section 5 concludes with our summary and discussions.

2. SYSTEM MODEL AND ENVELOPE POWER

The simplified transmitter structure of MC-CDMA is portrayed in Figure 1. Although the more attractive family of Quadrature Amplitude Modulation (QAM) schemes [11] can be readily used for generating the information symbols $\{b_l\}$ from the source data bits, here we limit the symbol mapping scheme to M-ary phase shift keying (MPSK). Furthermore, we limit our choice of the spreading sequences to those having elements satisfying $|c_l[n]| = 1$. The normalised complex envelope of an MPSK modulated multi-code MC-CDMA signal may be represented for the duration of a symbol period T as:

$$s(t) = \frac{1}{\sqrt{N}} \sum_{l=0}^{L-1} \sum_{n=0}^{N-1} b_l \, c_l[n] \, e^{j2\pi F n \frac{t}{T}} , \qquad (1)$$

where L is the number of simultaneously transmitted symbols, and F is the subcarrier separation parameter [1] invoked for mapping the

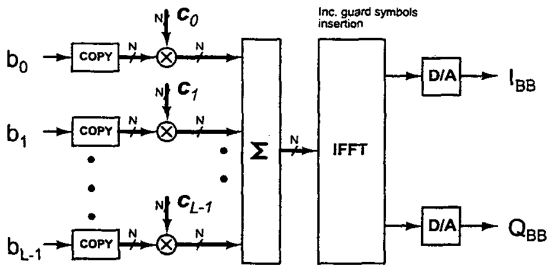


Figure 1: Multi-code MC-CDMA transmitter schematic

spread information symbol to N subcarriers that are sufficiently far apart in the frequency domain, in order to experience independent fading of the different subcarriers. The envelope power $|s(t)|^2$ of the MPSK modulated multi-code MC-CDMA signal was expressed as [10]:

$$|s(t)|^2 = \frac{2}{N}\mathrm{Re}\left[\sum_{n=0}^{N-1}(A[n]+X[n])\,e^{j2\pi Fn\frac{t}{T}}\right], \qquad (2)$$

where the combined auto-correlation $A[n]$ and the combined cross-correlation $X[n]$ are given as:

$$A[n] \triangleq \sum_{l=0}^{L-1} A_l[n] \;\; \text{for } n \neq 0, \;\; A[0] \triangleq NL/2 \qquad (3)$$

$$X[n] \triangleq \sum_{l=0}^{L-1}\sum_{l'=0, l'\neq l}^{L-1} b_l b_{l'}^* \, X_{l,l'}[n]\,. \qquad (4)$$

The aperiodic autocorrelations of the l-th spreading code $A_l[n]$ in (3) are defined by

$$A_l[n] \triangleq \sum_{i=0}^{N-n-1} c_l[i]\, c_l^*[i+n]\,, \qquad (5)$$

while the aperiodic cross-correlations between the l-th and l'-th spreading codes $X_{l,l'}[n]$ in (4) are defined by

$$X_{l,l'}[n] \triangleq \sum_{i=0}^{N-n-1} c_l[i]\, c_{l'}^*[i+n]\,, \qquad (6)$$

where the term 'aperiodic' indicates that the correlation is calculated as if the sequences were transmitted as one-shot sequences followed by all-zero sequences.

Again, the crest factor (CF) of the multi-code MC-CDMA signal $s(t)$ is defined as the ratio of the peak to the RMS amplitude [4]:

$$\mathrm{CF} \triangleq \frac{||s||_\infty}{||s||_2}\,, \qquad (7)$$

where $||s||_\infty = \max_t |s(t)|$ and $||s||_2 = \sqrt{\frac{1}{T}\int_T |s(t)|^2 dt}$. CF values are often represented in terms of dB as $20\log_{10}(\mathrm{CF})$.

3. SHAPIRO-RUDIN BASED SPREADING SEQUENCES

In Section 2 we have seen that the envelope power of the multi-code MC-CDMA signal depends on the combined aperiodic autocorrelation $A[n]$ of (3) and on the combined aperiodic crosscorrelation $X[n]$ of (4). In this section, we characterise the CF properties of Shapiro-Rudin code based spreading sequences, which exhibit zero or low values of $A[n]$ and $X[n]$.

