Probability of Error Calculation of OFDM Systems With Frequency Offset

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***Abstract*—Orthogonal frequency-division multiplexing (OFDM) is sensitive to the carrier frequency offset (CFO), which destroys orthogonality and causes intercarrier interference (ICI). Previously, two methods were available for the analysis of the resultant degradation in performance. Firstly, the statistical average of the ICI could be used as a performance measure. Secondly, the bit error rate (BER) caused by CFO could be approximated by assuming the ICI to be Gaussian. However, a more precise analysis of the performance (i.e., BER or SER) degradation is desirable. In this letter, we propose a precise numerical technique for calculating the effect of the CFO on the BER or symbol error in an OFDM system. The subcarriers can be modulated with binary phase shift keying (BPSK), quaternary phase shift keying (QPSK), or 16-ary quadrature amplitude modulation (16-QAM), used in many OFDM applications. The BPSK case is solved using a series due to Beaulieu. For the QPSK and 16-QAM cases, we use an infinite series expression for the error function in order to express the average probability of error in terms of the two-dimensional characteristic function of the ICI.**

***Index Terms*—Bit error rate, carrier frequency offset, intercarrier interference, orthogonal frequency-division multiplexing, symbol error rate.**

I. INTRODUCTION

ORTHOGONAL frequency-division multiplexing (OFDM) has been accepted for several wireless LAN standards, as well as mobile multimedia applications [1]. It is, however, sensitive to carrier frequency offset (CFO) and phase noise; for a large number of subcarriers these imperfections will destroy subcarrier orthogonality and introduce intercarrier interference (ICI). The CFO is caused by misalignment in carrier frequencies or Doppler shift. Several methods have been developed to reduce the effect of the CFO on OFDM [2]–[5].

In these studies, the effect of the CFO is calculated in two ways. It may be derived as a loss in the ratio between the signal power and the statistical average of the ICI. Alternatively, computer simulations may be used to obtain the performance degradation caused by the ICI. However, it is also both interesting and useful to know the precise correlation between the bit error rate (BER) or signal error rate (SER) and the CFO. In [6], the approximate degradation in signal-to-noise ratio (SNR) caused by the CFO is derived using simple algebraic manipulation.

In this letter, we propose a precise technique for calculating the effect of the CFO on the BER or SER in an OFDM system. The subcarriers are modulated with binary phase-shift keying (BPSK), quaternary phase-shift keying (QPSK), or 16-ary quadrature amplitude modulation (16 QAM), which are common modulation formats used in OFDM applications. The BPSK case is solved using a series from Beaulieu. For the QPSK and 16-QAM cases, we use an infinite series expression for the error function (again from Beaulieu) to express the average probability of error in terms of the two-dimensional (2-D) characteristic function (CHF) of the ICI. The technique readily achieves an accuracy in the region of 12 significant figures.

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II. ASSUMPTIONS AND PROBLEM STATEMENT

We assume an ideal additive white Gaussian noise (AWGN) channel. The CFO does not change during one OFDM symbol. The sampled signal for the kth subchannel after the receiver fast Fourier transform processing can be written as [2]–[6]

$$y_k = X_k S_0 + \sum_{l=0,\, l\neq k}^{N-1} S_{l-k} X_l + n_k, \qquad k = 0, 1, \ldots, N-1 \quad (1)$$

where X_k denotes the transmitted symbol for the kth subcarrier, n_k is a complex Gaussian noise sample (with its real and imaginary components being independent and identically distributed with variance σ^2), and N is the number of subcarriers. The second term in (1) is the ICI term caused by the CFO. The sequence S_k (ICI coefficients) depends on the CFO and is given by [3]–[5]

$$S_k = \frac{\sin \pi(k+\varepsilon)}{N \sin \dfrac{\pi}{N}(k+\varepsilon)} \exp\left[j\pi \left(1 - \frac{1}{N}\right)(k+\varepsilon)\right] \quad (2)$$

where ε is the normalized frequency offset which is the ratio between the CFO and the adjacent subcarrier spacing. For zero frequency offset, S_k reduces to the unit impulse sequence.

