An MGF-Derivative based Unified Analysis of Incoherent Diversity Reception of M-ary Orthogonal Signals over Fading Channels

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ABSTRACT — Exact error probability expressions for noncoherent M-ary frequency-shift-keying (MFSK) systems that employ postdetection equal-gain diversity over Rayleigh, Rician and Nakagami-m channels are derived using a Laplace derivative formula. Both independent and generically correlated fading cases are considered. For independent fading, closed-form solutions are also derived for both Nakagami-q fading (either with identical or dissimilar fading statistics) and mixed fading cases. Previous results are shown to be specific instances of our general expressions. Additionally, a new concise, derivative formula is obtained for calculating the bit error rate of square-law detected multichannel binary differential phase-shift-keying (BDPSK) signals. All these expressions are applicable in many cases of practical interest and provide accurate predictions of the performance of both binary and M-ary orthogonal signalling over generalized fading channels (with arbitrary fading parameters). Several numerical examples are presented to illustrate the application of the theory, including the investigation into optimal diversity order in energy-sharing communications, characterization of block orthogonal codes with soft-decision decoding (i.e., MFSK may also be viewed as a form of repetition coding) and analysis of MFSK with space (antenna) diversity.

I. INTRODUCTION

Noncoherent detection is a viable signalling scheme for wireless communications, particularly when the channel time variation is so rapid that it precludes an accurate estimation of the complex-valued channel parameters. However, such an implementation will also result in an increased SNR per bit requirement in order to satisfy a quality of service contract, when compared with coherent detection. Diversity mechanisms may be used to recover this penalty, and in addition, will improve the outage error rate performance over fading channels. Furthermore, square-law combining (also known as post-detection equal gain combining) has a simple implementation because noncoherent detection circumvents the need to co-phase and weight the diversity branches.

While the problem of square-law combining of MFSK signals over fading channels has been considered by many researchers over the past forty years, in the context of both flat-fading and spread-spectrum communications, the closed-form average bit error rate (ABER) formulas have not been reported for many important practical cases such as nonidentical fading and correlated fading situations. Recently, [1] obtained an integral expression for the ABER of MFSK using square-law combining over several common fading channel models. A similar expression has been derived in [2], but it was obtained using a different mathematical approach (i.e., by exploiting the Parseval theorem to transform two product integrals into the frequency domain). In addition to this, finite-range integral expressions were provided for the performance analyses of BDPSK, NCFSK and MFSK, when employing postdetection EGC over generically correlated Rician fading channels. Overall, previous studies on noncoherent MFSK, with the exception of [1] and [2], have assumed an idealized channel model (i.e., i.i.d fading statistics across the diversity branches) or consider specific correlation models. Unfortunately, the results for ABER in generalized fading channel models are not in closed-form. This is exactly the problem that will be addressed in this paper. Specifically, we derive a unified derivative formula for the ASER of MFSK with multichannel receiver over generalized fading channels. This concise expression involves the evaluation of an L-th order derivative of the moment generating function (MGF) of the combiner output SNR, where L denotes the diversity order. The generic closed-form expression can be further simplified into a finite polynomial for all common fading channel models: Rayleigh, Rician, Nakagami-m and Nakagami-q. Moreover, exact finite-range integral expressions for both MFSK and binary DPSK can be readily obtained from various new forms of the conditional error probabilities (CEPs) derived in [4]. Thus, this study provides the most comprehensive analysis to-date dealing with MFSK and DPSK modulation in conjunction with post-detection diversity. A by-product of this investigation includes the derivation of a simple, closed-form derivative formula for both square-law detected binary orthogonal FSK and differentially coherent PSK, over generalized fading channels. These exact closed-form solutions are of practical interest since they provide useful insights into the system behaviour and facilitate design parameter optimization.

It is worth mentioning that a flurry of recent papers (e.g., [1]) have shown that error rates for a multitude of digital modulation schemes can be analyzed using an MGF approach. The key idea is to find an exponential-type integral representation (with finite integration limits) for the CEP, so that the averaging over the fading distribution can then be represented in terms of an *integral* of the MGF of the combiner output SNR. In contrast, the performance analysis problem considered in this paper requires only the knowledge of the *derivatives* of this MGF. This is in fact a good news because derivatives are, in general, easier to be evaluated in closed

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form while integrals are not. As a consequence, exact closed-form expressions for the error rates can be derived for all common fading channel models, without involving any tedious manipulations.