While studying an absolute bound for a sum of trigonometric polynomials having binary coefficients, *Shapiro* found [5] a pair of binary sequences having a low bound. These sequences are referred to as Shapiro-Rudin sequences, which are defined recursively as:

$$\mathbf{s}_0 = \mathbf{c}_0 = 1\,, \qquad (8)$$

$$\mathbf{s}_{n+1} = \mathbf{s}_n\mathbf{c}_n\,, \qquad (9)$$

$$\mathbf{c}_{n+1} = \mathbf{s}_n\bar{\mathbf{c}}_n\,, \qquad (10)$$

where the sequence $\mathbf{ab} = (a_1a_2\cdots a_{N-1}b_1b_2\cdots b_{N-1})$ represents a concatenation of sequences $\mathbf{a} = (a_1a_2\cdots a_{N-1})$ and $\mathbf{b} = (b_1b_2\cdots b_{N-1})$, and $\bar{\mathbf{a}} = (-a_1 - a_2\cdots - a_{N-1})$ denotes the negation of the original sequence $\mathbf{a}$. Later it was recognised [12] that Shapiro-Rudin sequences constitute a special case of *Golay*'s complementary sequences [13]. A pair of equally long sequences is said to be a complementary pair, if their combined aperiodic autocorrelation defined in (3) for $L = 2$ is zero, except at zero shift. As we mentioned before, this is the property we want to retain in order to maintain low crest factors. The crest factor property of the Shapiro-Rudin pair in the context of a two-code BPSK modulated MC-CDMA scheme is presented in the form of the next theorem.

Theorem 1 *The crest factor of the two-code BPSK modulated MC-CDMA signal employing a pair of Shapiro-Rudin sequences as its spreading sequence is bounded by 3dB.*

The proof of this and the other theorems is beyond of the scope of this paper and can be found in [14, Ch. 4]. However it is worth mentioning that since a Shapiro-Rudin sequence pair is a complementary pair [13], the combined aperiodic auto-correlation of (3) becomes $A[n] = 0$ for $n > 1$. Interestingly, the corresponding combined aperiodic cross-correlation of (4) also becomes $X[n] = 0$ for $n > (N/2)$ [14, Ch. 4].

As an example, $\mathbf{s}_4$ and $\mathbf{c}_4$ of length $N = 2^4 = 16$ can be obtained by applying (8) through (10) repeatedly, and the corresponding sequences are given as:

$$\mathbf{s}_4 = (1\;1\;1\;{-1}\;1\;1\;{-1}\;1\;1\;1\;1\;{-1}\;{-1}\;{-1}\;1\;{-1}) \qquad (11)$$

$$\mathbf{c}_4 = (1\;1\;1\;{-1}\;1\;1\;{-1}\;1\;{-1}\;{-1}\;{-1}\;1\;1\;1\;{-1}\;1)\,. \qquad (12)$$

The associated phasor transition diagram is depicted in Figure 2(a) for the two-code BPSK modulated MC-CDMA signal $s(t)$ of (1) employing $\mathbf{s}_4$ and $\mathbf{c}_4$ of (11) and (12), respectively. It can be observed that the magnitude is always bounded by 2 and the corresponding crest factor is given by CF=$\sqrt{2}$=3dB.

Another interesting pair of orthogonal spreading sequences is constituted by the so-called sub-complementary sequences. Sivaswamy introduced sub-complementary sequences in the context of radar signal design [15]. A pair of sequences of length $N = 2^k N_o$, $k \geq 1$, is referred to as a sub-complementary pair, if the combined aperiodic