We assume the data symbols X_k's are independent and identically distributed random variables (RVs). For M-ary signaling, X_k is equally likely to assume one out of M levels. Without any loss in generality, we will consider the error rate for the zeroth subcarrier (i.e., $k = 0$). So the problem at hand is to determine, for a given symbol sent on the zeroth subcarrier, the probability that an incorrect decision will be made.

A. Average Probability of Error

1) BPSK Modulation: For BPSK modulation, $X_k \in \{1, -1\}$ and without incurring a loss in generality, it will be sufficient to consider the real part of (1). The CHF of $\Re(y_k)$ can be expressed as

$$\phi(\omega) = e^{j\omega X_k \Re\{S_0\} - (\omega^2\sigma^2/2)} \prod_{l=0,\, l\neq k}^{N-1} \cos(\omega \Re\{S_{l-k}\}) \quad (3)$$

where $\Re\{z\}$ denotes the real part of z. From the Beaulieu Series [7], [8], the cumulative distribution function (CDF) of an RV can be expressed using its CHF, i.e.

$$\Pr(X < x) = \frac{1}{2} - \sum_{n \in N_o}^{\infty} \frac{2\Im\{e^{-jn\omega_0 x}\phi(n\omega_0)\}}{n\pi} + \epsilon(x, \omega_0) \quad (4)$$

where $\Im\{z\}$ denotes the imaginary part of z, $\omega_0 = 2\pi/T$, T is a parameter governing the sampling rate in the frequency domain, and $\epsilon(x, \omega_0)$ is an error term.[1] The index set $N_0 = \{1, 3, \ldots\}$ is the set of all positive odd integers. Without loss of generality, we consider the first subcarrier and the transmitted symbol $X_0 = 1$. Now, a decision error occurs if the real part of y_0 (1) is less than zero. Thus, the probability of a bit error is

$$P_b = \frac{1}{2} - \sum_{n \in N_o}^{\infty} \frac{\sin[n\omega_0 \Re\{S_0\}] e^{-(1/2)\omega_0^2\sigma^2}}{n\pi} \prod_{l=1}^{N-1} \times \cos(n\omega_0 \Re\{S_l\}) + \epsilon(0, \omega_0). \quad (5)$$

Here, the error term can be made negligibly small by selecting a sufficiently large T. The numerical evaluation of (5) gives the exact probability of a bit error.

It is worthwhile to explain how $\epsilon(x, \omega_0)$ originates. The CDF of an RV X may be written exactly as an infinite integral using the Gil–Pelaez theorem. In (4), this infinite integral is replaced by a series, which can be viewed as approximating the integral by a trapezoidal sum. Thus, $\epsilon(x, \omega_0)$ is the error introduced due to this discretization process. Reference [8] shows that $\epsilon(x, \omega_0) = \sum (-1)^{n+1} f(x + nT/2)$ where $f(x)$ is the probability density function (PDF). Most PDFs for physical applications have rapidly decaying tails. As a result, for the problem at hand, $\epsilon(x, \omega_0)$ can be made arbitrarily small provided T is large compared to the tails of $x = y_k$. Although y_k is not strictly bounded, the probability that real or imaginary part of y_k exceeding $T_1 = \max_l |X_l \sum_k |S_k| + 3\sigma$ is very small. Thus, T can be chosen to be sufficiently larger than T_1.

2) QPSK Modulation: For the QPSK case, the above approach cannot be applied directly because it deals with one-dimensional (1-D) signaling. Moreover, as can be seen below, the error probability is now a product involving the error function. Using an infinite series representation of the error function, we therefore transform the error probability into a form that is suitable for averaging.