Aside from presenting a new theoretical framework for analyzing the MFSK and DPSK in conjunction with a quadratic multichannel receiver, we shall also discuss selected numerical examples as an application of the theory: (a) investigation into the trade-off between diversity gain with additional diversity branches and the combination losses in an energy-sharing communication; (b) characterization of block orthogonal codes with soft-decision decoding (since MFSK may also be viewed as a form of repetition coding, for which the order of diversity is equal to the number of times a symbol is repeated in the block orthogonal code and the combining technique represents a soft-decision decoding of the repetition code); and (c) mitigating deep fades using space diversity; Further details can also be found in [4], a much enhanced version of this paper.

II. GENERIC ERROR RATE EXPRESSIONS FOR SQUARE-LAW COMBINING OF BINARY AND M-ARY SIGNALS

In this section, we first derive the CEP for square-law detected MFSK signals (i.e., performance in an AWGN channel) in closed-form. Two equivalent forms of the CEP are also provided in Appendix B of [4]. Subsequently, three generic derivative formulas are obtained by averaging the CEP over the probability density function (PDF) of the quadratic combiner output SNR.

A. AWGN Channel

Consider M-ary orthogonal signalling with square-law detection and the combination of the signals on L diversity branches. The decision variables are given by

$$\begin{aligned} U_1 &= \sum_{l=1}^{L} |2\xi\alpha_l + N_{l1}|^2 \\ U_m &= \sum_{l=1}^{L} |N_{lm}|^2, \; m = 2, 3, \ldots, M \end{aligned} \tag{1}$$

where $\{N_{lm}\}$ are complex-valued zero-mean Gaussian random variables with variance $\sigma^2 = 2\xi N_0$. The probability of symbol error is then [5, Eq. (12-1-22)]

$$P_M = 1 - \int_0^{\infty}\left[1 - \exp\left(\frac{-u_1}{4\xi N_0}\right)\sum_{k=0}^{L-1}\frac{1}{k!}\left(\frac{u_1}{4\xi N_0}\right)^k\right]^{M-1} p(u_1)du_1 \tag{2}$$

where $p(u_1)$ denotes the PDF of U_1.

Expanding the power term in (2) binomially, and then carrying out the integration term by term, we obtain

$$P_M = \sum_{n=1}^{M-1}\binom{M-1}{n}(-1)^{n+1}\sum_{k=0}^{n(L-1)}\beta_{kn}\left(\frac{-1}{4\xi N_0}\right)^k \Phi_1^{(k)}(s)\Big|_{s=\frac{n}{4\xi N_0}} \tag{3}$$

where $\Phi_1^{(k)}(s)$ denotes the k-th order derivative of the MGF of U_1 and the MGF $\Phi_1(s)$ is defined as [5, Eq. (2-1-117)]

$$\Phi_1(s) = \frac{1}{(1 + s4\xi N_0)^L}\exp\left(\frac{-s4\xi^2}{1 + s4\xi N_0}\sum_{l=1}^{L}\alpha_l^2\right) \tag{4}$$

Also, the coefficients β_{kn} in (3) can be calculated using the multinomial theorem, viz.,

$$\beta_{kn} = \sum_{i=k-L+1}^{k}\frac{\beta_{i(n-1)}}{(k-i)!}I_{[0,(n-1)(L-1)]}(i) \tag{5}$$

where $I_{[a,b]}(i) = \begin{cases} 1 & a \le i \le b \\ 0 & \text{otherwise} \end{cases}$, $\beta_{00} = \beta_{0n} = 1$, $\beta_{1n} = n$ and $\beta_{k1} = 1/k!$.

Note that a specific case of the above general development has been done in [5]. That is, $p(u_1)$ is explicitly given for Rayleigh fading (see [5, Eq. (14-4-44)]). As such, the result is a performance analysis tailored to the Rayleigh fading environment. In contrast, the use of the derivatives of the MGF, in our case, ensures that the analysis is applicable to any fading environment. Moreover, other related studies on MFSK did not attempt to derive a closed-form expression for the CEP. Since the instantaneous bit error probability (BEP) can be related to the instantaneous symbol error probability (SEP) via the relationship

$$P_b(\gamma) = \frac{2^{\log_2 M - 1}}{2^{\log_2 M} - 1}P_M = \frac{M}{2(M-1)}P_M, \tag{6}$$

we have a closed-form solution for the CEP as a derivative formula:

$$P_b(\gamma) = \frac{M}{2(M-1)}\sum_{n=1}^{M-1}\binom{M-1}{n}\sum_{k=0}^{n(L-1)}\beta_{kn}(-1)^{n+k+1}\psi^{(k)}(s)\big|_{s=n} \tag{7}$$

where

$$\psi(s) = \Phi_1\left(\frac{s}{4\xi N_0}\right) = \frac{1}{(1+s)^L}\exp\left(\frac{-s\gamma}{1+s}\right) \tag{8}$$

and $\gamma = \frac{\xi}{N_0}\sum_{l=1}^{L}\alpha_l^2$ is the SNR per symbol[1].

In a fading environment, the statistical average of $\psi^{(k)}(.)$ need to be computed. However, the derivatives of the MGF with respect to s are much simpler than those of $\overline{\psi}(.)$, because the latter involves a product of two terms which have dependence on the parameter s (see (13)). Consequently, it is desirable to transform (7) so that the derivatives of the MGF can be directly used. For this we can expand the exponential term in $\psi(s)$ as an infinite series of $1/(1+s)^n$. Although this infinite series expansion of $\psi(.)$ appears to be more complicated, the resultant series for the derivatives can be reduced to a finite polynomial using Kummer transformation (see Appendix A of [4] for details). In [4, Appendix B], we have shown that the CEP can be further simplified as

$$P_b(\gamma) = \frac{M}{2(M-1)}\sum_{n=1}^{M-1}\binom{M-1}{n}\sum_{k=0}^{n(L-1)}\beta_{kn}\sum_{i=0}^{k}\binom{L+k-1}{k-i} \times \frac{(-1)^{n+1}k!}{i!(1+n)^{L+k+i}}\gamma^i\exp\left(\frac{-n\gamma}{1+n}\right), \tag{9}$$

or alternatively, as

$$P_b(\gamma) = \frac{M}{2(M-1)}\sum_{n=1}^{M-1}\binom{M-1}{n}(-1)^{n+1}\sum_{i=0}^{n(L-1)}\frac{\gamma^i}{i!(1+n)^{2i+L}} \times \exp\left(\frac{-n\gamma}{1+n}\right)\left[\sum_{k=0}^{n(L-1)-i}\frac{\beta_{(k+i)n}(k+i)!}{(1+n)^k}\binom{L+k+i-1}{k}\right] \tag{10}$$

1. The SEP and BEP can be expressed in terms of the SNR per bit γ_b by simply replacing γ with $\gamma_b \log_2 M$.

by computing the k-th order derivative of $\psi(s)$ in a closed-form. Note that (10) is obtained from (9) after reordering the summations. To the best of the authors' knowledge, (7), (9) and (10) are all new. Also, for the particular case of $L = 1$, (9) reduces to the familiar expression given in [5, Eq. (5-4-46)]. An equivalent form for the CEP (expressed in terms of the confluent hypergeometric function) is also derived in [4, Appendix B].

Now, recalling that the CEP for multichannel binary orthogonal FSK and binary differential PSK (DPSK) will have the same functional dependence on the combined SNR, with the exception that the noise variance for the binary orthogonal FSK will be twice that of the binary DPSK, it is straight-forward to show that the BEP for these binary signalling schemes is given by

$$P_b(\gamma) = \sum_{k=0}^{L-1} \frac{(-1)^k}{k!} \underset{\sim}{\psi}^{(k)}(s)|_{s=1} \quad (11)$$

where $\underset{\sim}{\psi}(s)$ is defined similarly to (8) except that γ is now replaced by $g\gamma$, $g = 1$ for binary orthogonal FSK and $g = 2$ for binary differential PSK.

B. Generalized Fading Channels

The ABEP performance of MFSK over fading channels can be readily obtained by averaging the CEP over the combiner output SNR. Thus, using (7), we have

$$\overline{P_b} = \frac{M}{2(M-1)} \sum_{n=1}^{M-1} \binom{M-1}{n} \sum_{k=0}^{n(L-1)} \beta_{kn} (-1)^{n+k+1} \overline{\psi}^{(k)}(s)|_{s=n} \quad (12)$$

where $p_\gamma(.)$ is the PDF of the quadratic combiner output SNR, and

$$\overline{\psi}(s) = \frac{1}{(1+s)^L} \int_0^\infty \exp\left(\frac{-s\gamma}{1+s}\right) p_\gamma(\gamma) d\gamma = \frac{1}{(1+s)^L} \phi_\gamma\left(\frac{s}{1+s}\right) \quad (13)$$