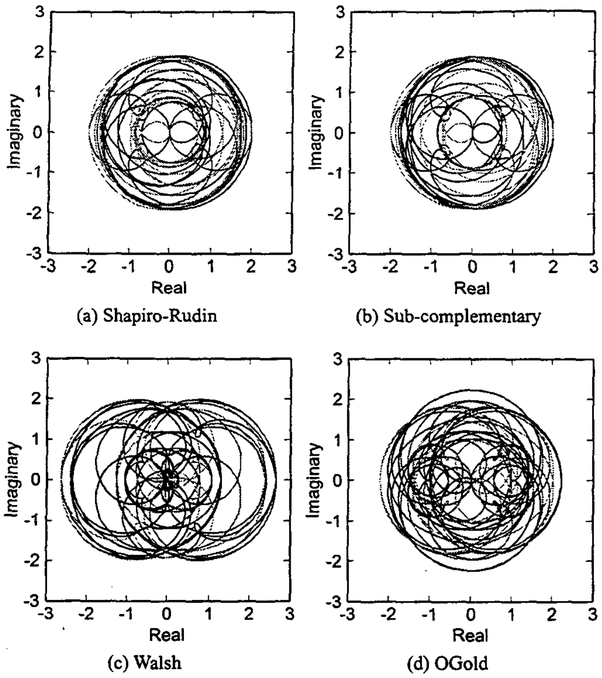


Figure 2: Phasor diagrams of two-code BPSK modulated MC-CDMA signal employing various spreading sequences of length $N = 16$

autocorrelation $A[n]$ of (3) is zero for $n > N_o$. A method of constructing sub-complementary pairs was also given in [15] as follows. Let $\mathbf{s}_{m-1}$ be any sequence of length $N/2$. Then, a pair of sequences $\mathbf{s}_m$ and $\hat{\mathbf{s}}_m$, constructed as [15]:

$$\mathbf{s}_m = \mathbf{s}_{m-1}\mathbf{s}_{m-1} \quad (13)$$

$$\hat{\mathbf{s}}_m = \mathbf{s}_{m-1}\bar{\mathbf{s}}_{m-1} \, , \quad (14)$$

is known to form a sub-complementary pair [15]. Sivaswamy's sub-complementary pairs exhibit similar crest factor characteristics to those of the Shapiro-Rudin sequences in the context of two-code BPSK modulated MC-CDMA systems.

Theorem 2 *If we apply a Shapiro-Rudin sequence [5] of length $N/2$ as the base sequence $\mathbf{s}_{m-1}$, then the sub-complementary pair [15] $\mathbf{s}_m$ and $\hat{\mathbf{s}}_m$ of (13) and (14) produces a crest factor of less than or equal to 3dB, provided that they are applied in two-code BPSK modulated MC-CDMA systems.*

The corresponding combined aperiodic correlation terms exhibit dual properties in comparison to those of the Shapiro-Rudin pair. In other words, the combined aperiodic cross-correlation becomes $X[n] = 0$ for all n and the corresponding autocorrelation becomes $A[n] = 0$ for $n > (N/2)$. An important consequence of $X[n] = 0$ for all n is that the envelope power does not depend on the message bits.

For example, when we take $\mathbf{s}_3$ of (9) as the base sequence of Sivaswamy's sub-complementary pair, we have:

$$\mathbf{s}_4 = (1\,1\,1\,-1\,1\,1\,-1\,1\,1\,1\,1\,-1\,1\,1\,-1\,1) \quad (15)$$

$$\hat{\mathbf{s}}_4 = (1\,1\,1\,-1\,1\,1\,-1\,1\,-1\,-1\,-1\,1\,-1\,-1\,1\,-1)\, . \quad (16)$$

The corresponding phasor transition diagram is depicted in Figure 2(b) for the two-code BPSK modulated MC-CDMA signal $s(t)$ of (1) employing $\mathbf{s}_4$ and $\hat{\mathbf{s}}_4$ of (15) and (16), respectively. Similarly to the Shapiro-Rudin pair based scheme, it can be observed that the corresponding crest factor is bounded by 3dB. Figure 2(c) and 2(d) illustrate the phasor diagrams of Walsh and orthogonal Gold (OGold) spread schemes having the sequence length of $N = 16$, respectively. There exists $\binom{N}{L} = \binom{16}{2} = 120$ possible ways of selecting a pair of spreading sequences out of 16 Walsh codes or 16 OGold codes. The phasor transition diagrams in Figure 2(c) and 2(d) correspond to the best pair of sequences resulting in the lowest corresponding crest factors of 5.45dB and 3.96dB for Walsh- and OGold-spread schemes.