Let us define the noise-free received signal point conditional on the ICI for the kth subcarrier as

$$z = X_k S_0 + \sum_{l=0,\, l \neq k}^{N-1} S_{l-k} X_l \quad (6)$$

where $X_l \in \{\pm 1 \pm j\}$ for QPSK signaling. The variable z is simply the difference $y_k - n_k$ in (1) and the subscript k has been dropped for notational brevity. The 2-D CHF of z is given by

$$\phi(\omega_I, \omega_Q) = \left\langle e^{j[\omega_I \Re\{z\} + \omega_Q \Im\{z\}]} \right\rangle \quad (7)$$

[1]Note that the error term depends on both x and T. For a given x, the error can be arbitrarily reduced by increasing T. In practice, once a suitable T is selected, it is used for all possible values of x.

where $\langle . \rangle$ denotes the average value over the distribution of z. Without loss of generality, we will assume that the transmitted symbol in the first subcarrier is $X_0 = (1 + j)$. Since X_ls are independent and identically distributed RVs, the CHF becomes

$$\phi(\omega_I, \omega_Q) = e^{j[\omega_I(\Re\{S_0\} - \Im\{S_0\}) + \omega_Q(\Im\{S_0\} + \Re\{S_0\})]} \cdot \prod_{l=1}^{N-1} \left(\cos(\omega_I \Re\{S_l\} + \omega_Q \Im\{S_l\}) \times \cos(\omega_I \Im\{S_l\} - \omega_Q \Re\{S_l\})\right). \quad (8)$$

The probability of a correct decision is the probability that $z + n$ is inside the first quadrant in the complex plane. Since the real and imaginary components of the additive noise are independent and identically distributed with a variance of σ^2, the correct decision probability is given by

$$\begin{aligned} P_c&(z + n \in D_1 | X_0 = 1 + j, z) \\ &= \int_0^\infty \frac{1}{\sqrt{2\pi}\sigma} e^{((-(x - \Re\{z\})^2)/2\sigma^2)} dx \\ &\quad \times \int_0^\infty \frac{1}{\sqrt{2\pi}\sigma} e^{((-(y - \Im\{z\})^2)/2\sigma^2)} dy \\ &= Q\left(\frac{-\Re\{z\}}{\sigma}\right) Q\left(\frac{-\Im\{z\}}{\sigma}\right) \end{aligned} \quad (9)$$

where D_1 is the first quadrant in the complex plane and $Q(x) = \int_x^\infty (e^{-t^2/2}/\sqrt{2\pi})\, dt$ denotes the Gaussian CDF and has the series representation [7], [9]

$$Q(x) = \frac{1}{2} - \frac{2}{\pi} \sum_{m \in N_o}^{\infty} \frac{\exp(-m^2\omega_0^2/2)\sin(m\omega_0 x)}{m} + \epsilon(x, \omega_0) \quad (10)$$

where $\epsilon(x, \omega_0)$ is the approximation error. Neglecting it and substituting (10) into (9), we find (11) (shown at the bottom of the next page). Next we need the average of this probability, $P_c(z + n \in D_1 | X_0 = 1 + j, z)$, over the distribution of z, which readily follows from the 2-D CHF in (7). Since $\sin\theta = \Im\{e^{j\theta}\}$, for instance, the average of $\sin(\omega \Re\{z\})$ can be given by $\Im\{\phi(\omega, 0)\}$. Consequently, the average of the first and second infinite series in (11) can be obtained using $\Im\{\phi(m\omega_0/\sigma, 0)\}$ and $\Im\{\phi(0, m\omega_0/\sigma)\}$. The sine product term in the double infinite series can be written as a sum of two cosine terms and (7) can again be used to get the average. Therefore, the average probability of correct decision may be expressed as (12) (shown at the bottom of the next page). Subsequently, the probability of symbol error is given by