Eq. (12) is obtained by recognizing that the integration with respect to γ is simply the MGF of the combiner output SNR $\phi_\gamma(.)$ (i.e., Laplace transform of the PDF). Notice that the application of (12) only requires an evaluation of the derivatives of $\overline{\psi}(.)$. Alternatively, using (9) and/or (10) and the Laplace transform identity

$$\int_0^\infty \gamma^k \exp(-\lambda\gamma) p_\gamma(\gamma) d\gamma = (-1)^k \phi_\gamma^{(k)}(s)|_{s=\lambda}, \quad (14)$$

we can obtain yet another generic formula for computing the ABEP of multichannel MFSK over fading channels:

$$\overline{P_b} = \frac{M}{2(M-1)} \sum_{n=1}^{M-1} \binom{M-1}{n} \sum_{k=0}^{n(L-1)} \beta_{kn} \sum_{i=0}^{k} \binom{L+k-1}{k-i} \times \frac{(-1)^{n+1+i} k!}{i!(1+n)^{L+k+i}} \phi_\gamma^{(i)}(s)\Big|_{s=\frac{n}{1+n}} \quad (15)$$

or equivalently,

$$\overline{P_b} = \frac{M}{2(M-1)} \sum_{n=1}^{M-1} \binom{M-1}{n} \sum_{i=0}^{n(L-1)} \frac{(-1)^{n+1+i}}{i!(1+n)^{L+2i}} \phi_\gamma^{(i)}(s)\Big|_{s=\frac{n}{1+n}} \times \left[\sum_{k=0}^{n(L-1)-i} \frac{\beta_{(k+i)n}(k+i)!}{(1+n)^k} \binom{L+k+i-1}{k} \right] \quad (16)$$

It is interesting to note that (16) is usually preferred over (15) for error rate calculations because it has significantly fewer terms that involve the derivatives of $\phi_\gamma(.)$, for practical values of M and L (i.e., the terms in the square brackets are independent of the MGF). The generic error rate expressions (12), (15) and (16) are powerful results because they can handle all common fading channel models (Rayleigh, Rician, Nakagami-m and Nakagami-q) as well as mixed-fading cases. It is also fairly easy to program and evaluate (12) and (16) in common mathematical software such as Maple. Their applications for the computation of ABEP of multichannel noncoherent MFSK systems in different fading environments (for both independent and correlated diversity branches cases) can be found in Section III through VI of [4]. As an illustrative example, we will next consider the ABEP calculation over a correlated Nakagami-m channel with arbitrary branch correlations. When the diversity branches are correlated, the analysis can proceed in a similar manner to that of independent fading with dissimilar statistics [4]. This is because the MGF of the combiner output SNR in correlated Rayleigh, Rician or Nakagami-m environments can be decomposed into a product of MGFs which closely resembles that of the independent fading cases. For instance, in a generically correlated Nakagami-m fading environment (with the assumption that the fading severity index is common to all the diversity branches), the MGF of the post-detection combiner output SNR, $\gamma = \sum_{l=1}^{L} \gamma_l$, can be written in the form [2]

$$\phi_\gamma(s) = \det(I + sR\Lambda)^{-m} = \prod_{l=1}^{L} \frac{1}{(1+s\lambda_l)^m} \equiv \prod_{l=1}^{L} \varnothing_l(s) \quad (17)$$

where I is the $L \times L$ identity matrix, Λ is a positive definite matrix of dimension L (determined by the branch covariance matrix), R is a diagonal matrix that is defined as $R = \mathrm{diag}(\bar{\gamma}_1/m, \ldots, \bar{\gamma}_L/m)$, m denotes the fading parameter, and λ_l is the l-th eigen value of matrix $R\Lambda$. As such, $\phi_\gamma^{(i)}(.)$, which is needed in (15) and (16), can be evaluated using identity [4, Eq. (C.3)] as

$$\phi_\gamma^{(i)}(s) = \sum_{n_1=0}^{i} \binom{i}{n_1} \varnothing_1^{(i-n_1)}(s) \sum_{n_2=0}^{n_1} \binom{n_1}{n_2} \varnothing_2^{(n_1-n_2)}(s) \times \sum_{n_3=0}^{n_2} \binom{n_2}{n_3} \varnothing_2^{(n_2-n_3)}(s) \ldots \sum_{n_{L-1}=0}^{n_{L-2}} \binom{n_{L-2}}{n_{L-1}} \varnothing_{L-1}^{(n_{L-2}-n_{L-1})}(s) \varnothing_L^{(n_{L-1})}(s) \quad (18)$$

where the n-th order derivative of the l-th product term $\varnothing_l^{(n)}(.)$ is given by

$$\varnothing_l^{(n)}(s) = \frac{(-\lambda_l)^n (m+n-1)!}{(1+s\lambda_l)^{m+n}(m-1)!} \quad (19)$$