So far we have concentrated our attention on the cases, when two symbols are transmitted simultaneously. Let us now extend our discussions to the scenario, where four spreading sequences are used simultaneously. Tseng and Liu [16] introduced a wider range of complementary sets of sequences. A set of sequences of equal length is said to be a complementary set, if the sum of autocorrelations of all the sequences in that set is zero, except for the peak term in zero-shift position [16]. Sivaswamy [15] proposed a method for constructing a complementary set of 2^{n+1} number of sequences of length $2^n N$, where $n \geq 1$ and N is the length of the base complementary pair. When $n = 1$, we can obtain a complementary set of four sequences of length N from a base complementary pair $\mathbf{s}_0$ and $\mathbf{c}_0$ of length $N/2$ using the following method [15]:

$$\mathbf{S}_1 = \mathbf{s}_0\mathbf{s}_0 \quad (17)$$

$$\mathbf{S}_2 = \mathbf{s}_0\bar{\mathbf{s}}_0 \quad (18)$$

$$\mathbf{S}_3 = \mathbf{c}_0\mathbf{c}_0 \quad (19)$$

$$\mathbf{S}_4 = \mathbf{c}_0\bar{\mathbf{c}}_0 \, . \quad (20)$$

Theorem 3 *If we apply a pair of Shapiro-Rudin sequences [5] of length $N/2$ as the base sequences $\mathbf{s}_0$ and $\mathbf{c}_0$, the set of four complementary sequences, given by (17) - (20), produces a crest factor of less than or equal to 3dB, provided that the sequences $\mathbf{S}_1$, $\mathbf{S}_2$, $\mathbf{S}_3$ and $\mathbf{S}_4$ are applied to four-code BPSK modulated MC-CDMA systems.*

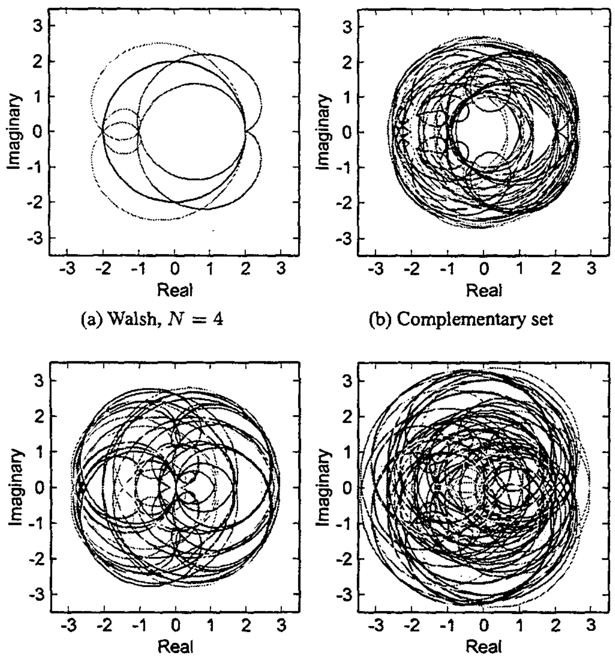
A special case of Sivaswamy's complementary set is a set of four Walsh codes of length $N = 4$. When we take a pair of Shapiro-Rudin sequence having a length of $N = 2$, *ie* $\mathbf{s}_0 = (1\ 1)$ and $\mathbf{c}_0 = (1\ -1)$, we have four Walsh codes of length $N = 4$ by applying (17) - (20). Figure 3(a) depicts the corresponding phasor diagrams for all the possible message sequences. The half of the total number of message sequences results in a circular phasor transition, corresponding to the ideal CF of 1 or 0dB.

