New ICI Reduction Schemes for OFDM System

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Abstract— Orthogonal Frequency Division Multiplexing (OFDM) is sensitive to the carrier frequency offset (CFO). We introduce the Peak Interference-to-Carrier Ratio (PICR) to measure the resulting intercarrier interference (ICI). This paper shows that the PICR can be reduced by coding to select only those messages with low PICR as valid codewords. While explicit constructions of higher-rate, ICI-suppressing codes are rather difficult, we propose two new codes that map data onto three and four adjacent subcarriers. We also propose a new ICI reduction scheme, called Tone Reservation, that inserts optimized pilot tones in the data frame. Computing these pilot tones at the transmitter is a simple optimization technique.

Keywords— Orthogonal Frequency Division Multiplexing, Carrier Frequency Offset, Intercarrier Interference

I. INTRODUCTION

Managing the explosive growth in mobile users and the demand for new services, such as wireless multimedia services and wireless Internet access, requires high-data-rate wireless communications systems. OFDM is a radio transmission technique for such applications, with sufficient robustness to handle radio channel impairments. OFDM has been accepted for several wireless LAN standards and mobile multimedia applications [1]. OFDM is, however, sensitive to the CFO, which is caused by misalignment in carrier frequencies, the Doppler shift or phase noise. The CFO violates the orthogonality of the subcarriers and results in ICI [1–4]. The BER (bit error rate) performance degrades as a result.

The CFO issue is considered as a limiting factor in the commercial application of OFDM for high-rate mobile communications. This is particularly true where low-cost mobile hand sets that cannot employ very accurate frequency estimators because of the cost involved. In the open literature, several techniques have been proposed to overcome this problem. Among these techniques, cancellation scheme [3,4] has received much attention due to its high robustness to CFO errors and simplicity despite a loss of 50% data throughput. In this paper, we will refer to this method as rate-half repetition coding.

Controlling the peak-to-average power ratio (PAR) of an OFDM signal has recently received much attention. The use of various coding strategies to reduce the PAR is a key idea (see [5–9] for several examples of this nature). As few data frames result in large signal peaks, block codes can be designed so that such data frames are never used. Consequently, the PAR is reduced. Motivated by the success of these approaches, this paper considers controlling ICI of OFDM signals by eliminating, from the set of all data frames, those which have high PICR. We also propose two new ICI cancellation schemes with improved data throughput. First, rate $\frac{2}{3}$ and $\frac{3}{4}$ ICI-suppressing codes are constructed that maps data onto three and four adjacent subcarriers. Second, ICI is suppressed by Tone Reservation that inserts optimized pilot tones in the data frame.

The organization of this paper is as follows: Section II presents ICI analysis and coding technique to reduce ICI. We explain the rate $\frac{2}{3}$ and $\frac{3}{4}$ ICI-suppressing codes in Section III and the ICI-suppression by Tone Reservation in Section IV. concluding remarks are presented in Section VI.

II. INTERCARRIER INTERFERENCE (ICI) ANALYSIS

A. OFDM Signalling

The complex baseband OFDM signal may be represented as

$$s(t) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} c_k e^{j2\pi k \Delta f t}, 0 \leq t \leq T. \quad (1)$$

where $j^2 = -1$, N is the total number of subcarriers, c_k is the data symbol for the k-th subcarrier. The frequency separation between any two adjacent subcarriers is $\Delta f = 1/T$ where T is OFDM symbol duration. Each modulated symbol c_k is chosen from the set $F_q = \{\lambda_1, \lambda_2, .., \lambda_q\}$ of q distinct elements, called *signal constellation* of the q-ary modulation scheme.

We assume that $s(t)$ is transmitted on an additive white Gaussian noise channel, and so the received signal sample for the k-th subcarrier after FFT demodulation can be written as [3,4]

$$y_k = c_k S_0 + \sum_{l=0,l \neq k}^{N-1} S_{l-k} c_l + n_k; k = 0, ..., N-1 \quad (2)$$

where n_k is a complex Gaussian noise sample (with its real and imaginary components being independent and identically distributed with variance σ^2). We shall refer to $\mathbf{c} = (c_0, c_1, ..., c_{N-1})$ as a data frame or codeword, as appropriate and is a constellation symbols from a encoder. The second right term in (2) is the ICI term attributable to the CFO. The sequence S_k (the ICI coefficients) depends on the CFO and is given by [3,4]

$$S_k = \frac{\sin \pi(k + \varepsilon)}{N \sin \frac{\pi}{N}(k + \varepsilon)} \exp \left[j\pi(1 - \frac{1}{N})(k + \varepsilon) \right] \quad (3)$$

where ε is the normalized frequency offset defined as a ratio between the frequency offset (which remains constant over each symbol period) and the subcarrier spacing. For a zero frequency offset, S_k reduces to the unit impulse sequence. The ICI term (2) can be expressed as

$$I_k = \sum_{l=0,l \neq k}^{N-1} S_{l-k} c_l, \quad \text{for} \quad 0 \leq k \leq N-1. \quad (4)$$