Substituting (18) and (19) in (16) (or (15)), we obtain the desired ABER formula. Although the correlated Rayleigh fading case can be treated as a special instance of Nakagami-m fading by letting $m = 1$ in (17) and (19), a much more concise expression for $\phi_\gamma^{(i)}(.)$ can be derived using the partial fraction approach:

$$\phi_\gamma^{(i)}(s) = \sum_{l=1}^{L} \frac{i!(-\lambda_l)^i}{(1+s\lambda_l)^{1+i}} \left[\prod_{k=1, k\neq l}^{L} \frac{\lambda_l}{\lambda_l - \lambda_k} \right] \quad (20)$$

Lastly, from (11), we can write the ABEP for the binary signalling schemes in a very compact form:

$$\overline{P}_b = \sum_{k=0}^{L-1} \frac{(-1)^k}{k!} \underline{\psi}^{(k)}(s)|_{s=1} \tag{21}$$

where $\underline{\psi}(s) = \dfrac{1}{(1+s)^L}\phi_\gamma\left(\dfrac{gs}{1+s}\right)$.

Table 1. MGF of the SNR of the k-th diversity branch $\phi_k(.)$ and its n-th order derivative $\phi_k^{(n)}(.)$ for several fading channel models.

Channel Model	MGF $\phi_k(s)$ and its n-th derivative w.r.t. s, $\phi_k^{(n)}(.)$
Rayleigh	$\phi_k(s) = \dfrac{1}{1+s\bar{\gamma}_k}$ $\phi_k^{(n)}(s) = \dfrac{(-\bar{\gamma}_k)^n n!}{(1+s\bar{\gamma}_k)^{1+n}}$
Rician $K \geq 0$	$\phi_k(s) = \dfrac{1+K_k}{1+K_k+s\bar{\gamma}_k}\exp\left(\dfrac{-sK_k\bar{\gamma}_k}{1+K_k+s\bar{\gamma}_k}\right)$ $\phi_k^{(n)}(s) = \dfrac{(-\bar{\gamma}_k)^n(n!)^2(1+K_k)}{(1+K_k+s\bar{\gamma}_k)^{1+n}}\exp\left(\dfrac{-sK_k\bar{\gamma}_k}{1+K_k+s\bar{\gamma}_k}\right)$ $\times\sum_{i=0}^{n}\dfrac{1}{(i!)^2(n-i)!}\left(\dfrac{K_k(1+K_k)}{1+K_k+s\bar{\gamma}_k}\right)^i$
Nakagami-q $-1 \leq b = \dfrac{1-q^2}{1+q^2} \leq 1$ $0 \leq q \leq \infty$	$\phi_k(s) = \dfrac{1}{\sqrt{[1+s\bar{\gamma}_k(1+b_k)][1+s\bar{\gamma}_k(1-b_k)]}}$ $\phi_k^{(n)}(s) = \dfrac{n!}{2^{2n}\sqrt{[1+s\bar{\gamma}_k(1+b_k)][1+s\bar{\gamma}_k(1-b_k)]}}$ $\times\left[\dfrac{-\bar{\gamma}_k(1+b_k)}{1+s\bar{\gamma}_k(1+b_k)}\right]^n\sum_{w=0}^{n}\dfrac{(2w)![2(n-w)]!}{[w!(n-w)!]^2}$ $\times\left[\dfrac{(1-b_k)[1+s\bar{\gamma}_k(1+b_k)]}{(1+b_k)[1+s\bar{\gamma}_k(1-b_k)]}\right]^w$
Nakagami-m $m \geq 0.5$	$\phi_k(s) = \left(\dfrac{m_k}{m_k+s\bar{\gamma}_k}\right)^{m_k}$ $\phi_k^{(n)}(s) = \dfrac{(-\bar{\gamma}_k)^n m_k^{m_k}\Gamma(m_k+n)}{(m_k+s\bar{\gamma}_k)^{m_k+n}\Gamma(m_k)}$

III. COMPUTATIONAL RESULTS

In this section, we present selected numerical curves that characterize the performance of noncoherently detected MFSK signals in different fading environments and help to assess the advantage of higher order signal alphabet sizes relative to a binary alphabet.