For the sequence length of $N = 16$, let us take a pair of Shapiro-Rudin sequences of length $N/2 = 8$ as the base sequences, which can be obtained using (8) to (10) as:

$$\mathbf{s}_0 = (1\,1\,1\,-1\,1\,1\,-1\,1) \quad (21)$$

$$\mathbf{c}_0 = (1\,1\,1\,-1\,-1\,-1\,1\,-1)\, . \quad (22)$$

When we apply $\mathbf{s}_0$ and $\mathbf{c}_0$ of (21) and (22) to (17) - (20), we arrive at



(a) Walsh, $N = 4$ (b) Complementary set

(c) Walsh (d) OGold

Figure 3: Phasor diagrams of four-code BPSK modulated MC-CDMA signal employing various spreading sequences of length $N = 16$ except for (a) where $N = 4$

a Sivaswamy complementary set of four spreading sequences:

$$\mathbf{S}_1 = (1\,1\,1\,-1\,1\,1\,-1\,1\,1\,1\,1\,-1\,1\,1\,-1\,1) \quad (23)$$
$$\mathbf{S}_2 = (1\,1\,1\,-1\,1\,1\,-1\,1\,-1\,-1\,-1\,1\,-1\,-1\,1\,-1) \quad (24)$$
$$\mathbf{S}_3 = (1\,1\,1\,-1\,-1\,-1\,1\,-1\,1\,1\,1\,-1\,-1\,-1\,1\,-1) \quad (25)$$
$$\mathbf{S}_4 = (1\,1\,1\,-1\,-1\,-1\,1\,-1\,-1\,-1\,-1\,1\,1\,1\,-1\,1)\,. \quad (26)$$

We can readily verify that they form an orthogonal set and hence these four sequences can be used as the spreading sequences of four-code BPSK modulated MC-CDMA systems. The corresponding phasor diagram is depicted in Figure 3(b), where we can observe that the magnitude is bounded by $2\sqrt{2}$. Also shown are the phasor diagrams of the best case Walsh- and OGold-spread MC-CDMA schemes. In order to arrive at the best case sequence combination, a set of four sequence pairs was chosen out of the $\binom{16}{4} = 1820$ possible combinations, resulting in the lowest crest factor. Comparing the phasor diagrams in Figure 3 we can observe the advantage of the four-code MC-CDMA scheme employing Sivaswamy's complementary set in comparison to Walsh- and OGold-spread MC-CDMA schemes.

Figure 4 depicts the peak-factors of some of Sivaswamy's complementary sets constituted by $L = 4$ sequences. Since the sequences generated using (17) - (20) constitutes a complementary set, the corresponding combined aperiodic auto-correlation becomes $A[n] = 0$ for $n > 0$. On the other hand, the cross-correlation counter-part $X[n]$ has three different values, depending on the message sequence, resulting in different crest factors represented by three different markers in Figure 4. Half the number of the message sequences belong to the case represented by the marker ○ and a quarter to those denoted by each of the marker □ and ⋆. We can observe in Figure 4 that the peak factors associated with the marker ○ are the lowest among the three values, when the sequence length N is $2^2 = 4$, $2^4 = 16$ or $2^6 = 64$. An ideal peak factor of 1 was achieved for the sequence length of $N = 2^2 = 4$, representing a constant power envelope. The fact that all the crest factors in Figure 4 are bounded by 3dB corroborates Theorem 3.

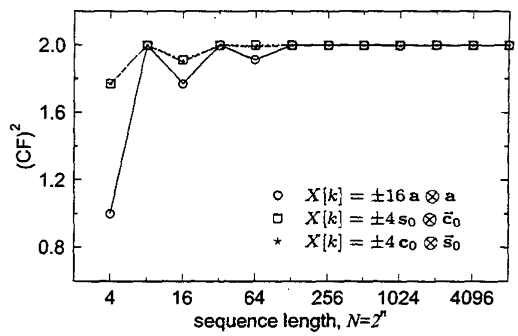


Figure 4: The crest-factors of Sivaswamy's complementary set of sequences. The number of simultaneously used sequences is $L = 4$ and the sequence length of $N = 2^n$ is between $2^2 = 4$ and $= 2^{13} = 8192$.