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Note that I_k is a function of both $\mathbf{c}$ and ε. In the sequel, we would be interested in reducing the peak magnitude of I_k.

The admissible sequences $\mathbf{c} \in F_q^N$ are called codewords, and the ensamble of all possible codewords is a code $\mathcal{C}$ of rate

$$R = \log_2\left(|\mathcal{C}|\right)/(N\log_2(q)) \qquad (5)$$

where $|\mathcal{C}|$ is the number of codewords of $\mathcal{C}$. We also denote by d the minimum Euclidean distance of $\mathcal{C}$.

B. PICR Problem

Before defining the PICR, we will also introduce the average interference-to-carrier ratio (ICR). The ICR and its reciprocal have previously been used by many authors to study the effects of ICI. The ICI for subcarrier k can be expressed as

$$\mathrm{ICR}(k) = \frac{E\left\{\left|\sum_{l=0,l\neq k}^{N-1} S_{l-k}c_l\right|^2\right\}}{|c_k S_0|^2} = \frac{\sum_{l=0,l\neq k}^{N-1} |S_{l-k}|^2}{|S_0|^2} \qquad (6)$$

where $E\{\cdot\}$ denotes the expected value over the distribution of data, $|c_k|^2 = 1$ for unit amplitude constellation such as PSK. Note also the numerator of (6) is approximately $(1 - |S_0|^2)$. Hence, the average for normal OFDM varies very little with k and does not depend on the data frame. We thus have

$$\mathrm{ICR}(k) \approx \frac{(\pi\varepsilon)^2}{3}. \qquad (7)$$

We define the Peak Interference-to-Carrier Ratio (PICR) as

$$\mathrm{PICR}(\mathbf{c}) = \frac{\max\limits_{0\leq k\leq N-1}\left|\sum_{l=0,l\neq k}^{N-1} S_{l-k}c_l\right|^2}{|S_0 c_k|^2}. \qquad (8)$$

Note that the PICR is a function of both $\mathbf{c}$ and ε.

Using the Couchy-Schwarz bound $|\sum a_k b_k|^2 \leq \sum |a_k|^2 \sum |b_k|^2$, we have

$$\mathrm{PICR}(\mathbf{c}) \leq \frac{\max\limits_{0\leq k\leq N-1}(N-1)\sum_{l=0,l\neq k}^{N-1} |S_{l-k}|^2}{|S_0|^2} \qquad (9)$$

which is a bound independent of $\mathbf{c}$.

By neglecting the higher order terms in the expansion of sinc function, $|S_0|^2$ is approximately equal to $(1 - \frac{(\pi\varepsilon)^2}{3})$. Further, $\sum_{l=0}^{N} |S_{l-k}|^2 \simeq 1$ for large N. Then (9) reduces to

$$\mathrm{PICR}(\mathbf{c}) \leq \frac{N-1}{\frac{3}{(\pi\varepsilon)^2} - 1}. \qquad (10)$$

Despite being rather loose, this bound shows that the PICR increases with increasing N and ε. Note that (10) reduces to zero for $\varepsilon = 0$.

To reduce ICI effects, (8) should be minimized and is zero for ICI-free channels. Interestingly, our definition in (8) is similar

to Peak-to-Average Power Ratio (PAR) issue in OFDM [5–9]. For a code $\mathcal{C}$, we define

$$\mathrm{PICR}(\mathcal{C}) = \min_{\mathbf{c}\in\mathcal{C}} \mathrm{PICR}(\mathbf{c}). \qquad (11)$$

We next state the main problem arising from these definitions.
What is the achievable region of triplets $(R, d, \mathbf{PICR}(\mathcal{C}))$ for length N?
This problem differs from the PAR issue in several ways:
- ICI occurs at the receiver side, whereas high PAR values affect the transmitter.
- Exact computation of the PAR requires oversampling, whereas max $|I_k|$ is obtained from N samples.
- As the transmitter does not know ε a priori, a code can be designed only on the basis of a worst-case value. Therfore, the performance of the code should hold independent of ε.