A. Optimal diversity order in energy-sharing communications

Fig. 1 depicts the ASEP performance of the square-law detected quaternary orthogonal signals on a Rician fading channel as a function of the order of diversity. The analysis of multichannel reception of M-ary orthogonal signals for energy-sharing communications (i.e., where the same information-bearing signal is transmitted on L diversity channels) is of interest because it indicates the optimum number of diversity orders for any given fixed total SNR that is defined as $\bar{\gamma}_T = L\bar{\gamma}$. The optimum value for L (denoted as L_{opt}) exists for a fixed $\bar{\gamma}_T$ owing to the combination losses inherent in noncoherent detection schemes. A careful examination of these curves reveals that the ASEP $\overline{P_M}$ is at a minimum when the average SNR/symbol/channel $\bar{\gamma} = \bar{\gamma}_T/L_{opt} \approx 5$ for Rice factor $K = 5$. Although not shown here, we also found that this result is independent of the alphabet size. Similar observations were also made in [5] for the Rayleigh fading case, except that the ratio $\bar{\gamma}_T/L_{opt} \approx 3$ when $\overline{P_M}$ is at a minimum. Since fade distributions affect the ratio $\bar{\gamma}_T/L_{opt}$ which minimizes $\overline{P_M}$, in Fig. 2, we plot ASEP curves as a function of the order of diversity for several different values of Nakagami-m fading severity indices, while $\bar{\gamma}_T$ remains fixed. It is clear that for $m = 1$, $\bar{\gamma} \approx 3$ when $\overline{P_M}$ is at minimum, which agrees with [5, pp. 791]. We also observe that $\bar{\gamma}$ which minimizes $\overline{P_M}$ decreases as the fading becomes more severe because L_{opt} shifts to the left as m increases. While these observations are not obvious from the closed-form solutions for the ASEP, more insight can be had from a Chernoff bound [4]. Details are omitted here for brevity.

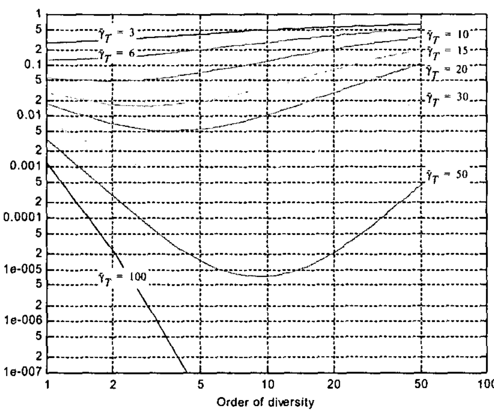

Fig. 1. Performance analysis of square-law detected quaternary orthogonal signals ($M = 4$) in a Rician fading ($K = 5$) as a function of diversity order.

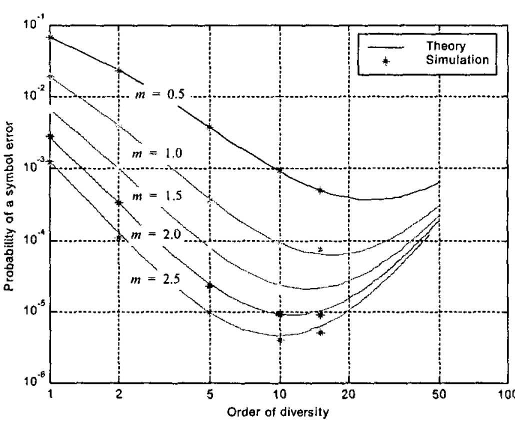

Fig. 2. Performance of square-law detected binary orthogonal signals on Nakagami-m fading channels when the total SNR is fixed at $\bar{\gamma}_T = 50$.