$$P_e = 1 - P_{c,\,\mathrm{av}}. \quad (13)$$

3) 16-QAM Modulation: The 16-QAM signal alphabet is the set $\Lambda_{16} = \{\pm 1 \pm j, \pm 3 \pm j, \pm 1 \pm 3j, \pm 3 \pm 3j\}$. Without loss of generality, we assume the transmitted symbol X_0 to be in the first quadrant; $X_0 \in \{1 + j, 3 + j, 1 + 3j, 3 + 3j\}$. Just as with the QPSK case, the correct decision probability can be written as a product of the error function. It is therefore convenient to define a function

$$\Psi(\alpha, \beta) = Q\left(\frac{\alpha - \Re\{z\}}{\sigma}\right) Q\left(\frac{\beta - \Im\{z\}}{\sigma}\right) \quad (14)$$

where α and β represent the decision region boundaries. It will soon become evident that the required error probabilities can be expressed in terms of this function.

The CHF of z in this case is

$$\phi(\omega_I, \omega_Q) = e^{j[\omega_I \Re\{S_0 X_0\} + \omega_Q \Im\{S_0 X_0\}]} \times \prod_{l=1}^{N-1} \frac{1}{16} \sum_{X_l \in \Lambda_{16}} e^{j[\omega_I \Re\{S_l X_l\}) + \omega_Q \Im\{S_l X_l\})]}. \tag{15}$$

We can now evaluate the average value of (14) using this CHF. Substituting (10) into (14) and using the same approach as in (13), it can readily be shown that

$$\begin{aligned}\langle\Psi(\alpha, \beta)\rangle &= \frac{1}{4} - \sum_{m \in N_o}^{\infty} \frac{\exp(-m^2\omega_0^2/2)\Im\left\{e^{j(m\omega_0\alpha/\sigma)}\phi\left(\frac{-m\omega_0}{\sigma}, 0\right)\right\}}{m\pi} \\ &\quad - \sum_{n \in N_o}^{\infty} \frac{\exp(-n^2\omega_0^2/2)\Im\left\{e^{j(n\omega_0\beta/\sigma)}\phi\left(0, \frac{-n\omega_0}{\sigma}\right)\right\}}{n\pi} \\ &\quad + \sum_{n \in N_o}^{\infty} \sum_{m \in N_o}^{\infty} \frac{2\exp\left(-(n^2+m^2)\omega_0^2/2\right)}{nm\pi^2} \\ &\quad \times \Re\left\{e^{j(m\omega_0\alpha - n\omega_0\beta)/\sigma)}\phi\left(\frac{-m\omega_0}{\sigma}, \frac{n\omega_0}{\sigma}\right) - e^{j(m\omega_0\alpha + n\omega_0\beta/\sigma)}\phi\left(\frac{-m\omega_0}{\sigma}, \frac{-n\omega_0}{\sigma}\right)\right\}.\end{aligned} \tag{16}$$

Note that, for $\alpha = \beta = 0$, this reduces to (12). This, (16), can be used to find the probability of a correct decision given the transmission of a particular signal constellation point.

For instance, consider $X_0 = 1 + j$. The correct decision region in the complex plane is now a square denoted by D_s with vertices at (0, 0), (0, 2), (2, 0), and (2, 2). Thus, we need the probability that $z + n$ lies inside this square. This conditional probability can be expressed as

$$\begin{aligned}&P_c(z + n \in D_s | X_0 = 1 + j, z) \\ &\quad = \int_0^2 \frac{1}{\sqrt{2\pi}\sigma} e^{((-(x-\Re\{z\})^2)/2\sigma^2)\big)} dx \\ &\quad\quad \times \int_0^2 \frac{1}{\sqrt{2\pi}\sigma} e^{((-(y-\Im\{z\})^2)/2\sigma^2)\big)} dy \\ &\quad = \left[Q\left(\frac{-\Re\{z\}}{\sigma}\right) - Q\left(\frac{2-\Re\{z\}}{\sigma}\right)\right] \\ &\quad\quad \times \left[Q\left(\frac{-\Im\{z\}}{\sigma}\right) - Q\left(\frac{2-\Im\{z\}}{\sigma}\right)\right] \\ &\quad = \Psi(0, 0) - \Psi(0, 2) - \Psi(2, 0) + \Psi(2, 2).\end{aligned} \tag{17}$$