So the problem at hand is to find out all the codewords in $\mathcal{C}$ that minimizes the PICR for a fixed ε and having error correcting capability with high code rate. Moreover, the code should be independent of CFO. Once the code is designed for a worst case CFO, it should be robust to any low CFO.

C. ICI Reduction by Block Coding

Since PICR is a function of $\mathbf{c}$, coding to select only those data frames with low PICR as valid codewords in the scheme can reduce ICI effects. To the best of our knowledge, such an approach for ICI reduction has not been proposed or studied before. Here we show the existence of such PICR-reduction codes by computer simulation.
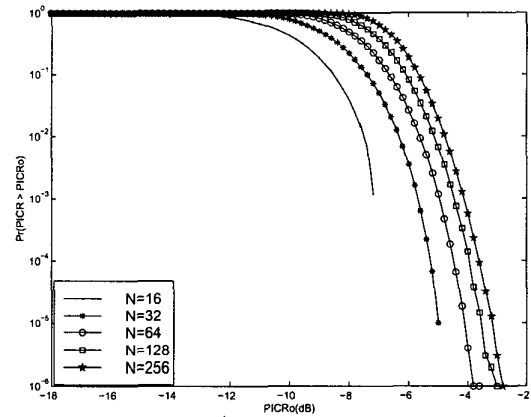

Fig. 1. CDF of PICR of an OFDM System with $\varepsilon = 0.1$

Fig. 1 shows the cumulative distribution function (CDF) of the PICR as a function of N for $\varepsilon = 0.1$. The subcarriers are modulated with binary phase shift keying (BPSK). For $N = 16$, the PICR exceeds -4 dB for only 1 out 10^6 of all OFDM blocks.

To show that ICI-suppressing codes exist, we consider an OFDM system with $N = 4$ and BPSK-modulated subcarriers. The PICR for all data frames is given Table I in ascending order. Ten data frames result in the maximum PICR of -18.56 dB for $\varepsilon = 0.05$. Clearly, we can reduce the PICR by avoiding transmitting these frames. This can be done by block coding the data such that 2 bits of data are mapped on to 4 best data frames.

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TABLE I

d_1	d_2	d_3	d_4	PICR (dB) $\varepsilon = 0.05$	PICR (dB) $\varepsilon = 0.1$
0	1	0	1	-28.06	-21.87
1	0	1	0	-28.06	-21.87
0	0	1	1	-20.97	-14.76
0	1	1	0	-20.97	-14.76
1	0	0	1	-20.97	-14.76
1	1	0	0	-20.97	-14.76
0	0	0	0	-18.56	-12.49
0	0	0	1	-18.56	-12.49
0	0	1	0	-18.56	-12.49
0	1	0	0	-18.56	-12.49
0	1	1	1	-18.56	-12.49
1	0	0	0	-18.56	-12.49
1	0	1	1	-18.56	-12.49
1	1	0	1	-18.56	-12.49
1	1	1	0	-18.56	-12.49
1	1	1	1	-18.56	-12.49

Fig. 2. CDF of PICR of CWC OFDM System for $N = 16$ and $\varepsilon = 0.1$

Such a block code would reduce the PICR by about 2dB. Note also that the complete rank-ordering of all codewords is independent of the frequency offset. That means, this block code's performance is independent of the CFO. This significant property is exactly what we need.

D. Constant Weight (CW) Codes

We have shown the existence of code sets to reduce ICI effects by simulation. However, we are not yet able to construct such codes algebraically. This study paves the way for further research on this direction. Interestingly, the first six code words with low PICR in Table 1 are Constant Weight Codes with weight $\frac{N}{2}$. This suggests that CW codes are suitable for reducing PICR.

Fig. 2 shows the CDF of PICR for the CW coded OFDM (CWC OFDM) system. For $N = 16$, all codewords of weight 8 are generated and ranked according to their PICR. Codes with rate increasing from $\frac{1}{2}$ to $\frac{13}{16}$ can be constructed this way. The rate $\frac{1}{2}$ CWC reduces PICR by nearly 7 dB. As more codewords are included and the rate increases, the PICR reduction decreases.