B. MFSK as time coding

M-ary orthogonal FSK may also be viewed as a form of repetition coding, for which the order of diversity is equal to the number of times a symbol is repeated in the block orthogonal code. By this observation, the combining technique represents a soft-decision decoding of the repetition code. In Fig. 3, ASEP curves are plotted as a function of SNR per bit $\bar{\gamma}_b = L\bar{\gamma}/\log_2 M$, with M and L as parameters. From this point of view, $\bar{\gamma}_b = \bar{\gamma}/R_C$, where $R_C = \log_2 M/L$ is the code rate. It is apparent from this figure that the receiver performance improves as L and M increases for a specified $\bar{\gamma}_b$. However, the improvement is much more drastic by increasing L compared to a corresponding increase in M. Since the bandwidth expansion factor is given by $LM/\log_2 M$, one can conclude that simple repetition coding with binary orthogonal FSK is more bandwidth efficient than a corresponding increase in the alphabet size. Nevertheless, it should be pointed out that the gain achieved by increasing M becomes more pronounced as L and/or m increases (since the spread between the curves for different alphabet sizes increases either with a larger L or as the channel condition improves).

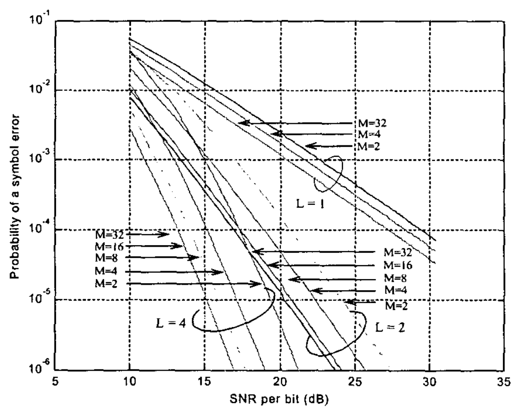

Fig. 3. Performance of noncoherent MFSK over a Nakagami-m fading channel ($m = 1.5$) with M and L as parameters.

C. MFSK with space (antenna) diversity

Next in Fig. 4, we examine the impact of branch correlations on the performance of noncoherent M-ary FSK in Nakagami-m fading channels. As an example, we consider a three element antenna array in triangular configuration with the branch covariance matrix is given by

$$R_r = \begin{pmatrix} 1 & 0.727 & 0.913 \\ 0.727 & 1 & 0.913 \\ 0.913 & 0.913 & 1 \end{pmatrix} \tag{22}$$

in accordance with the empirical results of Lee [6]. The branch covariance matrix was obtained with the assumptions that two adjacent antennas have equal separations (of the order of 8 times the operating frequency wavelength), the signals impinge upon the triangular array from the broadside of the line linking antennas and incident angles of these antennas to the third antenna is $60°$. For the sake of simplicity, we also assume that all the diversity branches have same SNR. As such, the positive definite matrix $R\Lambda$ in (17) is related to the branch covariance matrix R_r as $R\Lambda = (\bar{\gamma}\sqrt{R_r})/m$. It is evident that branch correlations significantly reduce the diversity gain and also that the degradation is more severe for larger alphabet sizes.

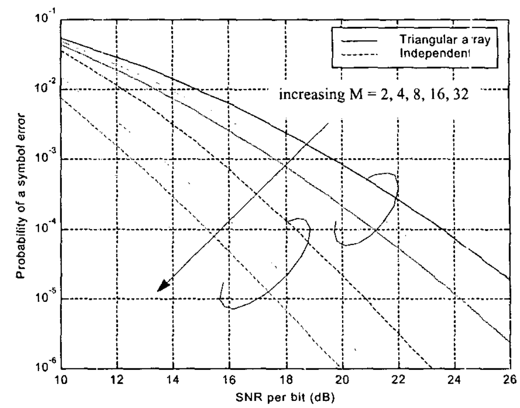

Fig. 4. Comparison between receiver performance of square-law detected M-ary orthogonal signals in independent and correlated Nakagami-m fading channels ($m = 1.5$) with a three element antenna array in a triangular configuration.

IV. CONCLUSION

This article presents unified derivative formulas for multichannel receiver performance of MFSK and DPSK signalling schemes over generalized fading channels. These generic closed-form expressions can be further simplified into a finite polynomial for all common fading channel models: Rayleigh, Rician, Nakagami-m and Nakagami-q. It is also straight-forward to extend these results and analyze the performance of MFSK and DPSK schemes in conjunction with a reduced complexity post-detection selection/equal-gain diversity quadratic receiver on i.i.d. Rayleigh and Nakagami-m fading channels using (12) or (16), along with an appropriate formula for $\phi_\gamma(.)$.

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