As a result, the average $P_{c,av}(X_0 = 1 + j)$ is readily obtained from (16). The same technique can be used to find the probability of a correct decision associated with the other three constellation points in the first quadrant. The total probability of a correct decision can then be given by

$$\begin{aligned}P_{c,av} = \tfrac{1}{4}\{&P_{c,av}(X_0 = 1 + j) + P_{c,av}(X_0 = 1 + 3j) \\ &+ P_{c,av}(X_0 = 3 + j) + P_{c,av}(X_0 = 3 + 3j)\}\end{aligned}$$

$$\begin{aligned}&P_c(z + n \in D_1 | X_0 = 1 + j, z) \\ &\quad = \frac{1}{4} + \frac{1}{\pi} \sum_{m \in N_0}^{\infty} \frac{\exp(-m^2\omega_0^2/2)\sin(m\omega_0\Re\{z\}/\sigma)}{m} \\ &\quad\quad + \frac{1}{\pi} \sum_{n \in N_o}^{\infty} \frac{\exp(-n^2\omega_0^2/2)\sin(n\omega_0\Im\{z\}/\sigma)}{n} \\ &\quad\quad + \frac{4}{\pi^2} \sum_{m \in N_o}^{\infty} \sum_{n \in N_o}^{\infty} \frac{\exp\left(-(n^2+m^2)\omega_0^2/2\right)\sin(m\omega_0\Re\{z\}/\sigma)\,\sin(n\omega_0\Im\{z\}/\sigma)}{nm}.\end{aligned} \tag{11}$$

$$\begin{aligned}P_{c,\mathrm{av}} &= \frac{1}{4} + \frac{1}{\pi} \sum_{m \in N_o}^{\infty} \frac{\exp(-m^2\omega_0^2/2)\Im\left\{\phi\left(\frac{m\omega_0}{\sigma}, 0\right)\right\}}{m} \\ &\quad + \frac{1}{\pi} \sum_{n \in N_o}^{\infty} \frac{\exp(-n^2\omega_0^2/2)\Im\left\{\phi\left(0, \frac{n\omega_0}{\sigma}\right)\right\}}{n} \\ &\quad + \frac{2}{\pi^2} \sum_{n \in N_o}^{\infty} \sum_{m \in N_o}^{\infty} \frac{\exp\left(-(n^2+m^2)\omega_0^2/2\right)\Re\left\{\phi\left(\frac{-m\omega_0}{\sigma}, \frac{n\omega_0}{\sigma}\right) - \phi\left(\frac{m\omega_0}{\sigma}, \frac{n\omega_0}{\sigma}\right)\right\}}{nm}\end{aligned} \tag{12}$$

TABLE I
QPSK OFDM ERROR PROBABILITIES FOR $\sigma^2 = 0.4$ AND $\epsilon = 0.05$

N	P_e	P_{exact}	Relative error
2	$7.4693663611 \times 10^{-6}$	$7.4693663612 \times 10^{-6}$	$3.7159210398 \times 10^{-12}$
3	$1.9927340483 \times 10^{-5}$	$1.9927340482 \times 10^{-5}$	$8.3570335859 \times 10^{-12}$
4	$2.9722979633 \times 10^{-5}$	$2.9722979633 \times 10^{-5}$	$6.5366605806 \times 10^{-12}$
5	$3.6800138543 \times 10^{-5}$	$3.6800138544 \times 10^{-5}$	$3.0168990350 \times 10^{-12}$
6	$4.2029145453 \times 10^{-5}$	$4.2029145453 \times 10^{-5}$	$1.3207775374 \times 10^{-11}$
7	$4.6024402437 \times 10^{-5}$	$4.6024402435 \times 10^{-5}$	$4.5229662114 \times 10^{-11}$
8	$4.9170074819 \times 10^{-5}$	$4.9170074819 \times 10^{-5}$	$1.0725140009 \times 10^{-11}$

$$= \tfrac{1}{4} \langle \Psi(0, 0) \rangle . \tag{18}$$

The probability of error can now be obtained using (13).