III. RATE $\frac{2}{3}$ AND $\frac{3}{4}$ ICI-SUPPRESSING CODES

The rate-half repetition coding technique is highly effective in canceling ICI, albeit at a loss of 50% data throughput. Since ICI coefficients (Eq. 3) vary slowly between adjacent subcarriers, a receiver will thus be highly effective in canceling ICI. In fact, symbol repetition and addition of wanted signal components achieve ICI reduction. This raises an interesting question: Can less redundancy be used for moderate ICI cancellation so that code rate can be traded against average CIR? Note that such codes do not achieve the same CIR performance as rate-half code. However, they will improve the data throughput.

For the new ICI cancellation schemes, modulated symbols (c_k) are repeated on the basis of expected code rate. For nor-
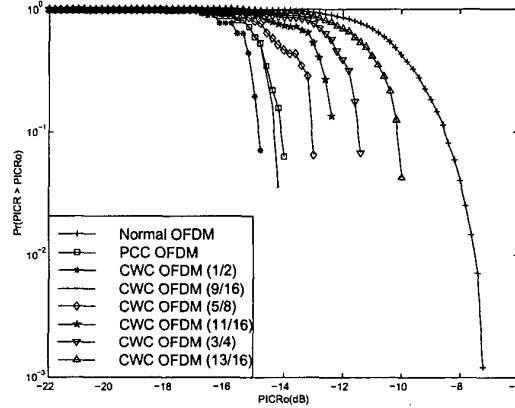
mal OFDM, data are mapped to individual subcarriers, whereas for half rate system data are mapped to pairs of adjacent subcarriers. Similarly, mapping two modulated symbols onto three adjacent subcarriers achieves a code rate of $\frac{2\lfloor \frac{N}{3} \rfloor + (N \bmod 3)}{N}$, that approximately reduces to $\frac{2}{3}$ for large value of N. In this case, the OFDM input block will be $\mathbf{c} = (c_0, c_1, -c_1, c_2, c_3, -c_3, ...)$. Similarly, a rate $\frac{3}{4}$ code can be obtained by mapping three modulated symbols onto four adjacent subcarriers.

If same modulated symbols are repeated on subcarrier k and $(k + 1)$, then the receiver computes the difference between the received samples, $(y_k - y_{k+1})$, as a decision variable. Otherwise, the received sample y_k alone is the decision variable. Hence, ICI reduction in each subcarrier is not uniform and we can expect two level of ICI reduction among these subcarriers, one for repeated symbols and the other for non-repeated symbols.
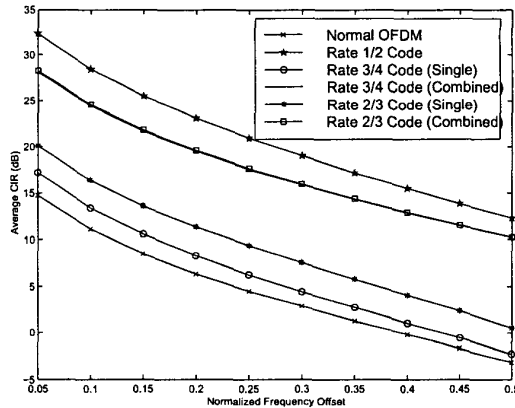

Fig. 3. Average CIR of Rate $\frac{2}{3}$ and $\frac{3}{4}$ Coded OFDM Systems with ε

Fig. 3 shows how CIR varies with the normalized frequency offset. OFDM system with $N = 16$ and BPSK modulation scheme in AWGN channel is assumed in this simulation. Normal OFDM and the rate-half code are also compared with the new codes. The rate-half code performs better than others since every symbol is repeated to cancel ICI. However, the new codes show significant CIR gain over normal OFDM with less data

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throughput loss. The rate $\frac{2}{3}$ code offers 5 to 13 dB gain in CIR over normal OFDM and that of $\frac{3}{4}$ code is 2.5 to 13 dB. Combining the repeated symbols at the receiver offers same CIR in rate $\frac{2}{3}$ and $\frac{3}{4}$ codes.

IV. ICI-SUPPRESSION BY TONE RESERVATION

Tone reservation approach has been proposed for the reduction of PAR of OFDM signals [9]. This method is based on adding a symbol dependent time domain signal to the original OFDM symbol to reduce ICI. The transmitter does not send data on a small subset of carriers, which are used to insert the optimized tones. The complex baseband signal may now be represented as

$$s(t) = \sum_{k \in I_{info}}^{N-1} c_k e^{j2\pi k \Delta f t} + \sum_{k \in I_{tones}}^{N-1} c_k e^{j2\pi k \Delta f t}, 0 \le t \le T.$$
$$(12)$$

wherer I_{info} and I_{tones} are two disjoint sets such that

$$I_{info} \cup I_{tones} = \{0, 1, \ldots, N-1\}.$$

These tones can be computed efficiently at the transmitter and can be easily stripped off at the receiver.