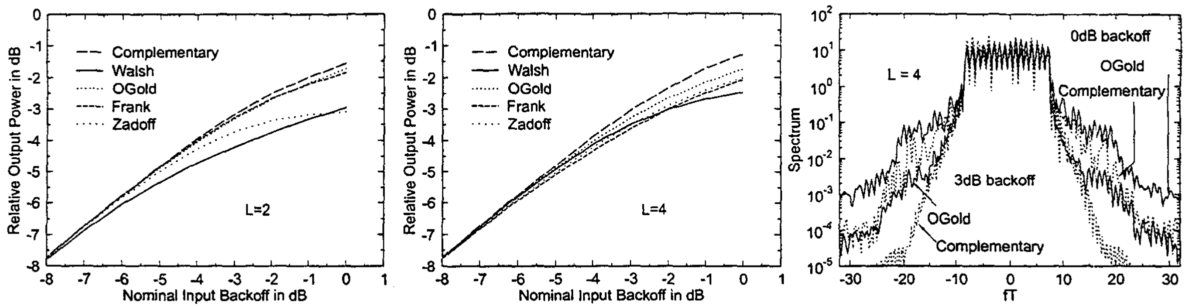
4. EFFECTS OF CLIPPING

In the previous section we observed that MC-CDMA signals exhibit a fluctuating envelope power and hence when they are subjected to non-linear amplification, they produce spurious out-of-band emissions. The extreme peaks of the signal inevitably suffer from so-called 'clipping effects', when they enter the saturation region of the amplifier, resulting in the loss of effective transmission power as well as in signal distortion, unless the gain of the amplifier is reduced to a level, where the amplifier's maximum output level is sufficiently high for the signal peaks not to be clipped. Many portable wireless devices use Solid-State-Power-Amplifiers (SSPA). The amplitude transfer characteristics or the AM-AM curve $g(x)$ of an SSPA can be modeled as [17, 18]:

$$g(x) = \frac{x}{(1 + x^{2p})^{1/2p}}\,, \quad (27)$$

where x is the input magnitude and p is the smoothness factor during its transition from the linear region to the saturation region. Considering that the output power of typical SSPAs at the 1dB compression point is about 0.7 dB below the saturation power, the value of smoothness factor in (27) becomes $p = 3.58$.

The power loss due to the above mentioned 'clipping effects' of a nonlinear amplifier model of (27) can be observed in Figure 5. For the two-code BPSK modulated MC-CDMA schemes seen in Figure 5(a), the MC-CDMA signal employing a complementary pair of Shapiro-Rudin codes showed the lowest power loss of 1.6dB for the set of five spreading codes investigated. A pair of OGold codes and a pair of Frank codes also exhibited similar behaviour, showing an associated power loss of 1.7dB and 1.8dB, respectively. Finally, the Walsh



(a) Power loss: $L = 2$ (b) Power loss: $L = 4$ (c) Spectrum : $L = 4$

Figure 5: Relative output power versus input back-off in multi-code BPSK modulated MC-CDMA employing various spreading codes of length $N = 16$

codes and Zadoff-Chu codes exhibited a high power loss of around 3dB at 0dB input back-off. The spreading code pairs were selected on the basis of maintaining the lowest crest-factors for each family of spreading codes.

When four spreading codes were used simultaneously, as seen in Figure 5(b), the effective transmission power losses at 0dB input back-off were 1.3dB for Sivaswamy's complementary set, 1.8dB for OGold codes, 2.0dB for Zadoff-Chu codes, 2.1dB for Frank codes and 2.5dB for Walsh codes. Considering that the best combination of Walsh codes exhibited a lower maximum CF than that of OGold codes in Section 3, the Walsh code related high power loss is surprising. This implies that the Walsh-spread four-code MC-CDMA signal still exhibits a relatively poor magnitude distribution.

The non-linear amplification of MC-CDMA signals generates high out-of-band power spectrum. Figure 5(c) compares the power spectra of four-code BPSK modulated MC-CDMA signals employing OGold and Sivaswamy's complementary sets of length $N = 16$. A raised cosine shaping was used for reducing the out-of-band power. All the message sequences were applied and their corresponding spectra were averaged for generating Figure 5(c). The spectrum corresponding to Sivaswamy's complementary set exhibits the lowest out-of-band power spillage. The spectrum of the Walsh-spread scheme fell between that of the OGold and Sivaswamy's complementary sets.