B. Gaussian Approximation

If the ICI is assumed to be a Gaussian-distributed RV with a zero mean, both the BER and SER can readily be computed for various modulation formats. The approximate error rates then can be expressed in terms of the $Q(.)$ function and the effective SNR for the kth subcarrier as

$$\gamma_{\text{eff}} = \frac{|S_0|^2 \gamma_s}{(1 + \gamma_s \sigma_{ICI}^2)} \tag{19}$$

where $\gamma_s = E|X_k|^2/\sigma^2$ which is the SNR for the kth subcarrier in the absence of a CFO. The variance of the signal constellation, $E|X_k|^2$, will be independent of k if all subcarriers use the same modulation format, which is the normal case. The variance of the ICI on the kth subcarrier can be given by

$$\sigma_{\text{ICI}}^2 = \sum_{l=0,\, l \neq k}^{N-1} |S_{l-k}|^2. \tag{20}$$

This effective SNR (19) can be used directly to calculate the probability of error approximately. We will use our precise formulas to verify the accuracy of this approach.

1) Approximate SNR Degradation: In [6], the effect of the CFO is characterized as a loss of SNR. Using the approach of [6] and our notation, this SNR degradation in decibels can be expressed as

$$D = \frac{10}{\ln 10} \, \frac{1}{3} \, (\pi \varepsilon)^2 \gamma_s. \tag{21}$$

Note that this approximation is derived using $\ln(1+x) \approx x$ which holds for $x \ll 1$. Hence, the above is accurate only for $(\pi\varepsilon)^2 \ll 3$. Now $\gamma_{\text{eff}} = \gamma_s - D$ (all quantities in decibels).

III. NUMERICAL RESULTS

Table I contains the QPSK error probabilities for small values of N. The error probability P_e is calculated using (12) and (13). The exact error probability P_{exact} is calculated by averaging (9) over all possible 4^{N-1} values of z. The relative error is defined as $|P_e - P_{\text{exact}}|/P_{\text{exact}}$. It is evident that the solution (13) is extremely accurate (for 11 to 12 significant figures).

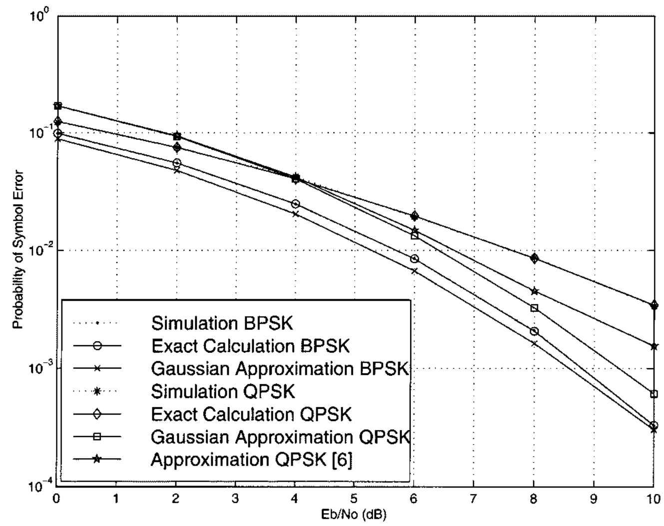

Fig. 1. Probability of symbol error for BPSK and QPSK. Normalized frequency offset $=0.1$.