The ICI term can be denoted in vector form as

$$\mathbf{I} = \mathbf{Sc} \qquad (13)$$

where $\mathbf{S}$ is $N \times N$ dimensional vector with $S_{ij} = S_{j-i}$ and $S_{ii} = 0$.

Note that I_k should be zero for all k to completely eliminate ICI. To achieve that, we set $I_{tones} = \{0, m, 2m, .., (p-1)m\}$ ($m = \lfloor \frac{N}{P} \rfloor$) and let $\mathbf{c}_p = [c_0, c_m, c_{2m}, \ldots, c_{(P-1)m}]$. Therefore, I_k needs to be zero for only $(N-P)$ subset of k, that belongs to the data. Thus, for ICI free channel, $\mathbf{c}_p$ should satisfy the following condition.

$$\mathbf{S}_o \mathbf{c} = \mathbf{0} \qquad (14)$$

where $\mathbf{S}_o$ results from eliminating P rows, corresponding to pilot tone positions, from $\mathbf{S}$ and $\mathbf{c} = \mathbf{c}_d \cup \mathbf{c}_p$ is the IFFT input consisting data ($\mathbf{c}_d = (c_0, \ldots)_{1 \times (N-P)}$) and pilots ($\mathbf{c}_p$). Now, the computation of $\mathbf{c}_p$ from (14) requires an optimization technique and we will consider two different approaches. Moreover, CFO should be known at the transmitter for the computation of $\mathbf{S}_o$. However, worst case CFO can be assumed for the computation. The simulation results show that any CFO less than the worst case perform same as if the computation is done with that CFO. That means computation of pilot tones is independent of CFO if those CFO are less than the worst case scenario. This is an interesting and significant phenomenon.

A. Least Square Error Optimization

This is a simple near optimization technique that minimizes the error. (14) can be written as

$$S_1 \mathbf{c}_p = -S_2 \mathbf{c}_d \qquad (15)$$

where S_1 is P columns of $\mathbf{S}_o$ for $\{0, m, 2m, \ldots, (p-1)m\}$ and S_2 is the remaining $(N-P)$ columns of $\mathbf{S}_o$. Thus, the least square error solution to (15) is [10]

$$\mathbf{c}_p = -(S_1^T S_1)^{-1} S_1^T S_2 \mathbf{c}_d \qquad (16)$$

where A^T denotes the conjugate transpose of A.

B. Standard Linear Programming Approach

In the least square error approach, we have focused on eliminating all ICI terms. This is a rather difficult optimization criterion and less ICI effects are tolerable. Hence, our optimization criterion is to minimize the maximum ICI. That is maximum of $|\mathbf{S}_o \mathbf{c}|$ should be minimized. However, this is a non-linear optimization problem. Therefore, we optimize t, where $|\mathbf{S}_o \mathbf{c}| \le t$ subject to following constraints.

$$\begin{aligned} |\Re\{S_1 \mathbf{c}_p + S_2 \mathbf{c}_d\}| &\le tU \\ |\Im\{S_1 \mathbf{c}_p + S_2 \mathbf{c}_d\}| &\le tU \end{aligned} \qquad (17)$$

where $\mathbf{U} = [1, 1, \ldots]_{(N-P) \times 1}^T$ and $\Re\{x\}$ and $\Im\{x\}$ denote the real part of x and imaginary part of x respectively. Then (17) can be expanded as

$$\begin{aligned} \Re\{S_1 \mathbf{c}_p\} &- tU &\le & \Re\{S_2 \mathbf{c}_d\} \\ - \Re\{S_1 \mathbf{c}_p\} &- tU &\le & - \Re\{S_2 \mathbf{c}_d\} \\ \Im\{S_1 \mathbf{c}_p\} &- tU &\le & \Im\{S_2 \mathbf{c}_d\} \\ - \Im\{S_1 \mathbf{c}_p\} &- tU &\le & - \Im\{S_2 \mathbf{c}_d\} \end{aligned} \qquad (18)$$