5. CONCLUSIONS

The crest factors of two- and four-code BPSK modulated MC-CDMA signals employing three different Shapiro - Rudin code based spreading sequences were shown to be bounded by 3dB. The corresponding low crest factor of the MC-CDMA signal employing Shapiro - Rudin based sequences resulted in lower power loss and lower out-of-band spectrum, when the signal was subjected to non-linear amplification, in comparison to Walsh- and OGold-spread MC-CDMA signals.

6. REFERENCES

[1] N. Yee, J. P. Linnartz, and G. Fettweis, "Multicarrier CDMA in indoor wireless radio networks," in *IEEE PIMRC'93*, pp. 109–113, 1993.

[2] K. Fazel and L. Papke, "On the performance of convolutionally-coded CDMA/OFDM for mobile communication system," in *IEEE PIMRC'93*, pp. 468–472, 1993.

[3] J. G. Proakis, *Digital Communications*. Mc-Graw Hill, 3rd ed., 1995.

[4] S. Boyd, "Multitone signals with low crest factor," *IEEE Trans. on Circuits and Systems*, vol. 33, pp. 1018–1022, October 1986.

[5] W. Rudin, "Some theorems on Fourier coefficients," *Proc. of American Mathematics Society*, vol. 10, pp. 855–859, December 1959.

[6] A. E. Jones, T. A. Wilkinson, and S. K. Barton, "Block coding scheme for reduction of peak to mean envelope power ratio of multicarrier transmission schemes," *Electronics Letters*, vol. 30, pp. 2098–2099, December 1994.

[7] R. L. Frank, "Phase shift pulse codes with good periodic correlation properties," *IRE Trans. on Information Theory*, vol. 8, pp. 381–382, October 1962.

[8] D. C. Chu, "Polyphase codes with good periodic correlation properties," *IEEE Trans. on Information Theory*, vol. 18, pp. 531–532, July 1972.

[9] R. L. Frank, "Comments on polyphase codes with good correlation properties," *IEEE Trans. on Information Theory*, vol. 19, p. 244, March 1973.

[10] B. J. Choi, E. L. Kuan, and L. Hanzo, "Crest–factor study of MC-CDMA and OFDM," in *IEEE VTC'99 (Fall)*, vol. 1, (Amsterdam, Netherlands), pp. 233–237, 19–22 September 1999.

[11] L. Hanzo, W. T. Webb, and T. Keller, *Single- and Multi-carrier Quadrature Amplitude Modulation*. New York, USA: IEEE Press-John Wiley, April 2000.

[12] B. M. Popović, "Synthesis of power efficient multitone signals with flat amplitude spectrum," *IEEE Trans. on Communications*, vol. 39, no. 7, pp. 1031–1033, 1991.

[13] M. J. E. Golay, "Complementary series,," *IRE Trans. on Information Theory*, vol. IT-7, pp. 82–87, 1961.

[14] B. J. Choi, *Multi Carrier Code Division Multiple Access*. PhD thesis, Department of Electronics and Computer Science, University of Southampton, UK, 2001.

[15] R. Sivaswamy, "Digital and analog subcomplementary sequences for pulse compression," *IEEE Trans. on Aerospace and Electronic Systems*, vol. 14, pp. 343–350, March 1978.

[16] C.-C. Tseng and C. L. Liu, "Complementary sets of sequences," *IEEE Trans. on Information Theory*, vol. 18, pp. 644–652, September 1972.

[17] E. Costa, M. Midrio, and S. Pupolin, "Impact of amplifier nonlinearities on OFDM transmission system performance," *IEEE Comm. Letters*, vol. 3, pp. 37–39, February 1999.

[18] G. Santella and F. Mazzanga, "A model for performance evaluation in M-QAM-OFDM schemes in presence of non-linear distortions," in *IEEE VTC 1995*, pp. 830–834, July 1995.