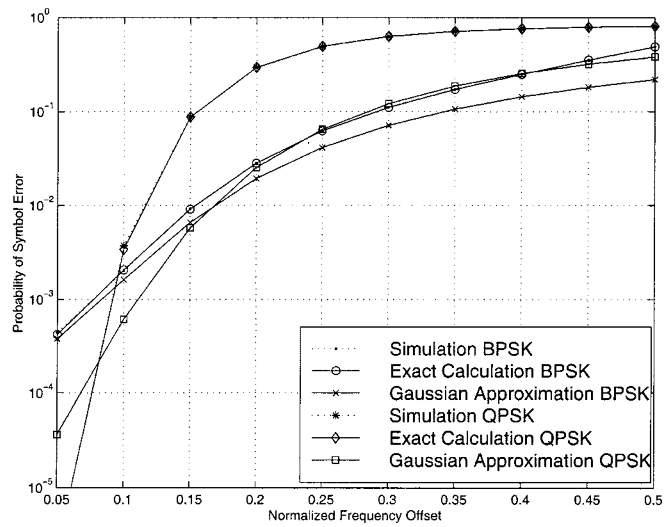

Fig. 2. Probability of symbol error with normalized frequency offset, $SNR = 8$ dB for BPSK and $SNR = 10$ dB for QPSK.

In Fig. 1, error rates are shown for an OFDM system with $N = 128$ for BPSK and QPSK modulation schemes. In the simulation results which follow, 10^7 random OFDM frames were generated to obtain each error rate point. The simulation results agree with those calculated using (5) and (13). While the Gaussian approximation yields optimistic estimates, it is acceptable for the BPSK case, since the difference in SNR between approximation and simulation is less than 0.4 dB. This difference increases for the QPSK modulation scheme. We may attribute this difference to the reduction of the minimum Euclidean distance between any two signal constellation points in QPSK.

The Gaussian approximation is acceptable for small values of CFO, though it does deviate from simulation results for increasing CFO (Fig. 2). Furthermore, the approximation is accurate for small value of SNR. In fact, Gaussian approximation is acceptable when noise dominates the ICI in the system. The approximation [6] is fairly close to the Gaussian approximation for small SNR values.

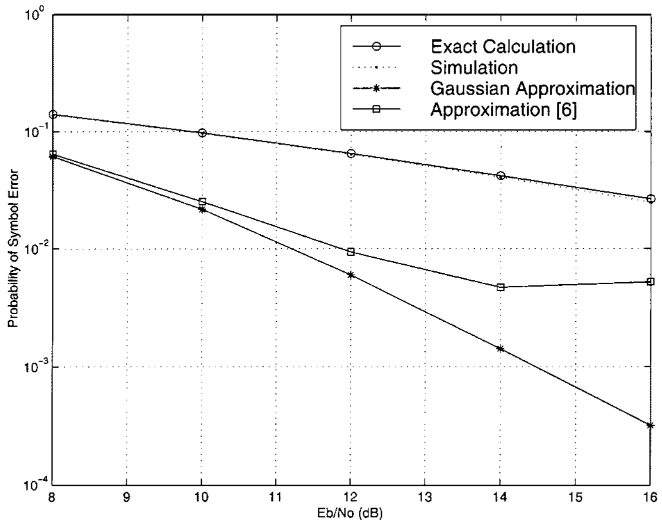

Fig. 3. Probability of symbol error for 16 QAM, normalized frequency offset $=0.05$.

Fig. 3 shows results for 16-QAM OFDM. Again, our technique is precise but the Gaussian approximation is much less accurate.

IV. Conclusion

In this letter, we have proposed a precise numerical technique for calculating the effect of the CFO on the BER or SER in an OFDM system. We considered BPSK, QPSK, and 16-QAM, which are common modulation formats used in OFDM applications. The BPSK case was solved using a series from Beaulieu. For the QPSK and 16-QAM cases, we used an infinite series expression for the error function in order to express the average probability of error in terms of a 2-D Fourier transform of the ICI. However, it appears difficult to generalize our method to M-ary PSK for $M > 4$. The reason for this is that the decision regions become nonrectangular and P_c cannot then be expressed using (14). The method from Reuter [10] may be applied in this case. Finally, our method can be used to study the use of error correcting codes to suppress the ICI caused by the CFO. This will be reported in a forthcoming paper.

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