Let us define

$$\widehat{\mathbf{c}} = \begin{pmatrix} \Re\{\mathbf{c}_p\} \\ \Im\{\mathbf{c}_p\} \end{pmatrix}, \quad S_{1r} = \begin{pmatrix} \Re\{S_1\} & -\Im\{S_1\} \end{pmatrix} \quad \text{and}$$
$$S_{1i} = \begin{pmatrix} \Im\{S_1\} & \Re\{S_1\} \end{pmatrix}$$

and then (18) can be expressed as

$$\begin{pmatrix} S_{1r} & -U \\ -S_{1r} & -U \\ S_{1i} & -U \\ -S_{1i} & -U \end{pmatrix} \times \begin{pmatrix} \widehat{c} \\ t \end{pmatrix} \le \begin{pmatrix} -\Re\{S_2 \mathbf{c}_d\} \\ \Re\{S_2 \mathbf{c}_d\} \\ -\Im\{S_2 \mathbf{c}_d\} \\ \Im\{S_2 \mathbf{c}_d\} \end{pmatrix} \qquad (19)$$

Now, (19) is a standard linear programming problem with $(2P+1)$ unknowns subject to $4(N-P)$ constraints [10]. Hence, pilot tones $\mathbf{c}_p$ can be computed.

C. Simulation Results

Fig. 4 shows the CDF of Peak Intercarrier Interference (PICI) per OFDM for $N = 128$, $P = 32$ and $\varepsilon = 0.1$ with BPSK modulation scheme. This illustrates how ICI effects can be reduced by inserting pilot tones in the data frame. There is 3 to 5 dB reduction in PICI for $P = 32$ and this is a sort of rate $\frac{3}{4}$ coding. In fact, ICI can be completely eliminated if half of the subcarriers are assigned for pilot tones, that becomes half-rate coding. Note that in the computation of $\mathbf{c}_p$, P unknowns were solved from $(N-P)$ equations ($P < N$) and exact solution is possible if $P = \frac{N}{2}$. Further, Linear Programming (LP) approach performs better than Least Square Error (LSE) solution method. However, the later is faster computation.

Fig. 5 shows the CDF of PICI per OFDM as a function of ε with LSE solution. It can be seen from Figure 5 that $\mathbf{c}_p$ can be computed by assuming worst case CFO and the optimization will not be affected by this assumption.

V. CONCLUSION

In this paper, we have presented the statistical distribution of the PICR. Our studies show that there exist codes offering reduced ICI independent of frequency offset errors. However, construction of such codes is a challenging task and it is an open
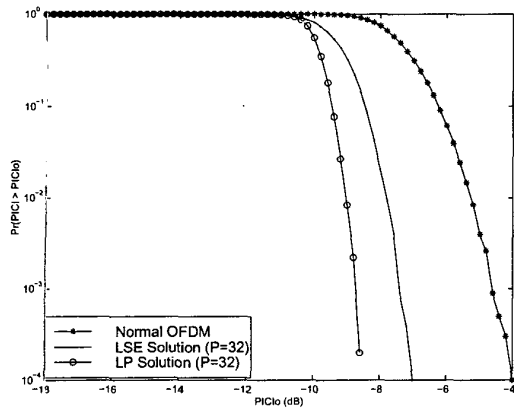
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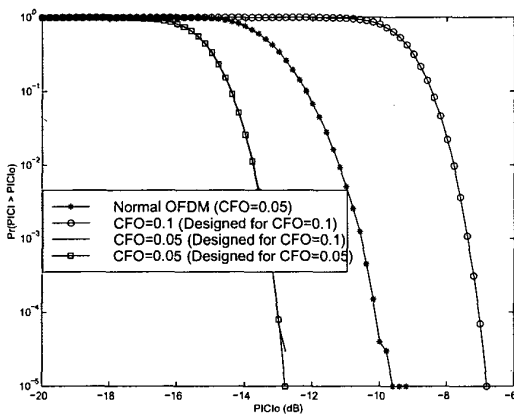Fig. 4. CDF of PICI of an OFDM System for $N = 128$ and $\varepsilon = 0.1$

Fig. 5. CDF of PICI of an OFDM System for $N = 128$ and $\varepsilon = 0.1$

research problem. Alternatively, we have proposed simple $\frac{2}{3}$ and $\frac{3}{4}$ ICI-suppressing codes. The proposed rate $\frac{2}{3}$ code offers 5 to 13 dB CIR gain over normal OFDM system and that of rate code $\frac{3}{4}$ is 2.5 to 13 dB. Using these simple ICI-suppressing codes, we may adaptively vary the data throughput based on ICI effects in the channel. We have also proposed a tone reservation method to reduce ICI. This scheme offers 3 to 5 dB gain in PICI over normal OFDM with 75% of data throughput. The price for this scheme is complex computation at the transmitter, which may not be an issue as faster computational devices are readily available.

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